APPENDIX A: DERIVATION OF THE BASIC EQUATIONS

In this appendix we derive the linearized equations governing the spiral waves in an inviscid disk [Eqs. (4a) and (4b)]. But first we explain the approximation (1) for the circularization radius. The specific angular momentum with respect to the primary of the stream as it crosses the L1 point at distance $r_{\rm L1}$ from the primary is $j_0 = \Omega r_{L1}^2$, where $\Omega = \sqrt{G(M_1+M_2)/a^3}$. After self-intersecting and shocking, the stream settles into an approximately circular orbit of radius $r_{\rm c}$. Insofar as the tidal field of the companion mass M_2 can be neglected, the angular momentum is then $j_{\rm c} = \sqrt{GM_1 r_{\rm c}}$. If one assumes that $j_{\rm c} = j_0$, then eq. (1) follows immediately. Of course the angular momentum of the stream is not strictly conserved. Eq. (1) overestimates Flannery (1975)'s numerical results by 21% for $q = 1/19$ and by 51% for $q = 19$; the relative error increases monotonically with q. Applying Flannery (1975)'s numerical method to a wider range of mass ratios,[1] we find that the following empirical formula, in which $\mu \equiv q/(1+q)$,

$$\frac{r_{\rm c}}{a} \approx (1+q)\left(\frac{r_{\rm L1}}{a}\right)^4 \exp\left(-0.5\mu^{1/3}\right) , \tag{A1}$$

has a maximum relative error $\lesssim 4\%$ for all positive q; the error is largest at $q \approx 3$.

We adopt a thin-disk approximation assuming that there is no vertical motion, and work in a non-rotating coordinate (r, ϕ, z) defined relative to the primary. Σ and P are the surface density and vertically integrated pressure respectively. The system is governed by the Euler equations,

$$\partial_t \Sigma + \nabla \cdot (\Sigma \mathbf{v}) = 0, \tag{A2a}$$

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{\nabla P}{\Sigma} = -\nabla(\Psi + \varphi_d). \tag{A2b}$$

Here $\Psi = -GM_1/r$ is the gravitational potential of the primary and φ_d is the tidal potential of the binary companion; we ignore the self gravity of the disc. The unperturbed disk has $v_r = 0, v_\phi = \Omega r$, with

$$\Omega^2 = \frac{GM_1}{r^3} + \frac{1}{r\Sigma_0}\frac{dP_0}{dr}. \tag{A3}$$

The subscript 0 denotes the unperturbed state. Linearizing (A2b) and (A2b) gives

$$\partial_t \delta\Sigma + \Omega \partial_\phi \delta\Sigma = -\frac{1}{r}\left[\partial_r(r\Sigma_0 \delta v_r) + \partial_\phi(\Sigma_0 \delta v_\phi)\right], \tag{A4a}$$

$$\partial_t \delta v_r + \Omega\partial_\phi \delta v_r - 2\Omega \delta v_\phi = -\left(\frac{1}{\Sigma_0}\partial_r \delta P - \frac{1}{\Sigma_0^2}\frac{dP_0}{dr}\delta\Sigma\right) - \partial_r \varphi_d, \tag{A4b}$$

$$\partial_t \delta v_\phi + \Omega \partial_\phi \delta v_\phi + \frac{\kappa^2 \delta v_r}{2\Omega} = -\frac{1}{r\Sigma_0}\partial_\phi \delta P - \frac{1}{r}\partial_\phi \varphi_d. \tag{A4c}$$

