Spin-Polarization of Composite Fermions and Particle-Hole Symmetry Breaking

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We study the critical spin-polarization energy ($\alpha_{\rm C}$) above which fractional quantum Hall states in two-dimensional electron systems confined to symmetric GaAs quantum wells become fully spin-polarized. We find a significant decrease of $\alpha_{\rm C}$ as we increase the well-width. In systems with comparable electron layer thickness, $\alpha_{\rm C}$ for fractional states near Landau level filling $\nu = 3/2$ is about twice larger than those near $\nu = 1/2$, suggesting a broken particle-hole symmetry. Theoretical calculations, which incorporate Landau level mixing through an effective three-body interaction, and finite layer thickness, capture certain qualitative features of the experimental results.

A hallmark of an interacting two-dimensional electron system (2DES) subjected to a strong perpendicular magnetic field (B) are the fractional quantum Hall states (FQHSs) which are observed predominantly at odd-denominator Landau level (LL) fillings ν [1, 2]. They stem from the strong Coulomb interaction energy between electrons, $V_{\rm C} = e^2/4\pi\epsilon l_B$, and signal the formation of incompressible electron liquid states with strong short-range correlations (ϵ is the dielectric constant and $l_B = \sqrt{\hbar/eB}$ is the magnetic length). The composite Fermion (CF) theory, in which two magnetic flux quanta are attached to each electron, maps the interacting electrons to a system of nearly non-interacting CFs [2–4], and explains many properties of FQHSs. For example, CFs have their own discrete, magnetic-field-induced, orbital energy levels, the so-called Λ-levels, whose filling factor is denoted by $\nu^{\rm CF}$. The integer quantum Hall states of CFs at $\nu^{\rm CF}$ manifest as FQHSs of electrons at filling $\nu = \nu^{\rm CF}/(2\nu^{\rm CF}+1)$ and $2-\nu^{\rm CF}/(2\nu^{\rm CF}+1)$.

The CFs also have a spin degree of freedom. In 2DESs confined to GaAs, because of the small Landé g-factor ($g^* = -0.44$), the Zeeman energy ($E_{\rm Z}$) is in fact comparable to the CF Λ-level separation, which is equal to a small fraction of $V_{\rm C}$. Thus, CFs might be partially spin-polarized when $E_{\rm Z} << V_{\rm C}$, and become fully spin-polarized only if $E_{\rm Z}/V_{\rm C}$ is larger than a ν-dependent critical value $\alpha_{\rm C} \simeq 0.02$ which should be intrinsic to the 2DES and independent of the sample quality [5, 6]. Moreover, in an *ideal* 2DES with zero layer-thickness and no LL mixing, the FQHSs that have the same $\nu^{\rm CF}$, e.g. $\nu = 2/3$ and $4/3$ ($\nu^{\rm CF} = -2$), are expected to have the same $\alpha_{\rm C}$ because of particle-hole symmetry $\nu \leftrightarrow 2-\nu$ [2]. Experimentally, the spin-polarization of CFs has been probed through measurements of transitions of FQHSs in both transport and optical studies [7–15]. In these studies, $E_{\rm Z}/V_{\rm C}$ in increased by either increasing the 2DES density or adding a parallel magnetic field. However, no systematic measurements of $\alpha_{\rm C}$ on samples with controlled parameters, such as layer thickness, are scarce [16].

Here we report measurements of $\alpha_{\rm C}$ for 2DESs confined to GaAs quantum wells (QWs) and with carefully controlled, *symmetric* charge distributions [17]. Moreover, in our experiments, to avoid charge distribution distortions that occur when a strong parallel magnetic field is introduced, we apply only perpendicular magnetic fields, and tune the ratio $E_{\rm Z}/V_{\rm C}$ at a fixed ν via changing the 2DES density. Our systematic measurements reveal that $\alpha_{\rm C}$ strongly depends on the charge distribution thickness. The thicker the 2DES is, the smaller $\alpha_{\rm C}$. We also find that, for a given electron layer-thickness, normalized to l_B, $\alpha_{\rm C}$ is much larger for the FQHSs around $\nu = 3/2$ compared to their particle-hole counterparts around $\nu = 1/2$, implying that particle-hole symmetry is broken. We present results of state-of-the-art microscopic calculations based on CF wavefunctions, The calculations include LL mixing through an effective three-body interaction, which affects fractions ν and $2-\nu$ differently and breaks the particle-hole symmetry. We find that, while the effect of LL mixing is typically small on the energy of any given FQHS, it has a substantial effect on the rather small energy *differences* that determine the $\alpha_{\rm C}$.

