Hybrid coronagraphic design: optimization of complex apodizers

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ABSTRACT

To spectrally characterize Earth-like planets around nearby stars with a coronagraph, an extreme adaptive optics (ExAO) system is mandatory. The correction of the amplitude and phase aberrations in the wavefront on both sides of the image plane and in sufficiently large bandwidths can be done with two deformable mirrors (DM) in a pupil mapping configuration. While this system is primarily intended to correct for aberrations, it can potentially be used to improve the contrast beyond the nominal value set by the coronagraph; the two DMs can be seen as a complex apodizer. We present solutions to two types of numerical optimization problems. Our first approach consists in maximizing the sum of the real and the imaginary parts of the electric field in the pupil plane, while constraining the intensity of the electric field in chosen regions of the the subsequent image plane to be less than a chosen extremum. The solutions can be translated in term of modulus and phase. The optimal modulus is very close to 1, and the high-contrast is induced by a binary phase shift, which cannot be induced with current deformable mirrors. Our second approach consists in directly optimizing the stroke commands sent to a deformable mirror. Solutions are computed by either solving successive linear optimizations or non-linear optimizations. For a telescope with a 30% central obscuration, a $3\lambda/D$ inner working angle and a $10\lambda/D$ outer working angle, a 10^{-6}-10^{-7} is reached after a dozen iterations, and the coronagraph has a 60-80% throughput. Shaped pupils are then computed to lower that contrast down to 10^{-9}-10^{-10}.

Keywords: Coronagraphy, High-Resolution Imaging, Exoplanet direct detection

1. INTRODUCTION

Many coronagraphic concepts have been proposed over the past 20 years to directly image and characterize exoplanets around nearby stars.

In addition to creating high-contrast with a coronagraph, it is mandatory to correct for wavefront aberrations, and maintain high contrast over a sufficiently long period of time.

To correct the phase and the amplitude errors on both sides of the image plane, at least two deformable mirrors (DM) must be used. These mirrors must be located in non-conjugate planes so that the phase that the first mirror induces can be turned into amplitude through free-space propagation. This two-DM combination is essentially a pupil mapping system.[1] As such it can potentially be used to do more than just correcting for phase and amplitude errors.[2]

In this paper is we present our attempts at optimally designing a complex apodizer that creates high contrast. This complex apodizer would be implemented using a combination of a shaped pupil and one or two DMs. For the scope of this paper, we focus our attention on single DM solutions. The phase and the amplitude can be computed one after the other, or at the same time.

The latter appears to be a better option, and we describe in Sec. 2 the joint optimization of the real and the imaginary parts of a complex apodizer. One of the major results of this type of optimization is that the optimal complex apodizer is actually a phase apodizer, and the optimal phase is a two-dimensional step function. By applying smoothness constraints onto the real and imaginary parts of the apodizer, the optimal phase becomes

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smooth as well, yet it cannot be accurately induced with a DM. To some extent these results are similar to the apodizing phase plates (APP[3,4]), although the phases of the APP are sums of N Zernike polynomials with optimal chosen coefficients.

In Sec. 3 we present alternative methods where the amplitude and the phase of a complex apodizer are computed one after the other. Computing the amplitude first limits the performance of the coronagraph, and performing the computation in the opposite order gives more satisfying results. We present combinations of DM and shaped pupils for clear and obscured apertures. Different optimization methods are described and compared. In particular, we show that only some optimal apodizations can be successfully combined with a shaped pupil.

Sec. 4 discusses these results and draws a conclusion to the paper.

2. OPTIMIZATION OF COMPLEX APODIZATION

Optimizing a shaped pupil is done by maximizing its transmission while constraining the extremum values of the electric field in the image plane.[5–7] This is a linear optimization problem that has a unique solution, and this solution can be computed efficiently.

The same basic principle can be applied to a complex apodizer: its real and imaginary parts can be maximized to ensure the maximization of its transmission, and the same constraints can be applied to the extremum values of the electric field in the image plane.

The main advantage of this method is that, contrary to the optimization of a phase apodizer, it is a linear problem. A unique, global solution can be found, and the real and imaginary parts can be afterwards converted into a phase and an amplitude.

2.1 Mathematical formalism

The complex transmission of the apodizer $A(x, y)$ can be written as the sum of a real and an imaginary part:

$$A(x, y) = A_R(x, y) + iA_I(x, y) \tag{1}$$

The electric field $E(u, v)$ in the image plane is the Fourier transform of $A(x, y)$. Assuming that the aperture has a diameter D, and that the Fourier transform is produced by a lens with a focal length f, it can be written:

$$E(u, v) = \mathcal{F}[A_R(x, y) + iA_I(x, y)] = \frac{e^{2i\pi F/\lambda}}{i\lambda F} \iint_{-D/2}^{D/2} (A_R(x, y) + iA_I(x, y)) e^{-2i\pi(ux+yv)/(\lambda F)}\, dx\, dy \tag{2}$$

The coordinates (u, v) in the image plane are physical lengths. It is customary to make a change of variable so as to work with angular coordinates: $u \to u\lambda F/D$, $v \to v\lambda F/D$. In addition, we change the coordinates (x, y) used in the pupil plane so as to write them in units of pupil diameter: $x \to xD$, $y \to yD$. The expression of $E(u, v)$ can thus be written as follows:

$$E(u, v) = D^2 \frac{e^{2i\pi F/\lambda}}{i\lambda F} \iint_{-1/2}^{1/2} (A_R(x, y) + iA_I(x, y)) e^{-2i\pi(ux+yv)}\, dx\, dy \tag{3}$$

To solve the optimization problem, the real and imaginary parts must be separated:

$$\begin{aligned}(A_R(x, y) + iA_I(x, y)) e^{-2i\pi(ux+yv)} &= A_R(x, y) \cos\big(2\pi(ux + yv)\big) + A_I(x, y) \sin\big(2\pi(ux + yv)\big) \\ &\quad -i\Big(A_R(x, y) \sin\big(2\pi(ux + yv)\big) - A_I(x, y) \cos\big(2\pi(ux + yv)\big)\Big)\end{aligned} \tag{4}$$

The complexity of the computation of $E(u, v)$ can be simplified if the integrand - which is a function of x, y, u, and v - can be written as the product of a function of x, u, and v, and a function of y, u, and v.

$$\begin{aligned}\cos\big(2\pi(ux + vy)\big) &= \cos(2\pi ux)\cos(2\pi vy) - \sin(2\pi ux)\sin(2\pi vy) = C_x C_y - S_x S_y \\ \sin\big(2\pi(ux + vy)\big) &= \cos(2\pi ux)\sin(2\pi vy) + \sin(2\pi ux)\cos(2\pi vy) = C_x S_y + S_x C_y\end{aligned} \tag{5}$$

