xD-Track: Leveraging Multi-Dimensional Information for Passive Wi-Fi Tracking

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ABSTRACT

We describe the design and implementation of xD-Track, the first practical Wi-Fi based device-free localization system that employs a simultaneous and joint estimation of time-of-flight, angle-of-arrival, angle-of-departure, and Doppler shift to fully characterize the wireless channel between a sender and receiver. Using this full characterization, xD-Track introduces novel methods to measure and isolate the signal path that reflects off a person of interest, allowing it to localize a human with just a single pair of access points, or a single client-access point pair. Searching the multiple dimensions to accomplish the above is highly computationally burdensome, so xD-Track introduces novel methods to prune computational requirements, making our approach suitable for real-time person tracking. We implement xD-Track on the WARP softwaredefined radio platform and evaluate in a cluttered office environment. Experiments tracking people moving indoors demonstrate a 230% angle-of-arrival accuracy improvement and a 98% end-toend tracking accuracy improvement over the state of the art localization scheme SpotFi, adapted for device-free localization. The general platform we propose can be easily extended for other applications including gesture recognition and Wi-Fi imaging to significantly improve performance.

Categories and Subject Descriptors

C.2.1 [Computer-Communication Networks]: Network Architecture and Design

1. INTRODUCTION

Passive localization and tracking without any device carried by or attached to a person has been an exciting area of recent interest, with important applications in security, elderly care, and retail business. While techniques based on visible light [8] and cameras [9] have been proposed, Wi-Fi based solutions possess unique advantages stemming from non-line-of-sight signal coverage and a pervasive existing deployment of Wi-Fi access points. Such systems receive and process Wi-Fi transmissions reflected off objects in the vicinity to extract essential information about nearby reflectors' locations and velocities. By its nature, passive tracking is



Figure 1: (a) Joint estimation in both time-of-flight (ToF) and angle-of-arrival (AoA) dimensions. **(b)** Joint estimation in ToF, AoA and Doppler shift parameter dimensions.

more challenging than localization of the transmitter itself because signal reflections are typically orders of magnitude weaker than the directly-received transmissions and arrive at the receiver superimposed with other reflections.

Wi-Fi radio mapping-based methods for passive localization [12] circumvent the difficulty in modeling the signal propagation in a rich multipath environment with a labor-intensive site survey involving CSI- or RSSI-based fingerprinting. But these methods require a high density of Wi-Fi access points and achieve only a coarse localization accuracy that may degrade with changes in the environment. An alternative approach is to directly estimate properties of the wireless channel such as angle-of-arrival (AoA) [2, 14] or time-of-flight (ToF) [13] in order to infer a target's location. While promising, this approach faces fundamental resolution limitations stemming from a limited number of antennas (in the case of AoA) and a limited radio frequency bandwidth (in the case of ToF). Recent attempts to overcome these limitations with client motion [2] or channel combining [13, 15, 16] work, but apply in fewer practical situations of interest.

In this paper, we propose x-Dimensional Track (xD-Track), a new approach to improve localization and tracking accuracy based on jointly estimating as many different properties of a wireless propagation path as possible, each in its own separate dimension. As we detail in §2, wireless signal propagation can be measured in frequency, time, and space, leading to estimates of a number of propagation parameters including AoA, ToF, angle of departure (AoD), Doppler shift, and signal attenuation. To illustrate the intuition behind our scheme, consider Figure 1, which depicts three signals arriving at a receiver on time (ToF) and space (AoA) axes. If the receiver were to estimate only the respective ToF values of the two signals S1 and S2, resolution limits imposed by the channel frequency bandwidth would result in the estimates merging into one, since the two signals' respective ToF values are too close to each other. But when the receiver jointly estimates ToF and AoA as shown in Figure 1(a), signals S1 and S2 become resolvable, since



Figure 2: Passive localization by jointly making use of multiple dimensions of information.

they arrive at sufficiently-separated angles. Conversely, signals S1 and S3 cannot be separated using AoA alone, but become resolvable when the receiver leverages a joint ToF-AoA estimate.