We made measurements on 2DESs confined to 31- to 65-nm-wide GaAs QWs flanked by undoped AlGaAs spacer layers and Si δ-doped layers, which were grown by molecular beam epitaxy. The 2DESs have densities (n) ranging from 3.5 to 0.34, in units of $\times 10^{11}$ cm^{-2} which we use throughout this report, and very high mobilities, $\mu \simeq 1{,}000$ to 250 m^2/Vs. Each sample is a 4×4 mm^2 cleaved piece, with alloyed InSn contacts at its four corners. We fit the samples with an In back-gate and Ti/Au front-gate. By carefully applying the front- and back-gate voltages, we can change n while keeping the QW symmetric. The measurements were carried out at temperature $T \simeq 30$ mK, and using low-frequency (< 40 Hz) lock-in techniques. We injected a very low measurement current ~ 10 nA to avoid polarizing the nuclear spins which might introduce an effective nuclear field to our 2DES thus affecting the electron Zeeman splitting [11–14]. We also checked the up and down magnetic field sweeps to ensure the absence of any noticeable hysteresis.

In Fig. 1, we present longitudinal magnetoresistance (R_{xx}) traces for a symmetric 65-nm-QW at different den-

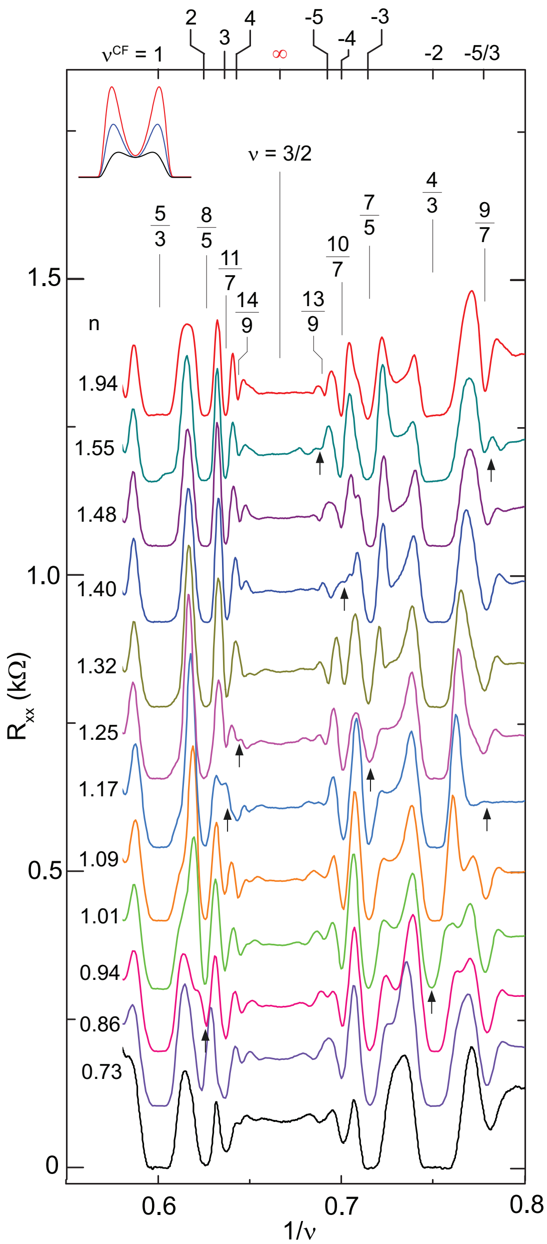

FIG. 1. (color online) R_{xx} traces for 2D electrons confined to a 65-nm-wide, GaAs QW near $\nu = 3/2$. The density for each trace is indicated (in units of 10^{11} cm^{-2}), and traces are shifted vertically for clarity. Top inset shows the ($B = 0$) calculated charge distributions at $n = 0.73$, 1.40 and 1.94×10^{11} cm^{-2}. Spin transitions of FQHSs, seen as a weakening or disappearing of R_{xx} minima, are marked with arrows.