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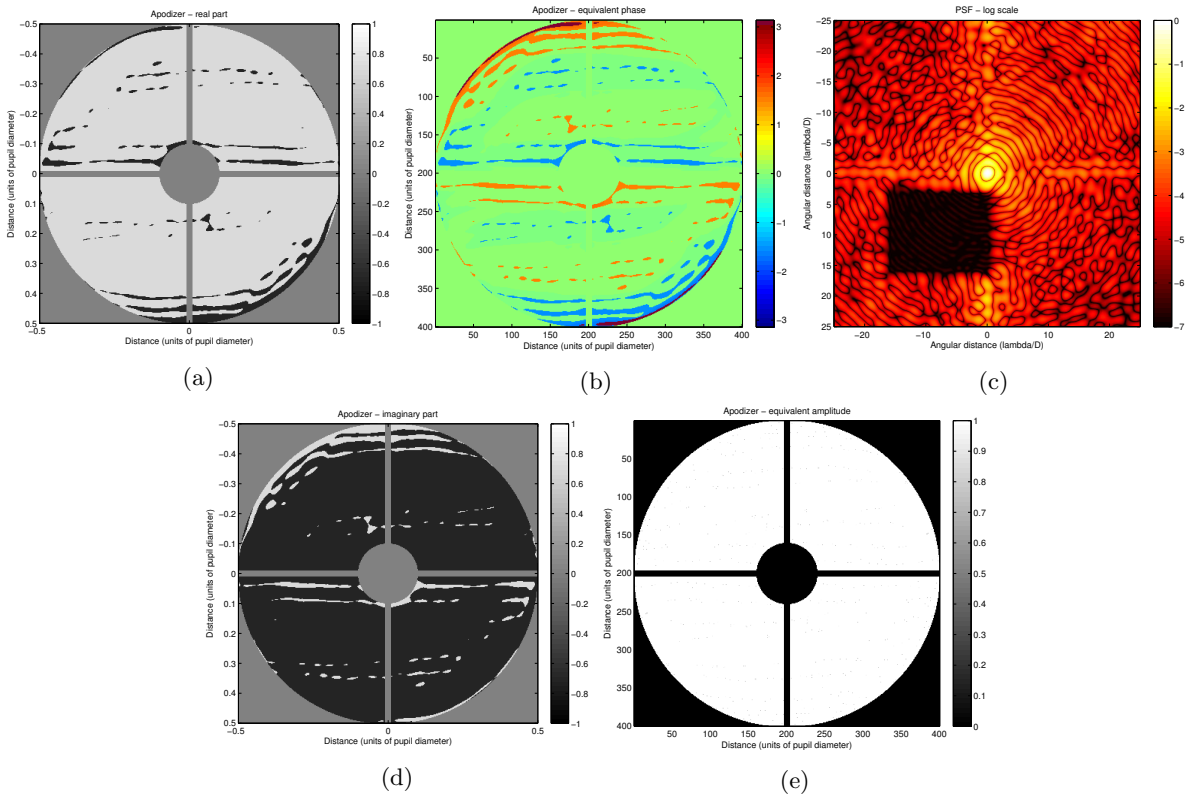

(a) (b) (c)

(d) (e)

Figure 1: (a) & (d): Real and imaginary parts of the apodizer designed to create high-contrast in one quadrant of the image plane. (b) & (e): Equivalent phase (in radians) and amplitude. (c): PSF (log scale)

where we have replaced $\cos(2\pi ux)$ with C_x, $\cos(2\pi vy)$ with C_y, and $\sin(2\pi ux)$ with S_x, $\sin(2\pi vy)$ with S_y. We can write the integrand in a way that allows it to be separated:

$$\begin{aligned}(A_R + iA_I)e^{-2i\pi(ux+yv)} &= A_R\big(C_xC_y - S_xS_y\big) + A_I\big(C_xS_y + S_xC_y\big) - i\Big(A_R\big(C_xS_y + S_xC_y\big) - A_I\big(C_xC_y - S_xS_y\big)\Big) \\ &= C_x\big(A_RC_y + A_IS_y\big) + S_x\big(A_IC_y - A_RS_y\big) - i\Big(C_x\big(A_RS_y - A_IC_y\big) + S_x\big(A_RC_y + A_IS_y\big)\Big)\end{aligned} \tag{6}$$

However, instead of performing four different integrations over y, the problem can be simplified by performing only two integrations. This is made possible by the fact that the same two expressions appear in the expressions of the real and the imaginary parts: $A_RC_y + A_IS_y$, and $A_IC_y - A_RS_y$.

These expressions must be discretized to be computed. This discretization process has been extensively described in previous papers,[7] and we do not reproduce it here.

2.2 Examples of two optimal complex apodizers

We present two different examples of optimal complex apodizers. The first apodizer creates a dark hole in one quadrant of the image plane. The second apodizer creates a two-sided dark hole.

The aperture for which the apodizer is computed has a 20% central obscuration, and 2% thick, orthogonal spiders.

2.2.1 One-sided dark hole

Fig. 1 shows the real and imaginary parts of an apodizer that creates 10^{-7} contrast in one quadrant of the image plane, from 3 to $16\lambda/D$ along the y axis, and from 0 to $16\lambda/D$ along the x-axis.