Here $\kappa^2 = 2\Omega r^{-1}\frac{d}{dr}(r^2\Omega)$ is the epicyclic frequency. Assuming an azimuthal dependence of $\exp(im\phi - im\omega t)$ (and implicitly dropping this dependence in the perturbed variables), the above equations become

$$-i\sigma\delta\Sigma = -\frac{1}{r}\left[\frac{d}{dr}(r\Sigma_0\delta v_r) + im\Sigma_0 \delta v_\phi\right], \tag{A5a}$$

$$-i\sigma\delta v_r - 2\Omega\delta v_\phi = -\left(\frac{1}{\Sigma_0}\frac{d}{dr}\delta P - \frac{1}{\Sigma_0^2}\frac{dP_0}{dr}\delta\Sigma\right) - \frac{d}{dr}W_m, \tag{A5b}$$

$$-i\sigma\delta v_\phi + \frac{\kappa^2\delta v_r}{2\Omega} = -\frac{im}{r\Sigma_0}\delta P - \frac{im}{r}W_m. \tag{A5c}$$

Here δ denotes Eulerian perturbation and $\sigma = m(\omega - \Omega)$ is the frequency of the tide in the corotating frame and W_m the $m^{\rm th}$ azimuthal harmonic of the tidal potential, with

$$\varphi_d(r,\phi,t) = \sum_{m=2}^{\infty} W_m(r)\exp(im\phi - im\omega t). \tag{A6}$$

In this paper, we only consider the quadrupole ($m = 2$) potential,

$$W_2 = -\frac{3}{4}\frac{GM_2}{a^3}r^2. \tag{A7}$$

We then write the equations in terms of the radial Lagrangian displacement ξ and Eulerian enthalpy perturbation $\delta K \equiv \delta\Sigma/\Sigma_0$. The radial velocity is related to ξ by $\delta v_r = (\partial_t + \Omega\partial_\phi)\xi = -i\sigma\xi$, and $\delta\Sigma$ is given by

$$\delta\Sigma = -\frac{d\Sigma_0}{dr}\xi + \frac{1}{c^2}\left(\Sigma_0\delta K + \frac{dP_0}{dr}\xi\right) = \frac{\Sigma_0}{c^2}\delta K - \Sigma_0 N_0^2\left(\frac{dK_0}{dr}\right)^{-1}\xi. \tag{A8}$$

Here $K_0 = \int \Sigma_0^{-1} dP_0$ is the unperturbed enthalpy and N_0 is the Brunt-Väisälä frequency defined by eq. (3). Eliminating δv_ϕ from (A5a) - (A5c) and expressing the derivatives of Σ_0, P_0 using dK_0/dr and N_0^2 gives the equation of motion (4a) and (4b).

[1] but we initialize the orbital integrations with the unstable linear mode near L1, whereas Flannnery starts exactly at L1 with a small $\dot{x}$

APPENDIX B: VALIDITY OF IGNORING DISK SELF GRAVITY

Consider the gravitational potential perturbation φ due to the density perturbation in the disk. Assume that the disk is 2D (i.e. zero thickness in z), we have

$$\nabla^2\varphi = 4\pi G\Sigma\delta(z). \tag{B1}$$

For $z \neq 0$, the WKBJ solution for φ is Goldreich & Tremaine (1979)

$$\varphi(r,z) = \Phi(r)\exp\left(-|k(r)z| + i\int^r k(s)ds\right). \tag{B2}$$

Here as always, the linearised quantities are implicitly proportional to $\exp[im(\phi - \omega t)]$. Now (B1) at $z = 0$ becomes

$$-2|k|\Phi\exp\left(i\int^r k(s)ds\right) = 4\pi G\Sigma. \tag{B3}$$

Mild self-gravity modifies the WKBJ wavenumber $k(r)$ from its value (D1), but nevertheless throughout most of the disk, $\sigma^2 \sim \kappa^2 \sim \Omega^2$, so that

$$k(r) \sim \Omega/c. \tag{B4}$$

Ignoring disk self gravity is a good approximation when $|\delta\varphi(r, z = 0)/\delta K(r)| \ll 1$. Since

$$\left|\frac{\delta\varphi(r,z=0)}{\delta K(r)}\right| \sim \left|\frac{4\pi G\delta\Sigma c/\Omega}{c^2\delta\Sigma/\Sigma_0}\right| \sim \Sigma_d\mathcal{M}, \tag{B5}$$

On the RHS we use the normalized units of 2.1, in which $4\pi G$, Ω, and r are $O(1)$, and $c \sim \mathcal{M}^{-1}$, when r is not too small compared to the size of the Roche lobe. Self-gravity can therefore be neglected if

$$\Sigma_d\mathcal{M} \sim (M_d/M_1)\mathcal{M} \ll 1, \tag{B6}$$

in which M_d is the disk mass. This condition is satisfied for typical CV disks in quiescence and in circumplanetary disks.