sities ranging from $n = 0.73$ to 1.94. In our GaAs samples, $E_Z \simeq 0.3B$[T] K while $V_C \simeq 50\sqrt{B[\mathrm{T}]}$ K. Therefore, the ratio E_Z/V_C increases as $\sqrt{B}$ or $\sqrt{n}$ at a fixed ν. At a critical density n_C, when $E_Z/V_C = \alpha_C$, a FQHS at a given ν makes a transition from partial to full spin-polarization, signaled by a weakening or disappearance of the R_{xx} minimum and its reappearance at larger n. We mark such transitions of FQHSs in Fig. 1 with arrows. The $\nu = 4/3$ FQHS ($\nu^{CF} = -2$), e.g., is strong at the lowest densities but becomes weak at $n = 1.01$ and strong again at higher n. We then identify 1.01 as n_C for the $\nu = 4/3$ FQHS ($\nu^{CF} = -2$). At somewhat higher $n = 1.25$ the $\nu = 7/5$ FQHS ($\nu^{CF} = -3$) exhibits a similar transition, followed by one for the $\nu = 10/7$ state ($\nu^{CF} = -4$) at yet a higher $n = 1.40$. On the other side of $\nu = 3/2$, we observe similar transitions for the FQHSs at $\nu = 8/5$, 11/7, and 14/9 ($\nu^{CF} = 2, 3, 4$) at $n = 0.94$, 1.17, and 1.25, respectively.

In Fig. 2 we summarize the measured n_C in the 65-nm-QW for each FQHS as it becomes fully spin-polarized (closed square symbols). The black dotted line represents the phase boundary above which all the FQHSs in this sample are fully spin-polarized. It is clear in Fig. 2 that n_C for this QW increases as $|1/\nu^{CF}|$ decreases, resulting a "tent"-like shape for the phase boundary, with a maximum at $\nu = 3/2$ ($\nu^{CF} = \infty$). This behavior has been observed previously for the spin- [9, 10] or valley-polarization [18] of the FQHSs, and is also predicted theoretically [5]. Below this boundary, FQHSs can show several transitions as the CFs become progressively more polarized with increasing E_Z/E_C [9]. An example of such multiple transitions is seen in Fig. 1, where we identify two transitions for the $\nu = 9/7$ FQHS ($\nu^{CF} = 5/3$) at densities $n = 1.17$ and 1.55, the latter corresponding to n_C for the transition to a fully spin-polarized state. Note in Fig. 2 plot that this n_C is higher than n_C for the $\nu = 4/3$ FQHS, suggesting that a second "tent" develops around the even-denominator filling $\nu = 5/4$. The FQHSs seen at fractional ν^{CF} correspond to higher-order CFs, and we will discuss their spin transitions elsewhere.

We performed similar experiments on the 31- and 42-nm-QWs, and summarize the measured n_C in Fig. 2. Clearly, n_C strongly depends on the QW well-width. For example, the $\nu = 7/5$ state becomes fully spin-polarized when $n \gtrsim 1.25$ in the 65-nm-QW, while it remains partially polarized until n reaches $\simeq 2.05$ in the 42-nm-QW or $\simeq 3.19$ in the 31-nm-QW. For the 42-nm-QW, we measured n_C in two different wafers with very different as-grown densities, 1.8 and 2.9, whose densities can be tuned from 1.4 to 2.0 and 2.0 to 3.0, respectively. The fact that both samples have very similar n_C at $\nu = 11/7$ confirms that, for a symmetric GaAs QW with a given well-width, n_C for a particular FQHS spin-polarization transition is indeed an intrinsic property of the 2DES.