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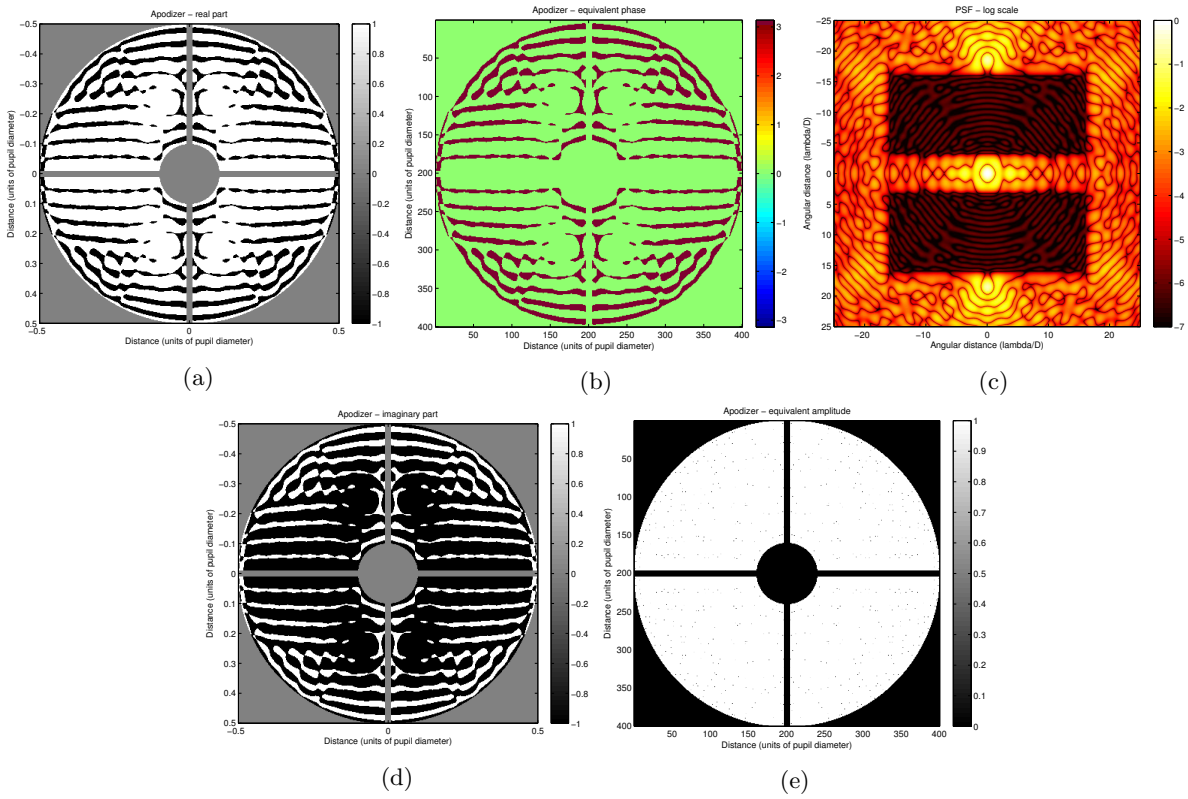

Figure 2: (a) & (d): Real and imaginary parts of the apodizer designed to create high contrast on both sides of the image plane. (b) & (e): Equivalent phase (in radians) and amplitude. (c): PSF (log scale)

It can be noticed that the imaginary part is a symmetric mapping of the real part: $A_R(x, y) = -A_I(-x, -y)$. This property can be used to simplify the computation of the point-spread function (PSF).

Fig. 1 also shows the equivalent phase and amplitude. The amplitude is almost 100% everywhere in the aperture. Replacing this amplitude with the original aperture does not change the contrast. Only the phase creates high-contrast, and instead of a complex apodizer, the result of these optimizations is a phase apodizer.

The phase is a 2D step function. The phase is either zero, $\pm\pi/2$, or close to π. Like the real and amplitude parts, an antisymmetry can be noted in the phase $\phi(x, y)$: $\phi(x, y) = -\phi(-x, -y)$.

While the apodizer only shifts the light that goes through it, the throughput of the apodizer is not 100%. The energy ratio between the inner part of the coronagraphic PSF (within $3\lambda/D$ of the star) and the original PSF equals 81.5%. The remaining light is diffracted in the outer region of the PSF.

2.2.2 Two-sided dark hole

Fig. 2 shows the real and imaginary parts of an apodizer that creates 5×10^{-7} contrast in two symmetric dark holes, between ± 3 and $\pm 16\lambda/D$ along the y-axis, and between -16 and $16\lambda/D$ along the x-axis.

The real and imaginary parts are the opposite of each other: $A_R(x, y) = -A_I(x, y)$. In both cases the x- and y-axes are axes of symmetry.

Fig. 2 also shows the equivalent phase and amplitude. The amplitude is here also almost 100% everywhere in the aperture, and replacing this amplitude with the original aperture does not change the contrast.

The phase is binary: it is either zero or π. This property can be used to greatly simplify the problem. In fact, it can be shown that the exact same solution can be obtained by optimizing a 2D shaped pupil whose amplitude is between -1 and 1, instead of being between 0 and 1.

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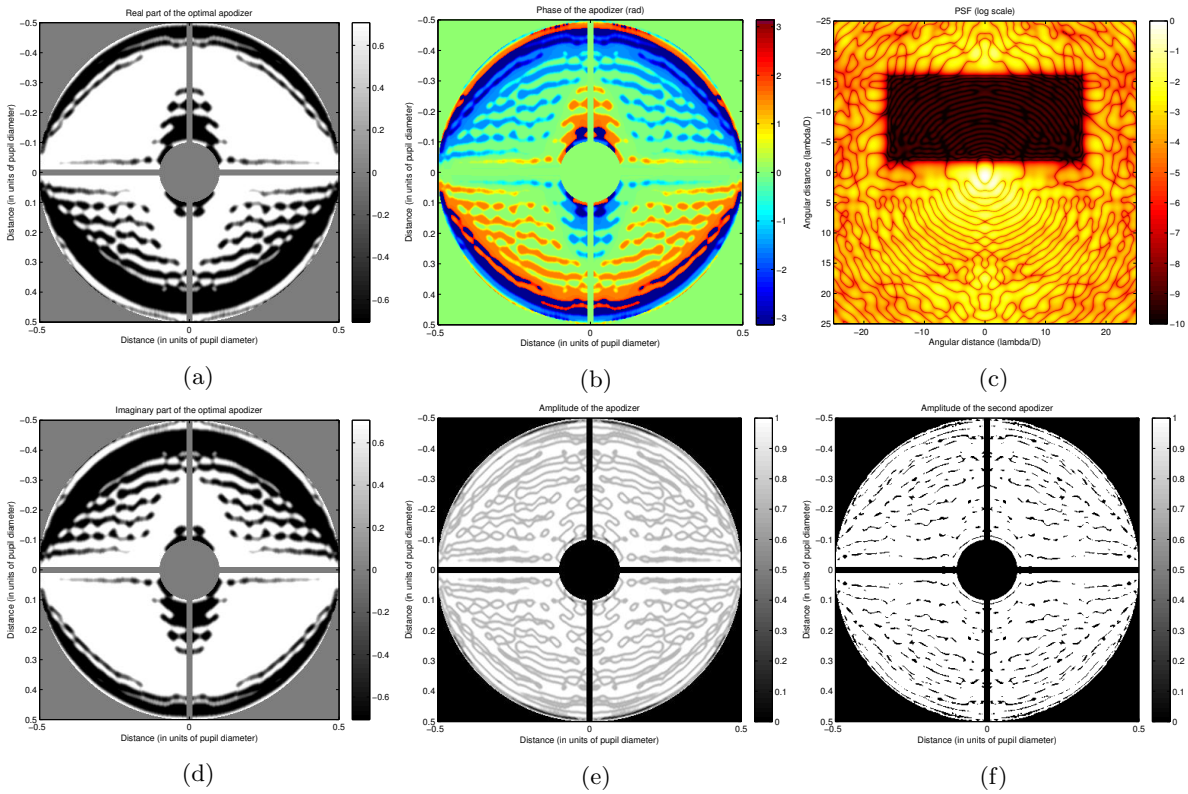

Figure 3: (a) & (d): Real and imaginary parts of the apodizer, designed to create high-contrast on side of the image plane, and with a smooth phase. (b) & (e): Equivalent phase (in radians) and amplitude. (c): PSF (log scale)

In this case the throughput of the coronagraph (as defined previously) equals 54.1%. Looking at the PSF displayed in Fig. 2, it can be seen that two relatively bright features are located next to the dark hole, along the y-axis. Both are 25% as bright as the core of the PSF.