One caveat is that in the analysis above we are considering only regions in the disk where the WKBJ approximation is applicable. As we show in the main text, however, density waves are mainly excited where the WKBJ approximation does not hold. Still, based on the agreement between our result (ignoring self gravity) and that of Goldreich & Tremaine (1979) (including self gravity) for wave excitation at an ILR, it is likely that self gravity in the wave excitation region will not significantly affect any nontrivial excitation.

APPENDIX C: FIRST-ORDER SYSTEM

When written in terms of the scales variables $\mathbf{y} = [\sqrt{r\Sigma_0}\xi, \sqrt{r\Sigma_0}(\delta K + W_m)]^T$, the first order equations (4a) and (4b) become

$$\frac{d\mathbf{y}}{dr} = \mathbf{M}\cdot\mathbf{y} + \mathbf{w}, \tag{C1a}$$

where

$$\mathbf{M} = \begin{bmatrix} -\left[\dfrac{2m\Omega}{r\sigma} + \dfrac{1}{2}\dfrac{d\ln(r\Sigma_0)}{dr} - N_0^2\left(\dfrac{dK_0}{dr}\right)^{-1}\right] & \dfrac{m^2}{r^2\sigma^2} - \dfrac{1}{c^2} \\ \sigma^2 - \kappa^2 - N_0^2 & +\left[\dfrac{2m\Omega}{r\sigma} + \dfrac{1}{2}\dfrac{d\ln(r\Sigma_0)}{dr} - N_0^2\left(\dfrac{dK_0}{dr}\right)^{-1}\right] \end{bmatrix}, \tag{C1b}$$

$$\mathbf{w} = \begin{bmatrix} \dfrac{1}{c^2}\sqrt{r\Sigma_0}W_m \\ N_0^2\left(\dfrac{dK_0}{dr}\right)^{-1}\sqrt{r\Sigma_0}W_m \end{bmatrix}. \tag{C1c}$$

Note that $\mathbf{M}$ is traceless. The eigenvalues of the system, therefore, are both purely real or both purely imaginary. Note that this is not the unique choice of $\mathbf{y}$ that gives a traceless $\mathbf{M}$. However, any different scaling will make $\mathbf{M}$ directly depend on Σ_0, while here $\mathbf{M}$ depends only on $1/H_r \equiv -d\ln\Sigma_0/dr$.

The eigenvalue is $\pm ik_0$ with

$$\begin{aligned}k_0^2 =&\frac{\sigma^2-\kappa^2-N_0^2}{c^2}\left(1-\frac{m^2c^2}{r^2\sigma^2}\right)-\left[\frac{2m\Omega}{r\sigma}+\frac{1}{2}\frac{d\ln(r\Sigma_0)}{dr}-N_0^2\left(\frac{dK_0}{dr}\right)^{-1}\right]^2\\ \approx&\frac{\sigma^2-\kappa^2-N_0^2}{c^2}-\left[\frac{1}{2H_r}+N_0^2\left(\frac{dK_0}{dr}\right)^{-1}\right]^2.\end{aligned}\tag{C2}$$

In the second line, all terms that are always $\lesssim O(1)$ are dropped. This gives the eigenvalue in (7).