This concept extends to even higher dimensions, as shown in Figure 1(b), where the receiver jointly and simultaneously estimates ToF, AoA, and Doppler shift. The reflectors of signals S4 and S5 are located close to each other and so exhibit similar AoA and ToF values, hence the receiver cannot separate them even with a two-dimensional estimate. However, S4's reflector may be a moving human, hence S4 would exhibit a non-zero Doppler shift. On the other hand, S5 may reflect off a static object, hence would exhibit a Doppler shift close to zero. In this case, we can easily separate these two signals with a three dimensional estimate, as shown in the figure. *In general, with a higher dimensional estimate, pairs of signals become more easily separated, increasing the resolvability of different signals without changing the resolution of any individual dimension.*

xD-Track is a general framework that incorporates the use of multi-dimensional signal propagation parameters in passive object tracking. Specifically, we develop a maximum-likelihood superresolution approach to jointly estimate multi-dimensional parameters. The estimation accuracy for each signal propagation path is significantly improved without requiring a large-bandwidth radio front end for high resolution observations in the time domain. and without requiring large numbers of antennas for high-resolution spatial signal profiling. We further reduce computational complexity and make xD-Track practical on commodity wireless signal processing systems (our prototype has been developed on the Rice WARP platform). Our trace-driven experiments demonstrate that the xD-Track prototype can localize a target with only one transmitter-receiver pair, greatly reducing deployment requirements. These preliminary results demonstrate a 230% improvement in AoA estimation compared with state-of-the-art SpotFi system, and promising results in ToF and Doppler shift estimation, which together translate into a 98% improvement of end-to-end object tracking accuracy. xD-Track can be extended to off-the-shelf commodity hardware, and our current work is moving towards such a prototype. Our multi-dimensional platform can easily be extended for other applications such as gesture recognition and Wi-Fi imaging.

2. FUNDAMENTAL RESOLUTION LIMITS

In this section, we briefly introduce the resolution limits for each path parameter and explain how each can help to localize the target.

Time of flight. The propagation time τ the signal takes to travel over a particular path from the transmitter to receiver is referred to as the *time of flight* (ToF) of that path. ToF can be used to compute the length of propagation path. In Figure 2, the length of "*Path-*I" can be computed using its ToF τ_1 by $\tau_1 \times c$. Therefore, the feasible locations of the targets (human) fall on the periphery of an ellipse whose foci are collocated with the sender and receiver. The resolution of ToF estimation $\Delta \tau$ varies in inverse proportion with the channel frequency bandwidth *B*, *i.e.* $\Delta \tau = 1/B$: the wider the bandwidth, the finer the resolution [13].

Angle of arrival. Only the signals with specific incidence angles ϕ can reach the receiver along a specific path. The angle ϕ is called the *angle of arrival* (AoA). In Figure 2, the target can be localized as the intersection point of AoA ϕ_1 and the ToF ellipse. The resolution of the AoA estimate is determined by the number of antennas in the receiver's array.

Angle of departure. Similar to AoA, only the signals moving in one specific outgoing angle φ can reach the receiver along one particular propagation path. Such an angle φ is called the *angle of departure* (AoD). The resolution of AoD estimation is determined by the number of antennas in the transmitter's array.

Doppler shift. When the reflector is mobile, it causes Doppler shift γ to its reflected signal. Doppler shift can not provide absolute location information of the target, but can estimate radio velocity of the target. For example, we can infer that "*path-1*" in Figure 2 must arise from a moving target since it has Doppler shift $\gamma_1 \neq 0$ and "*path-2*" should be associated with a static reflector like a furniture as its Doppler shift is zero. Doppler resolution $\Delta \gamma$ is related to the observation interval T_c by $\Delta \gamma = 1/T_c$: the longer the interval, the finer the resolution.

Complex attenuation. When the signal propagates over a distance in the environment, its amplitude is attenuated and its phase is changed: the complex number α is used to quantify these two phenomena. The power loss of such a path can be characterized as the norm of α . The power loss of the signal can be used together with a path loss model to roughly estimate path length and then localize the target.

3. DESIGN

In this section, we present the system design of xD-Track. We start with an overview of the system.