2.3 Smoothness constraint

The two complex apodizers already presented are, in fact, phase apodizers. The phases shifts are not smooth, and inducing this phase with a continuous face sheet DM does not appear possible. Non-continuous face sheet DM could potentially induce these phases, but the number of actuators required to do so is much higher that what current DMs offer.

In order to try and make this possible, we set a smoothness constraint on the real and the imaginary parts of the apodizer. The smoothness is chosen to match the minimum radius of curvature achievable with a 32x32 actuators DM such as the kilo-DM produced by Boston Micromachines.

Contrary to the previous case it is not possible to create high contrast on both sides of the image plane.

The apodizer is designed to create a 5×10^{-10} contrast at only $2\lambda/D$ from the star. Because it uses phase, this apodizer can actually move the core of the PSF, and forcing such a small IWA is not possible. As a consequence the PSF core moves $0.7\lambda/D$ away from the high-contrast region, and the effective IWA is close to $2.7\lambda/D$.

The result of this optimization is displayed in Fig. 3. As expected the smoothness of the real and the imaginary parts causes the phase of the apodizer to be smooth as well. The apodizer is not fully transmissive, however. The transmission of the apodizer is 78.9%, and the throughput of the coronagraph is 38.7% for a mean contrast of 1.6×10^{-7}.

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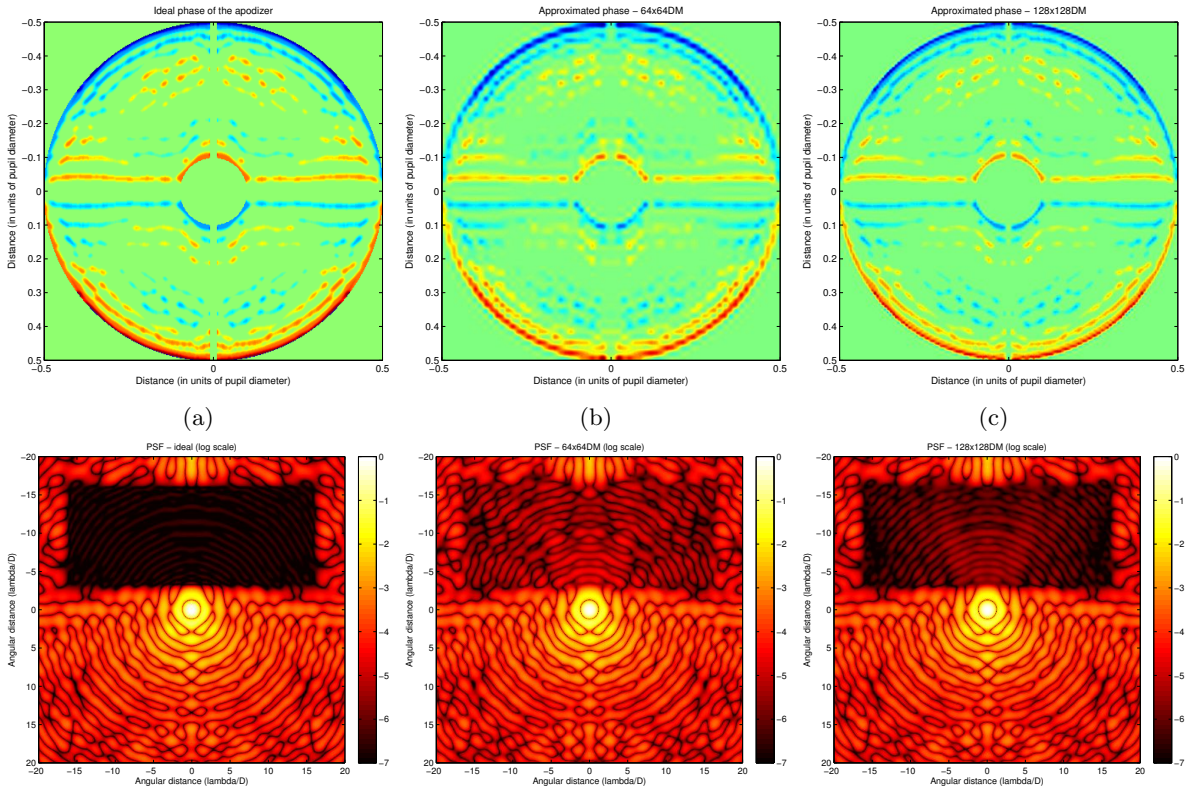

Figure 4: (a) & (d): Optimal phase component of a complex apodizer creating 10^{-7} contrast (the amplitude of the apodizer is not shown). (b) & (e): closest phase achievable with a 64x64 DM, and corresponding PSF. (c) & (f): closest phase achievable with a 128x128 DM, and corresponding PSF.

The amplitude of this complex apodizer is smooth too, but a binary equivalent - a shaped pupil - can be computed. Fig. 3f shows a shaped pupil that, combined with the previous phase, creates the same high-contrast region. Its transmission is 93%, and the throughput of the coronagraph is 47.9%.

One may wonder how the shaped pupil computed afterwards increases the throughput of the coronagraph. The reason is that the optimization of the real and the imaginary parts of the complex apodizer must assume that their absolute values is less than $1/\sqrt{2}$ so that their modulus is less than 1. This bound is applied separately on the real and the imaginary parts, which artificially limits the maximum throughput of the apodizer. When the shaped pupil is optimized, the conservation of the energy still applies, but since it is an amplitude apodizer and not a complex apodizer the amplitude itself is bounded.

While the phase of this complex apodizer is smooth, attempts at computing DM stroke commands to create high contrast have not been much successful. The approximated phases and the associated PSFs are showed in Fig. 4 for a 64-by-64 DM and a 128-by-128 DM. We assumed gaussian influence functions when computing the phase induced with the DMs.

The dark hole barely appears with the 64-by-64 DM, and 128-by-128 is not enough to recover the same high contrast. To truly design a complex apodizer whose phase can be induced with a DM, the optimization of its phase must be done directly.