Figure 3 summarizes α_C deduced from Fig. 2 data. In Fig. 3, we also include α_C reported for a GaAs/AlGaAs heterojunction sample [9]. The heterojunction sample has a very small layer-thickness, typically $\simeq 0.1\ l_B$, and is closer to an ideal, zero-thickness 2DES. It has a larger α_C and, in Ref. [9], an additional parallel magnetic was applied to enhance E_Z and reach the transition to full spin-polarization. In Fig. 3 it is clear that α_C decreases significantly as the 2DES layer-thickness increases. As we discuss below, this strong thickness dependence of the

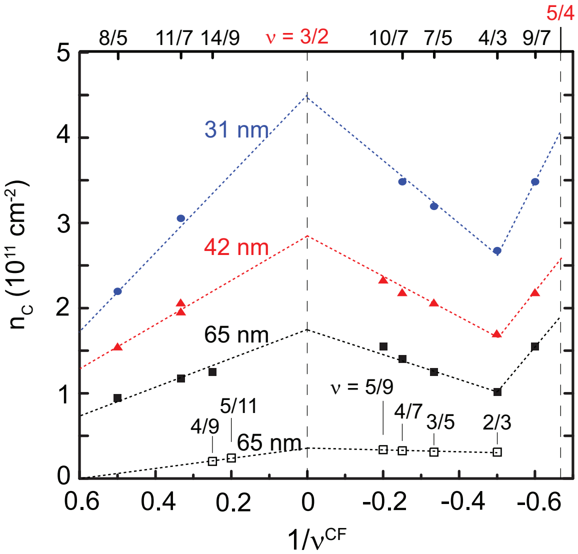

FIG. 2. (color online) Summary of the measured n_C above which the FQHSs become fully spin-polarized. The dotted lines, drawn as guides to the eye, represent the phase boundaries that separate the partially-polarized (below) from the fully-polarized states (above). Data are shown for FQHSs near $\nu = 3/2$ (closed symbols), and near $\nu = 1/2$ (open symbols). The error bar for n_C is about $\pm 5\%$ for FQHSs near $\nu = 3/2$ and $\pm 10\%$ for those near $\nu = 1/2$.

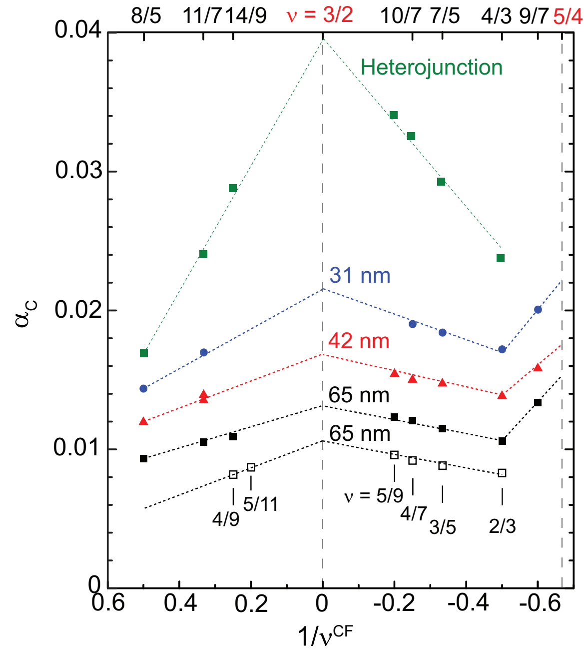

FIG. 3. (color online) Summary of α_C, the critical ratio of the Zeeman to Coulomb energies, needed to fully spin-polarize FQHSs at different ν and in different samples. Dotted lines represent the phase boundaries that separate the partially- (below) and fully-polarized (above) states. Closed (open) symbols are data measured near $\nu = 3/2$ ($\nu = 1/2$).

phase boundary stems from the softening of the Coulomb interaction when the electron layer thickness becomes comparable to or larger than l_B.