3.1 System overview

xD-Track incorporates three main components to achieve accurate passive localization:

Channel sounder. The sender works together with the receiver as a channel sounder. The transmitter emits a piece of wireless signal to probe the propagation environment. The receiver measures the signal propagation in multiple dimensions, including the time, frequency and space. The observation interval T_c , the instantaneous signal bandwidth B, and the antenna number of the sender and receiver array determines the sensing capability in time, frequency and space respectively and hence decides the overall sensing performance of the channel sounder. Channel measurements are used to estimate the multidimensional channel propagation parameters. Channel parameter estimator. This component estimates propagation parameters for all dominant signals. We assume there are L dominant signals travel along L paths to reach the receiver and $\theta_l = [\tau_l, \phi_l, \varphi_l, \gamma_l, \alpha_l]$ is a *path parameter vector* containing parameters that characterize the *l*th path. Specifically, τ_l is the *ToF* due to the path length, ϕ_l is the *azimuthal AoA* of the incident signal, φ_l is the *azimuthal AoD* of the outgoing signal, γ_l is the *Doppler shift* caused by the movement of the transmitter, receiver, or a reflecting object and α_l denotes the *complex attenuation*. The channel parameter estimator takes the channel measurement as input and estimates the $\Theta = [\theta_1, \theta_2, \dots, \theta_L]$ for all *L* dominant signal paths. Passive target locator. We jointly exploit the ToF, AoA, AoD, Doppler shift and complex attenuation of one signal to localize the passive target. The first step is to identify all signals that reflect off the moving human target. We achieve this goal by leveraging the Doppler shift value of the L signals. Any signal path with Doppler shift $\gamma_l \neq 0$ must arise from the non-static reflectors. The path parameter vector θ_l for signal *l* will be kept for further processing and may contain the location information of the target.

After we identify the propagation path that connects the transmitter, the moving target and the receiver, we need to localize the reflector using the multi-dimensional parameter θ_l we estimated. The idea is to find the location that can best fit the estimated values for ToF, AoA, AoD and the complex attenuation. Mathematically, we find the location (x_i, y_i) that minimizes the following objective function:

$$\Lambda_i = w_\tau (\tau_i - \tau_l)^2 + w_\phi (\phi_i - \phi_l)^2 + w_\phi (\varphi_i - \varphi_l)^2 + w_\alpha (\alpha_i^2 - \alpha_l^2)$$
(1)

where τ_i , ϕ_i , φ_i , α_i , are the ToF, AoA, AoD and complex attenuation that we will observe if the target is located at location (x_i, y_i) . The weighting factors w_{τ} , w_{ϕ} , w_{φ} , w_{α} are constants to unify the different scales of the corresponding dimensions, for example ToF values are measured in ns and AoAs are measured in radian.

3.2 Channel parameter estimator

In this section, we detail our design of the channel parameter estimator. We employ a joint maximum-likelihood based algorithm to estimate the channel parameters. Before we introduce the algorithm, we briefly justify the rationale behind our design choices.

Joint estimation. In order to localize the target with multi-dimensional information, a naive approach is to first estimate each dimension separately by performing multiple one-dimensional estimates and then combine the results for a final target location estimate. As illustrated in §1, one-dimensional estimates are often not able to achieve the required fine-grained resolution. Furthermore, the problem of associating path parameters from different dimensions is not easy. For example, we may obtain four estimated AoAs $\phi_1, \phi_2, ..., \phi_4$ and only three estimated ToFs $\tau_1, \tau_1, ..., \tau_3$. If two ToFs are too close to each other, they merge into one. Pairing each AoA with its respective path's ToF is non-trivial.

Instead of estimating path parameters separately and then combining the information, we apply joint estimation to handle the aforementioned problems. Naturally, joint estimation can estimate multi-dimensional information for each path simultaneously so the estimations from different dimensions are paired automatically for each path. Furthermore, when multiple dimensions are jointly estimated, the effective resolution is significantly increased: two signals can be effectively separated for higher resolution if they can be separated in any one dimension.