3. HYBRID CORONAGRAPHIC DESIGN

We refer to a hybrid coronagraph as one or several coronagraphic components (apodizer, focal plane mask, Lyot stop, etc.) used with a chosen set of deformable mirrors actively used to create high contrast. The DM is not

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intended to be used to only control the wavefront, but to actively improve the contrast beyond the nominal value set by the coronagraph.

As previously suggested,[2] we first tried - without much success - to optimize a phase apodizer that increases the contrast created with a previously designed shaped pupil. We write the phase as a sum of phases induced by the individual actuators of a deformable mirror. The influence functions are assumed to be gaussians with a 0.019 standard deviation σ.

The results we present in this section indicate that, on the contrary, optimizing the DM settings first, and a shaped pupil second is a better course of action. Making the DM (or a combination of two DMs) the primary coronagraphic component may seem counter-intuitive, but it can be seen as a new type of pupil mapping coronagraph.

3.1 Using a deformable mirror to increase a shaped pupil efficiency

Even if shaped pupils can be designed to create high contrast with arbitrary apertures, it is not possible to create arbitrarily low contrast while maintaining the throughput of the coronagraph arbitrarily high.

A shaped pupil can be designed to create the highest contrast possible with the original aperture of the telescope, say 10^{-7}, given a chosen IWA and minimum throughput, say 2.5-3λ/D and 20%. A deformable mirror can then potentially be used to further increase the contrast.

In practice optimal DM settings fail to achieve significantly lower contrast, however. It seems only possible to increase contrast by one order at most. For instance, starting from a 10^{-7} contrast, a 10^{-8} contrast can be obtained with a 32 by 32 DM.

Our understanding of this issue is that the regions where the phase should preferentially be shifted in order to increase the contrast are already masked by the shaped pupil. Fig. 5 illustrates this conflict by showing a shaped pupil and a phase apodizer both designed to create the same high contrast region. The phase of this apodizer is in this case directly optimized, and built so as to be induced with a 32 by 32 DM. The strongest phase shifts are located on the left and right sides of the pupil, and the same regions are mostly blocked by the shaped pupil.

3.2 Computing optimal DM settings to create high contrast

By blocking light the shaped pupil prevents the phase apodizer from efficiently increasing the contrast, but proceeding the other way around can solve the issue.

Optimizing the DM settings first and the shaped pupil second does not make the DM, or the combination of two DMs, a PIAA[8] (phase-induced amplitude apodization) which is designed to create spheroidal prolate or prolate-like apodizations. The optimal settings that we compute do make the DM a pupil mapper, however. The two main differences between our pupil mapping concept and the PIAA are that (a) we compute the DM shape to directly create high-contrast in the image plane (it is not a Lyot coronagraph), and (b) there is no conservation of the optical path here.

We write $A(x,y) = |A(x,y)|e^{i\phi(x,y)}$, and the direct optimization of the phase is obviously a non-linear problem to which it may be difficult to find a solution. A linear approximation can be made for small phases, however: $e^{i\phi(x,y)} = 1 + i\phi(x,y) + O(\phi(x,y)^2)$ if $\phi(x,y) << 1$.

Thanks to this approximation, we can consider a phase apodizer as a complex apodizer with a constant real part that equals 1, and a variable imaginary part that equals $\phi(x,y)$.

This approximation allows us to try and improve the contrast, within the limit set by the assumption made when making the approximation. In other words, one must progress iteratively towards a solution, so that at each iteration the phase increment is small enough to keep the linear approximation valid.

The algorithm we have used to iteratively optimize the phase is described in the following steps:

1. start with a prior phase (no linear approximation is made here, and it is set at 0 in the beginning).
2. compute a small phase increment by solving a linear optimization problem.

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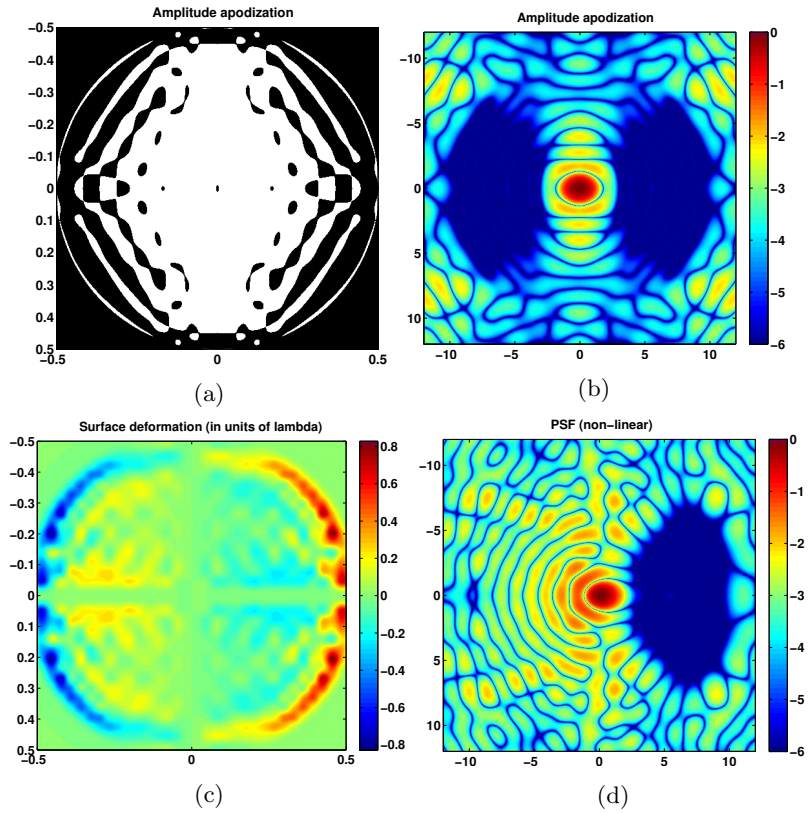

Figure 5: Comparison of (a) a shaped pupil and (c) an optimal phase apodizer that create similar high-contrast regions (the top PSF (b) is created by the shaped pupil and the other (d) by the phase apodizer. The phase deformation is measured in units of wavelength. Distances in the pupil plane are in units of pupil diameter. In the image plane, the angular distances are in units of λ/D. The PSF are showed in log scale.

3. check whether the result is satisfying.
   - if so: make the new prior phase the sum of the current prior phase and the phase increment, then iterate.
   - if not: relax the objective of the optimization, and restart with the unchanged prior phase

The optimization itself is a stroke minimization: $a_{i,j}$ is the stroke of an actuator (i, j) of an N-by-N DM, and we minimize $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{i,j}^2$. The constraint is set on the contrast of the electric field in the image plane. The principle of stroke minimization is often used in extreme adaptive optics,[1] although it is implemented in a different way to control wavefront.