An accurate assessment of the finite-layer-thickness effect requires taking the shape of charge distribution into account. For a semi-quantitative discussion we use the simple parameter λ/l_B, where λ is the standard deviation of the electron's transverse position, as a measure of the thickness. To determine λ, we performed calculations of the charge distribution (at $B = 0$) by solving the Schroedinger and Poisson equations self-consistently and show examples of the resulting charge distributions above Fig. 4. Note that when the QW width is large, the 2DES has a bilayer-like charge distribution at high densities but in all cases λ has a well-defined value. In Fig. 4, we plot our measured α_C as a function of λ/l_B for the $\nu = 7/5$ FQHS. For the heterojunction sample, which has a very thin layer-thickness, $\lambda/l_B \simeq 0.1$, while in our QW samples, λ is comparable to l_B. Figure 4 reveals that α_C for the 7/5 FQHS monotonically decreases from about 0.03 in the heterojunction sample ($\lambda/l_B \simeq 0.1$) to about 0.012 in the 65-nm-wide QW ($\lambda/l_B \simeq 1.1$). The same trend is also seen for the other FQHSs.

In Figs. 2 and 3, we include the measured n_C and α_C for several FQHSs near $\nu = 1/2$, namely those at $\nu = 2/3$, 3/5, 4/7, 5/9, 4/9 and 5/11. The data were taken in another 65-nm-QW with a very low as-grown density of 0.34. The charge distribution in this low-density sample is single-layer-like and thinner than the higher density 65-nm-QW sample (see Fig. 4 top panel), so that there is less softening of the Coulomb interaction. However, instead of having a larger α_C compared to their particle-hole counterparts near $\nu = 3/2$, the FQHSs near $\nu = 1/2$ have about 20% *smaller* α_C (see Fig. 3). In a more quantitative comparison, when we normalize λ to l_B, this discrepancy becomes even larger. This mismatch is seen vividly in Fig. 4 where we plot α_C for the $\nu = 3/5$ FQHS measured in heterojunction samples [8, 10], and in two QWs with $W = 60$ and 65 nm. Since the $\nu = 3/5$ and 7/5 FQHSs both correspond to $\nu^{CF} = -3$, they are expected to have the same α_C if particle-hole symmetry holds. However, as seen in Fig. 4, α_C for $\nu = 3/5$ is less than half of α_C for $\nu = 7/5$ at the same λ/l_B, implying that the particle-hole symmetry is broken. We add that a very similar discrepancy was recently observed for the *valley*-polarization energies of the FQHSs at $\nu = 2/3$ and 4/3 in 2DESs confined to AlAs QWs [19].

Next we present results of our theoretical calculations. To include the effect of finite W, we took $\cos^2(\pi z/W)$ as the shape of the density profile in the transverse direction, for which $\lambda \simeq 0.18\,W$. Note that this simple model disregards the double-humped density profile for large W and n (see Fig. 4 top panel). We obtain the energies of the fully and partially spin polarized states at different ν by extrapolating the finite system results to the ther-