Maximum-likelihood based estimation. Subspace-based methods, like MUSIC, have been widely adopted to estimate channel parameters. Such methods partition the eigenvectors of the array covariance matrix into signal and noise subspaces and leverage either of them to estimate the signal parameters. MUSIC is a one dimensional estimation algorithm. Even though SpotFi [7] extends MUSIC to two dimensions that can estimate ToF and AoA simultaneously, there is no general framework for such methods to incorporate many dimensions. Most importantly, subspace-based methods sacrifice accuracy for computational efficiency [4], resulting in sub-optimal localization. At the same time, the computational load of a two-dimensional extension of MUSIC is extremely high, hindering real-time operation.

On the other hand, maximum-likelihood (ML) methods demonstrate superior performance in estimation accuracy. ML-based algorithms also have straightforward extensions to incorporate an arbitrary number of dimensions, which is important for our applications. Furthermore, such methods can produce optimal estimation performance, approaching the Cramer-Rao lower bound (CRLB) on the estimation error variance [4]. Although the superiority of the methodology has rarely been questioned, the concern for the computation overhead arises if the applications require a solution in real time, just as most indoor localization systems. Fortunately, several computational optimizations can be applied to the ML approach to significantly reduce the computation overhead without sacrificing optimality.

3.2.1 Estimation algorithm

In this section, we present xD-Track's ML-based joint estimation algorithm. We assume that the transmitting AP has a array of *N* sending antennas and the listening AP has an array of *M* receiving antennas. As mentioned earlier, $\theta_l = [\tau_l, \phi_l, \varphi_l, \gamma_l, \alpha_l]$ is a *path parameter vector* containing parameters that characterize the *l*th path from the transmitter to receiver. If we denote the transmitted signals as $U(t) = [u_1(t), u_2(t), \dots, u_N(t)]$, then we can use the above parameters to analyze the signal received over the *l*th path as follows:

$$\mathbf{s}_{l}(t;\theta_{l}) = [s_{1}(t;\theta_{l}), s_{2}(t;\theta_{l}), \dots, s_{M}(t;\theta_{l})]^{\mathsf{T}}$$
$$= \alpha_{l} \exp\left\{j2\pi\gamma_{l}t\right\} \mathbf{c}_{\mathbf{r}}(\phi_{l})\mathbf{c}_{\mathbf{s}}(\varphi_{l})^{T}u(t-\tau_{l}) \qquad (2)$$

where the $\mathbf{c_r}(\phi_l)$ and $\mathbf{c_s}(\varphi_l)$ is the steering vector of the sender and receiver array respectively. The overall received signal at the antenna array is then the superposition of the signals received over the *L* paths:

$$\mathbf{Y}(t) = \sum_{l=1}^{L} \mathbf{s}(t;\theta_l) + \mathbf{N}(t),$$
(3)

where $\mathbf{N}(t)$ is an *M*-dimensional complex white Gaussian noise vector capturing the background noise.

With the superposed signal captured at the receiver, we need to first determine the number of dominant paths L. Recent empirical evidence [6] has shown that the dominant path number of indoor environment is limited. In our implementation, we first set Lto be a large number, *e.g.* 10, to ensure that we do not miss the weak paths as for passive localization, the signal reflected from the human target can be weak. On the other hand, if the real dominant path number in the environment is less than the number L we set, our algorithm can still accurately estimate parameters of those dominant paths with large power, but will generate some paths with extremely low power. We will adjust the path number by dropping those paths with signal strength below the dynamic range of the sensing devices.

Maximum likelihood estimation. Given $\mathbf{y}(t)$, one observation of $\mathbf{Y}(t)$ over the observation interval *T*, the objective of ML estimation is to estimate the parameters $\Theta = \{\theta_l\}_{l=1}^L$ for the *L* paths. The ML function inputs parameters $\theta_1, \theta_2, \dots, \theta_L$ and considers them jointly, measuring the power difference between guessed parameters and the received data:

$$\Lambda(\Theta; \mathbf{y}) = -\int_{T} \left\| \mathbf{y}(t) - \sum_{l=1}^{L} \mathbf{s}_{l}(t; \theta_{l}) \right\|^{2} dt.$$
(4)

For the given observation, to solve the ML problem, the objective is to maximize $\Lambda(\theta; \mathbf{y})$, *i.e.*, make it as close to zero as possible with respect to the path parameters:

$$\Theta_{ML} = \underset{\Theta}{\arg\max} \{ \Lambda(\Theta; y) \}.$$
(5)

Inspection of Equation 4 reveals that this is a non-linear least squares problem, so no closed-form solution exists to achieve a global maximum. Since the complex attenuation that maximizes $\Lambda(\theta; y)$ can



Figure 3: The placement of the transmitting and receiving APs in our indoor office environment testbed.

be analytically expressed as a function of other parameters, the computation of Θ 's ML estimate is essentially a 3 × *L*-dimensional brute-force search with high computational load due to the high dimensionality of Θ . Thus, to make it suitable for real-time applications such as location tracking indoors, we need to significantly reduce the computational load involved to make it suitable for real-time indoor motion tracking.