The result of the optimization is checked by making sure that (a) the extremum values of the stroke are within a chosen range, or (b) the contrast measured after computing a new PSF has improved over the previous iteration.

The phase apodizer showed first in Fig. 5 is the result of a stroke minimization where the results are checked by looking at the extremum values found for the stroke. The evolution of the contrast at each iteration is presented in Fig. 6, along with the evolution of the extremum values of the stroke. The contrast decrease is not steady, but a 4×10^{-7} contrast is reached after 11 iterations. More iterations do not see an improvement, and the contrast oscillates around 5×10^{-7}. The throughput of this phase apodizer is 74%. The PSF photocenter is not moved substantially in this case.

Another example is presented in Fig. 7. In this case the aperture has a 30% central obscuration, and 2% thick orthogonal spiders. The throughput of the phase apodized coronagraph is 57%. A contrast of 1.3×10^{-7} is

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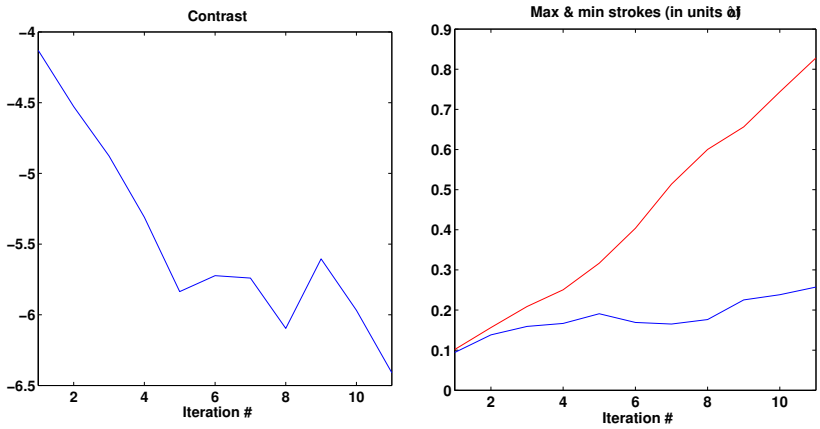

(a) (b)

Figure 6: Result of a stroke minimization where the result of each iteration is assessed by looking at the extremum values of the stroke. The optimal phase is found after 11 iterations, and it is showed with its associated PSF in Fig. 5. The DM has 32 by 32 actuators. (a): evolution of the contrast (log scale) over the first 11 iterations. (b): evolution of the extremum values of the stroke in the right half of the pupil plane (the red curve represents the positive values, and the blue curve represents the negative values). The stroke is measured in units of wavelength.

found after 17 iterations. The size and shape of the dark hole are identical to the previous case. The maximum stroke is substantially larger, and it equals 1.3λ (instead of 0.83).

With the same aperture but when checking the contrast by looking at the effective PSF, the contrast decrease is much more monotonic, as illustrated is Fig. 8. The contrast stalls at 2.5×10^{-7} after approximately 13 successful iterations. The throughput of this apodizer is close to 81%, which is significantly larger than what was obtained in the previous case. Moreover the maximum stroke is twice as low.

We attempted at reducing the initial contrast increment in order to obtain better contrast, but we did not notice a significant improvement when doing so. For instance proceeding with 50 iterations instead of 13 (with contrast increments about 5 times smaller at each iteration) does not lead to a different minimum contrast.

The two previous optimizations problems are linear. We have also tackled the original non-linear phase optimization problem. In this case we do not proceed to a stroke minimization, but to a contrast minimization. Given a chosen stroke range - $[-\lambda/2, \lambda/2]$ in this case - we constrain the normalized intensity of the electric field in the image plane to be less than a variable c that we minimize.

The throughput of this solution is close to 80%. The maximum deformation is slightly smaller, however, and it is just above 0.5λ. It is not surprising that the maximum surface deformation is higher than the bounds of the stroke range: the DM surface which is plotted in Fig. 9b is the sum of the individual strokes of each actuator (plotted in Fig. 9a), while the stroke range applies separately on each actuator.

The contrast which is found at the end on the non-linear optimization of the phase is $c = 2 \times 10^{-7}$, which is slightly better than the result of the series of linear optimizations performed for the same dark hole and the same aperture.

When comparing the two mirror deformations, it appears that while both display large strokes on the left and right sides of the pupil, as well as next to the left and right sides of the central obscuration, the phase patterns are different.

3.3 Computing shaped pupils to increase contrast

Once optimal settings have been found for a DM, one can try and decrease contrast from 10^{-6}-10^{-7} down to 10^{-9}-10^{-10} with an apodizer. As with previously shaped pupils and complex apodizers, the optimization is a transmission maximization under constraints set on the extremum values of the intensity in chosen regions of the image plane. The main originality of this problem is the presence of the optimal phase.

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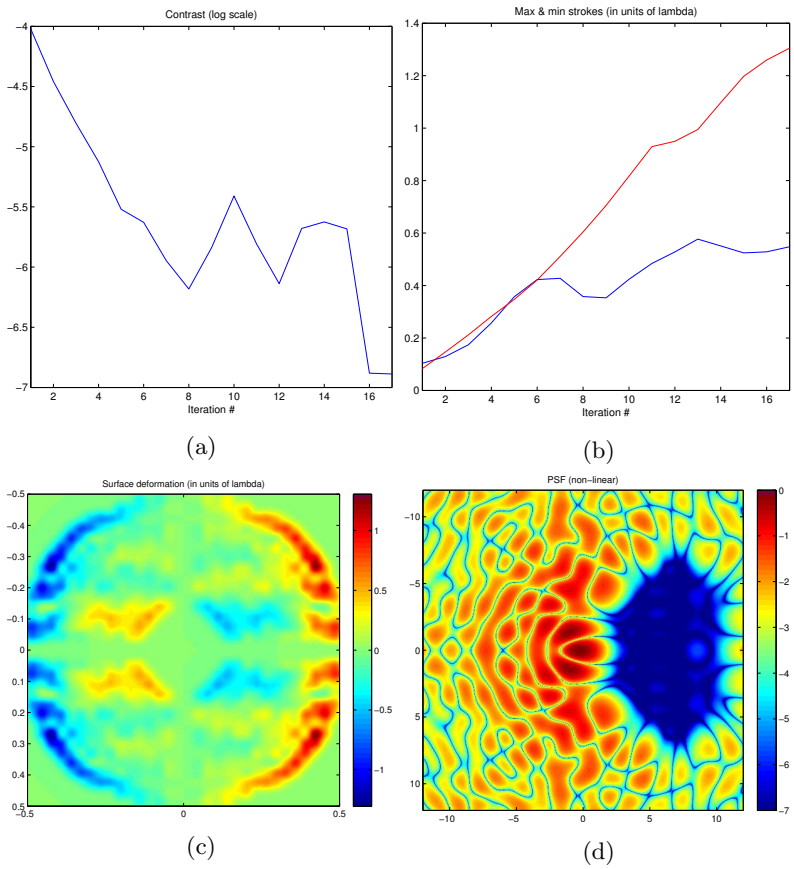