Either component of $\mathbf{y}$ can be eliminated in favor of the other to yield a single second-order equation for the remaining component, such as eq. (13). Although often convenient, this may introduce spurious singularities. For example, the coefficient $f(r)$ defined by eqs. (13) & (D1c) is singular where $M_{12}=0$, although this is not a singularity of the first-order equations (C1). Similarly, if one eliminates y_1 in favor of y_2 from the second row of eq. (C1a), a singularity appears where $M_{21}=0$, coinciding with the Lindblad resonances when $N_0^2=0$. Although these are not true singularities, they are very nearly turning points inasmuch as k_0^2 is dominated by the off-diagonal terms of $\mathbf{M}$. Corotation (where $\sigma=0$) is usually a physical singularity, however (Goldreich & Nicholson 1989).

The first-order form also allows a slightly different view of the WKBJ approximation. Let $\mathbf{y}=(q,p)^T$, where $q(r)$ and $p(r)$ are scalar functions. Since $\mathbf{M}$ is traceless ($M_{11}=-M_{22}$), eq. (C1a) is equivalent to the equations of motion generated by the hamiltonian

$$H(q,p;r)=\frac{1}{2}\left(M_{12}p^2+2M_{11}pq-M_{21}q^2\right)\ +w_1p-w_2q\,,\tag{C3}$$

with r playing the role of "time." The level curves of H in the plane of (q,p) are closed when the discriminant of the quadratic form is negative: namely, when $M_{11}^2+M_{12}M_{21}=-k_0^2<0$. In that case, the action

$$J(H;r)\equiv\frac{1}{4\pi}\oint\limits_{H=\text{const}}(pdq-qdp)$$

is an adiabatic invariant: it is conserved to exponential accuracy where the logarithmic derivatives of the coefficients $\{M_{ij}(r),\,w_i(r)\}$ that define the hamiltonian (C3) are small compared to the oscillation frequency $|k_0|$. This condition always fails at the turning points ($k_0^2=0$), but it may fail elsewhere if $\mathbf{M}$ or $\mathbf{w}$ varies rapidly.

APPENDIX D: THE SECOND ORDER EQUATION AND THE LOCAL MODEL FOR RESONANCES

D1 The second order equation

The first order equation (C1a) can be rewritten as a first order equation in y_1 in the form of (13), with

$$p(r)=-\frac{d\ln|M_{12}|}{dr}\approx\frac{d\ln c^2}{dr},\tag{D1a}$$

$$k^2(r)=M_{11}\left(\frac{d\ln M_{12}}{dr}-\frac{d\ln M_{11}}{dr}\right)+k_0^2\approx k_0^2,\tag{D1b}$$

$$\begin{aligned}f(r)&=M_{12}\frac{d(M_{12}^{-1}w_1)}{dr}+M_{11}w_1+M_{12}w_2\\ &=\frac{1}{c^2}\sqrt{r\Sigma_0}\left(\frac{dW_2}{dr}-\frac{2m\Omega}{r\sigma}W_2\right)-\frac{d\ln(1-m^2c^2/r^2\sigma^2)}{dr}\frac{1}{c^2}\sqrt{r\Sigma_0}W_2+\frac{m^2}{r^2\sigma^2}N_0^2\left(\frac{dK_0}{dr}\right)^{-1}\sqrt{r\Sigma_0}W_2\\ &\approx\frac{1}{c^2}\sqrt{r\Sigma_0}\left(\frac{dW_2}{dr}-\frac{2m\Omega}{r\sigma}W_2\right).\end{aligned}\tag{D1c}$$

The approximate final equalities in each of eqs. (D1), and the final form of k_0^2 in eq. (8), are accurate when $\mathcal{M}\gg1$ and $\mathcal{M}^2/r\gg1/H_r,N_0^2(dK_0/dr)^{-1}$.