3.2.2 Reducing computational complexity

Expectation maximization. Expectation maximization (EM) [3] is a conceptual framework for solving ML estimation problems, which works in an iterative way to maximize local approximations of the likelihood function and converge to the global maximum. We apply the EM algorithm here to decompose the above $3 \times L$ -dimensional non-linear optimization procedure into L three-dimensional optimization procedures that can be conducted in parallel, greatly reducing the time required to compute an estimate.

However, for each three-dimensional individual search, the computational load is still high for real-time applications. For instance, if the search space for a single θ_l path parameter is $\tau_l \in [0, 300]$ ns with a 0.5 ns step, $\phi_l \in [0, 360^\circ]$ with a half-degree step, and $\gamma_l \in [-20, 20]$ Hz with a 0.1 Hz search step, then we would need to search a prohibitive 1.75×10^7 possible combinations in each iteration to obtain the global optimum with EM.

SAGE. We apply the Space-Alternating Generalized Expectation-Maximization (SAGE) algorithm [5] to further reduce each threedimensional search into three one-dimensional searches. SAGE is actually an extension of EM, which updates the ML estimate of parameters sequentially. For example, to search the exactly the same space we mentioned for EM, we can reduce the candidate number of combinations from 1.75×10^7 to 1,420 after applying the SAGE algorithm, which makes it possible to obtain a solution in real-time.

4. IMPLEMENTATION

We implement xD-Track on the Rice WARP platform. Each WARP kit is attached with an FMC-RF-2X245 module, to enable four radios on each board. All data recorded is retrieved through Ethernet connections between the WARPs and a desktop server. We implement our multidimensional estimation algorithm on the server side using Matlab. Our xD-Track implementation currently supports four-dimensional estimation, jointly estimating the ToF, AoA, Doppler shift and complex attenuation of incoming signals. Accordingly, the transmitter in our system is equipped with one antenna and the receiver is equipped with four antennas.

5. EVALUATION

We conduct our experiments in two different types of environments: an indoor office spanning more than 1,500 sq. ft, just as Figure 3 shows, which is a multipath rich environment, and an indoor meeting room where direct path signal dominates. The transmitter and receiver are WARPs placed at fixed locations; a human target moves around in the environment whose trajectory is



Figure 4: (a): ToF and AoA estimates with no reflector present. (b): Histogram of AoA estimates with no reflector. (c): Histogram of ToF estimates with no reflector. (d): ToF and AoA estimates with a metal reflector. (e): Histogram of AoA with a metal reflector. (f): Histogram of ToA with a metal reflector.

recorded using 3 cameras as ground truth. The transmitter is configured to transmit packets every 20 ms using one 20 MHz channel in the 2.4 GHz frequency band. The receiver listens to channel that the transmitter is working on, records 40 sequential packets as one channel measurement which corresponds to an observation interval of 800 ms and uploads the channel measurements to server.

Resolving a reflection signal. We first test whether we can successfully resolve signals reflecting off objects in the environment. We send a signal to sense the environment. Then, we put a reflector (a metal board) in the environment, choosing its position to guarantee a signal reflection from the transmitter to the reflector and back to the receiver and sense the channel again. With the channel measurement, we estimate the AoA and ToF for all resolvable paths. Figure 4(a) plots the estimation results when there is no reflector the dashed line represents the ground truth AoA of the direct path. Clearly, we can observe four dominant paths including one direct path and three reflection paths. As a comparison, Figure 4(d) plots the AoA and ToF estimates after we place the metal reflector. We are now able to resolve five dominant paths-the one circled arises from the newly placed reflector and the dashed line gives us the ground truth AoA of the reflection path. Figures 4 (b) and (e) plot the histogram for AoA estimation and Figures 4 (c) and (f) plots the histogram for ToF estimation. We can see that xD-Track clearly resolves the signal reflected off an object in the environment.