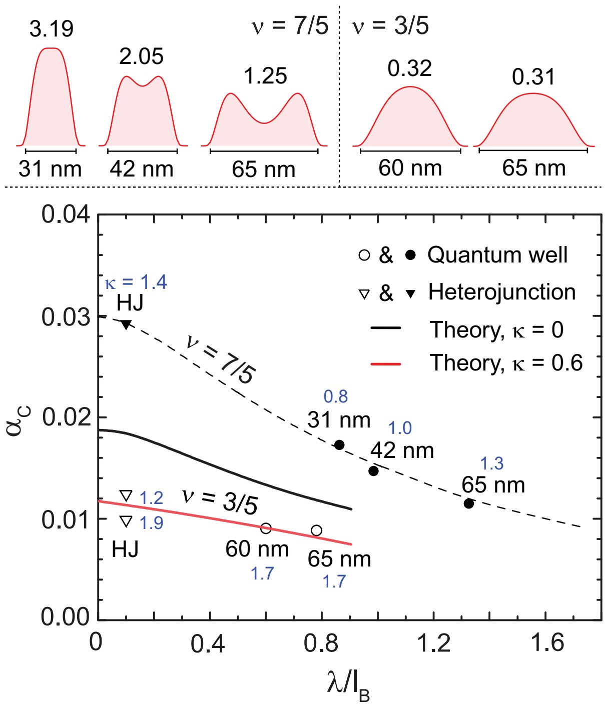

FIG. 4. (color online) Measured critical polarization energy α_C as a function of the layer-thickness, parameterized by λ/l_B, at filling factors $\nu = 7/5$ (solid symbols) and its particle-hole counterpart at $\nu = 3/5$ (open symbols). λ is the standard deviation of the electron position, deduced from self-consistent charge distribution calculations, examples of which are shown in the panel above the figure. The dashed curve is a guide to the eye. The solid curves represent the theoretically calculated α_C when $\kappa = 0$ (black) and 0.6 (red). In this plot, we also include the κ value (in blue) of experimental points.

modynamic limit; the resulting α_C is shown in Fig. 4 (solid black curve) for $\nu = 3/5$ or 7/5. It reproduces the overall trend of decreasing α_C with increasing λ/l_B, as expected from a softening of the Coulomb interaction due to the finite width. A quantitative discrepancy remains, however, the sign of which depends on ν.

We then include the effect of LL mixing. LL mixing modifies the two-body interaction, while also producing an effective three-body interaction (in a model that projects into the lowest LL); the latter is more relevant here as it breaks the particle-hole symmetry. The two- and three-body pseudopotentials have been obtained in a number of articles in a perturbative scheme in the parameter $\kappa = V_C/\hbar\omega_c$, where $\hbar\omega_c$ is the cyclotron energy separation between LLs [20–23]. We use the pseudopotentials given in Ref. [23], which are obtained perturbatively in κ for finite widths assuming a form of $\cos(\pi z/w)$ for the transverse wave function. We keep all three-body pseudopotentials $V^{(3)}_{S,m}$ up to relative angular momentum $m = 3$ for spins $S = 1/2$ and 3/2 [23]. To obtain the new α_C, we perform exact diagonalization including these short range ($m \leq 3$) triplet pseudopotentials for systems with $N = 5, 8, 11$ (9, 12, 15, 18) for partially (fully) spin polarized 3/5, and $N = 13, 20, 27$ (25, 32, 39, 46) for the corresponding 7/5 FQHSs. At 3/5, the correction to α_C is linear in κ for up to $\kappa \approx 1$. Furthermore, LL mixing *depresses* α_C. The calculated α_C for $\kappa = 0.6$, shown in Fig. 4 for $\nu = 3/5$ as a red curve, show good agreement with the experimental data. But note that $\kappa \gtrsim 1$ for the data, suggesting that the experiments are beyond the region where a perturbative treatment is applicable, and the correction presumably saturates with increasing κ.

The theoretical behavior is less clear for $\nu = 7/5$. Here, the correction in α_C is not proportional to κ even for $\kappa \approx 0.5$, implying that the ground-state itself is affected significantly by LL mixing for this value of κ. Furthermore, an irregular size dependence makes the extrapolation to the thermodynamic limit unreliable, suggesting the need for larger systems and perhaps also for pseudopotentials beyond $m = 3$. As a result, we are not able to ascertain reliably the correction to α_C for 7/5 that can be compared semi-quantitatively to experiments; nonetheless, our calculations clearly demonstrate that LL mixing causes at 7/5 an *increase* in α_C, in agreement with the experimental observation. We studied other states near $\nu = 1/2$ (2/5, 3/7, 2/3) and $\nu = 3/2$ (8/5, 11/7, 4/3) and found a behavior similar to 3/5 and 7/5. Our results imply that LL mixing causes a significant renormalization of the polarization mass of CFs (for a fixed λ/l_B), increasing it in the vicinity of $\nu = 1/2$ but lowering it near $\nu = 3/2$.

In summary, our systematic study of 2DESs confined to symmetric GaAs QWs reveal that the critical Zeeman energies where FQHSs become fully spin polarized depend substantially on finite layer thickness and, more importantly, on LL mixing, which breaks particle-hole symmetry. Our results thus provide fundamental insight into the nature of the three-body interaction terms induced by LL mixing.

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