Figure 7: Result of a stroke minimization where the result of each iteration is assessed by looking at the extremum values of the stroke. The optimal phase is found after 17 iterations. (a): evolution of the contrast (log scale) over the first 17 iterations. (b): evolution of the extremum values of the stroke in the right half of the pupil plane (the red curve represents the positive values, and the blue curve represents the negative values). The stroke is measured in units of wavelength. (c): phase induced by the 32-by-32 DM (also in units of wavelength). (d): PSF (log scale).

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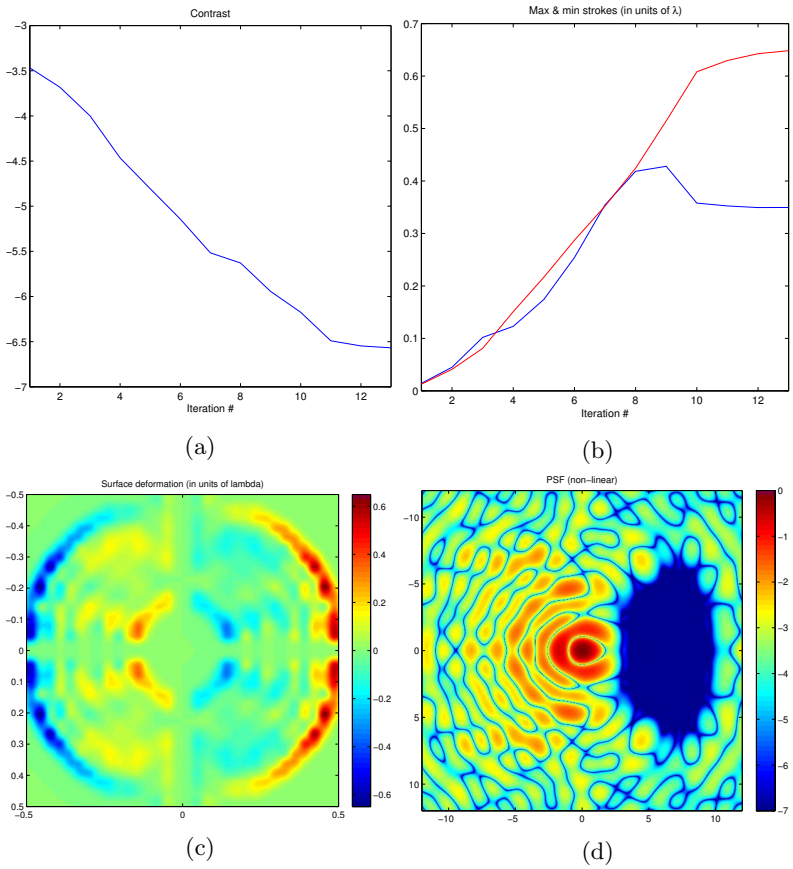
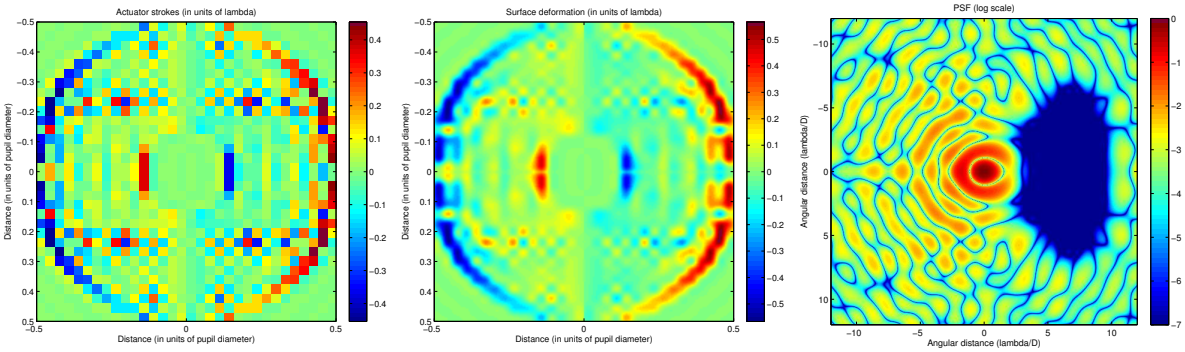


(a) (b) (c) (d)

Figure 8: Result of a stroke minimization where the result of each iteration is assessed by measuring the contrast in the dark hole. (a) The contrast decreases steadily and stalls after 13 iterations. (b) The maximum negative value of the stroke (blue line) increases during these first 13 iterations, while the maximum positive value of the stroke (red line) increases over the first 8-9 iterations then decreases and stalls after 4 more iterations. (c) The phase induced by the 32-by-32 DM in units of wavelength. (d) The point-spread function (in log scale), where the dark hole clearly appears.



(a) (b) (c)

Figure 9: Result of the non-linear contrast minimization.

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Note that we have already developed the formalism used to optimize shaped pupils together with a phase pattern. In a previous paper[9] we suggested that shaped pupils could be designed to create high contrast in spite of phase aberrations induced by surface errors. We presented a shaped pupil creating a one-sided, $3\lambda/D$ IWA, 10^{-5} high-contrast region with the aperture of the James Webb space telescope and simulated 136nm RMS phase aberrations.

The problem we propose to solve here is different, however: contrary to surface errors, the spatial frequency properties of the optimal phase may perturb the optimization of an amplitude apodizer. In other words, the issue described in Sec. 3.1 can still make it difficult to increase contrast.

We present in Fig. 10 a few shaped pupils designed to work with the already presented phase apodizers. The most obvious observation that can be made is that the transmissions of these shaped pupils are very different.

The first shaped pupil (see Fig. 10a) is designed to further decrease the contrast created by the phase apodizer showed in Fig. 7c (stroke minimization, check performed on the extremum stroke). Its transmission is 98%, and the throughput of the apodized coronagraph becomes 43% when the shaped pupil is added to the phase apodizer (instead of 57% without the shaped pupil). A mean contrast of 1×10^{-9} is created in the dark hole. Thus, the contrast is improved by a factor 100 at the price of a throughput loss of 14%.