Finding the mobile path. In this section, we want to verify the efficacy of using Doppler shift to identify a path that is associated with a moving target. In our experiment, we first measure the channel without the moving target. Then, we let a human walk in the environment and measure the channel again. Figures 5 (a) and (d) plot the Doppler shift and AoA estimation results for all resolvable paths, (b) and (e) plot the histogram of AoA estimation. Comparing (a) and (d) we can easily identify the paths associated with the moving target. And in Figure 5 (c) we can also observe that only the mobile path has non-zero Doppler shift values, which makes Doppler shift a good indicator of a reflector's mobility.

Estimating the AoA of a reflection path. In this experiment, we evalate xD-Track's parameter estimation accuracy. We put a reflector in the propagation environment and manually measure its ground-truth AoA. We compare the estimation result with the widely used MUSIC algorithm and the estimation scheme proposed in SpotFi [7]. Even though SpotFi is a active localization system, its parameter estimation algorithm can be directly used to estimate the reflection paths. To provide a fair comparison, we implement a



Figure 5: (a): Doppler and AoA estimations in static environment. (b): Histogram of AoA estimations. (c): Histogram of Doppler estimations. (d): Doppler and AoA estimations with human motion. (e): Histogram of AoA with human motion. (f): Histogram of Doppler estimations with human motion.



Figure 6: CDFs of the AoA estimation errors for reflection paths.

two-dimensional xD-Track so that we have the same number of dimensions as SpotFi. Figure 6 plots the CDF of the AoA estimation error. We can see that xD-Track's median error is about five degrees with only four antennas, outperforming the state-of-the-art SpotFi and MUSIC by 230% and 360% respectively. We can exploit the AoA and ToF estimation errors to project end-to-end localization performance in object tracking, which gives us a 98% and 213% improvement of end-to-end accuracy over SpotFi and MUSIC, respectively. The reason we can achieve this improvement is because of the optimality of our ML-based estimation algorithm. On the other hand, both MUSIC and SpotFi are subspace-based estimation algorithms, which are sub-optimal in terms of estimation accuracy.

6. RELATED WORK

Passive localization using diverse dimensions. Wi-Vi [2] uses one antenna to emulate a large antenna array and estimate AoA to track the direction of human motion. WiTrack [1] uses customized, dedicated hardware to transmit an ultra-wideband wireless signal to conduct channel sounding and estimate ToF for human tracking. Doppler shifts cannot provide absolute location information of a passive target, but instead gives us motion-related information. Hence, Wi-See [10] exploits it to classify different type of human gestures. While some recent systems [6, 7] consider the use of both AoA and ToF for object tracking, their algorithms are designed to estimate two-dimensional parameters, and cannot generalize in a straightforward way to estimate signals in higher dimensions.

Channel parameter estimation algorithms. Subspace-based algorithm has been widely used for channel parameter estimation, such as one dimensional MUSIC [11] and two dimensional SpotFi [7]. Such algorithms are appealing for their computation efficiency, which, however, sacrifice the optimality of accuracy. Besides, subspacebased algorithms have no elegant extensions to higher dimensional estimations. On the other hand, ML-based algorithms [3, 4] are theoretically optimal in terms of estimation accuracy and can be easily extended to higher dimensions. Furthermore, the ML-based EM [3] and SAGE [5] algorithms can localize efficiently without sacrificing optimality.

7. CONCLUSION

xD-Track is able to incorporate information from many possible signal dimensions to improve resolution. We believe this platform will open a new window to significantly improve the performance for applications besides localization including gesture recognition and Wi-Fi imaging.

8. ACKNOWLEDGMENTS

The research leading to these results has received funding from Singapore MOE AcRF Tier 1 MOE2013-T1-002-005, NTU NAP grant M4080738.020, the European Research Council under the EUs Seventh Framework Programme (FP/2007-2013)/ERC Grant Agreement NO. 279976 and the Google Doctoral Fellowship.

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