The second shaped pupil (see Fig. 10c) is optimized in the same way but for the phase apodizer showed in Fig. 8c (stroke minimization, check performed on the effective contrast). Its transmission is only 20.5%, and the coronagraph throughput is 4.9% (instead of 81%). The mean contrast is 7.5×10^{-9}, which is 33 times better than without the shaped pupil.

This large throughput loss is puzzling. It is currently difficult to understand why highly transmissive shaped pupils are computed in one case, and very poorly transmissive shaped pupils are computed in the other. The only difference between the two appears to be associated to the way the solutions are kept or discarded at each iteration of the algorithm described in the previous section.

Since the optimization problem is non-linear, multiple solutions (phase apodizations) may be found through the linear approximation that we have used, and it appears that only some of these solutions can be efficiently combined with shaped pupils.

In addition, the optimal phase apodizations computed by directly solving the non-linear optimization problem (see Fig. 9b) are also poorly compatible with shaped pupils.

4. DISCUSSION AND CONCLUSION

We have presented in this paper different approaches to the optimal design of hybrid apodizers, i.e., combinations of apodizers and deformable mirror(s) that create high-contrast by using amplitude and phase apodization. For simplicity reasons we have chosen to only use one deformable mirror. The mirror is not used to only correct the wavefront: it actively increases the contrast. A two-mirrors solution would effectively be a pupil mapper. This would have several advantages such as partial achromaticity, two-sided high-contrast regions, and potentially higher throughputs.

We first showed that the linear optimization of complex apodizers produces phase apodizations. Ideally an achromatic phase shifter would be required to induce these phases, but again we aim at using DMs to induce them. The optimal phases that are found are 2D step functions which cannot be induced with a deformable mirror, however. In the hope of making it possible we forced a smoothness constraint on the real and the imaginary parts of the complex apodizer so as to smooth its equivalent phase. This constraint makes the amplitude of the apodizer smooth too, although a shaped pupil can be designed to replace it. More importantly, currently available deformable mirrors fail to accurately induce these smooth phases, and this severely impacts the performance of the coronagraph.

As an alternative solution, we directly optimized the commands sent to the individual actuators of a 32-by-32 deformable mirror. Because the optimization of the phase is non-linear we first looked for solutions to this problem by making a linear approximation and increasing iteratively the contrast by small steps. We later tackled the non-linear problem, for which we successfully found solutions similar yet slightly different from the previous ones.

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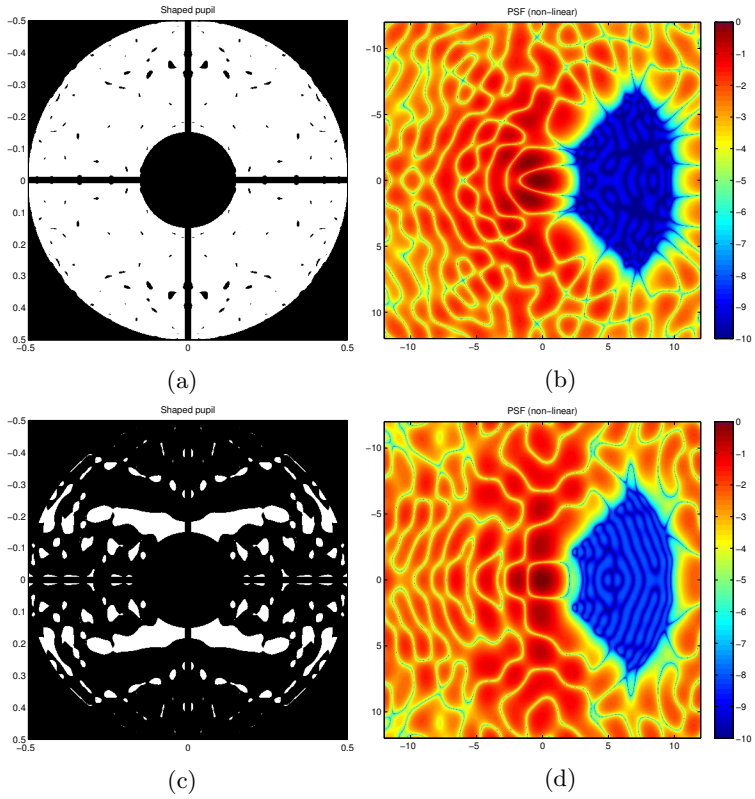

Figure 10: Optimal shaped pupils designed to decrease contrast in the dark hole created with the phase apodizers showed in Fig. 7c and Fig. 8c. Top, left: shaped pupil computed to assist the phase apodization obtained through stroke minimization with extremum stroke check. Right: associated PSF (log scale). Bottom: shaped pupil computed to assist the phase apodization obtained through stroke minimization with contrast check. Right: associated PSF (log scale).

To optimize the phase we have minimized either the total square of the stroke commands sent to the DM (while constraining the intensity in the image plane to be less than a chosen value), or the maximum intensity inside the dark hole (while constraining the extremum values of the stroke commands).

For the purpose of our first optimizations we chose to set the IWA to $3\lambda/D$ and the OWA to $10\lambda/D$. The optimal deformable mirror settings that have been found can create 10^{-6}-10^{-7} contrast. We have not yet characterized the influence that the number of actuators may have on the minimum contrast or the smallest IWA that can be obtained. The OWA is also likely to be set by the number of actuators, as it is the case with wavefront control. The solutions found for the same apertures but with different methods show significant differences of throughput (from 57% to 81%) and extremum strokes (1.3 and 0.5λ).

We have computed shaped pupils to further decrease that contrast to the level that is required for the direct detection and characterization of Earth-like planets around nearby stars, namely 10^{-9}-10^{-10}. We have found that in some cases the optimal shaped pupil has a very high transmission, which moderately decreases the throughput of the coronagraph. However, in other cases the optimal shaped pupil has a very low transmission. Non-linear problems usually have multiple solutions with different properties, and it is not surprising that depending on the method used to solve the non-linear phase optimization, different types of optimal phases are found.

The phase and the amplitude are currently optimized one after the other, which makes possible conflicts between the two approaches. Optimizing a shaped pupil first and a phase apodization second proves to be more inefficient: the regions that should be phase shifted to increase contrast are already blocked by the shaped pupil. An elegant solution to that problem could be the joint optimization of a phase and amplitude apodizer. This will be the subject of future research. So will be the optimization of the settings of two deformable mirrors located in non-conjugate planes.

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Finally, the DM settings that we have presented have been optimized for one wavelength. While the impact of broadband operation on the contrast has not yet been analyzed, we anticipate that optimizations for several wavelength will be necessary. This may not be difficult with only one DM, but two-DMs solutions could make it possible.

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