D2 Obtaining the local model for resonances

Near a resonance (i.e. a zero of k_0^2), the second-order equation can be further approximated as

$$\frac{d^2y_1}{dx^2}-xy_1=\lambda^2f(r),\tag{D2}$$

with $x\equiv(r-r_{\rm res})/\lambda$ and λ the lengthscale (25), provided that

$$\lambda\ll r,\qquad\left|\frac{d\ln M_{12}}{dr}\right|\ll\lambda^{-1},\qquad\left|\frac{d}{dr}\left[M_{11}\left(\frac{d\ln M_{12}}{dr}-\frac{d\ln M_{11}}{dr}\right)\right]\right|\ll\lambda^{-3},\tag{D3}$$

A sufficient condition for (D3) to hold is that $c^2, H_r, N_0^2, dK_0/dr$ (and $\sigma^2-\kappa^2$ when the resonance is not the ILR) are effectively constant over distances $\sim\lambda \ll r$.

Note, however, that we do not require Σ_0 to be slowly varying: indeed, its scale length H_r may be comparable to λ at an ACR or SACR, provided only that H_r itself varies slowly. The variation of $f(r)$ in eq. (D2) is then dominated by the factor $\sqrt{\Sigma_0}$ that it contains. Defining $\alpha\equiv\lambda/2H_r$ [so $\Sigma_0(x)\propto e^{-2\alpha x}$ approximately], we approximate eq. (D2) still further by putting

$$f(x)\approx f(r_{\rm res})e^{-\alpha x}, \qquad \alpha\equiv\lambda/2H_r\,, \tag{D4}$$

which gives eq. (26).

D3 Amplitude of the ingoing wave

At large negative x, the solution to eq. (26) should consist of an ingoing wave that satisfies the homogeneous Airy equation—plus a non-wavelike part that is in phase with the local forcing term

$$y_1(x)\approx A_{\rm in}[{\rm Ai}(x)+i{\rm Bi}(x)]\;+\;(-x)^{-1}\lambda^2 f(r_{\rm res})e^{-\alpha x}\quad x\to-\infty. \tag{D5}$$

The angular-momentum flux is proportional to the square of the first (wavelike) term, but the second term is much larger because of the exponential factor. In the forbidden zone, $y_1\to 0$ as $x\to+\infty$.

To solve (26) for the coefficient $A_{\rm in}$, we first adopt a new dependent variable $u(x)=e^{\alpha x}y(x)$, which satisfies

$$\frac{d^2u}{dx^2}-2\alpha\frac{du}{dx}+(\alpha^2-x)u(x)=\lambda^2 f(r_{\rm res}). \tag{D6}$$

The solution for $u(x)$ should tend to zero in both directions, so it will have a Fourier transform $\tilde{u}(q)$,

$$u(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{iqx}\tilde{u}(q)\,dq\,. \tag{D7}$$

The transform $\tilde{u}(q)$ satisfies the transform of eq. (D6),

$$\frac{d\tilde{u}}{dq}+(-iq^2+2\alpha q+i\alpha^2)\tilde{u}=2\pi i\delta(q)\lambda^2 f(r_{\rm res})\,, \tag{D8}$$

with initial condition $\tilde{u}(q)=0$ at $q<0$, since the phase of $u(x)$ should increase with x to have negative radial group velocity:

$$u(x)=i\lambda^2 f(r_{\rm res})\int_0^{\infty}\exp\left[\tfrac{1}{3}iq^3-\alpha q^2+iq(x-\alpha^2)\right]\,dq. \tag{D9}$$

This strongly resembles the standard integral representations for the Airy functions, with convergence for all x thanks to the factor $\exp(-\alpha q^2)$ in the integrand. For large negative x, two contributions dominate the integral. One of these comes from the lower endpoint $q\approx 0$:

$$u_{\rm nonwave}(x)\approx(\alpha^2-x)^{-1}\lambda^2 f(r_{\rm res})\;+O(x^{-3})\,. \tag{D10}$$

This matches the expected second term on the right side of eq. (D5) to leading order in $(-x)^{-1}$. The remaining contribution comes from the vicinity of the steepest-descent point, i.e. the point $q=q_0$ such that $\partial\psi(q,x)/\partial q=0$ if $\psi(q,x)$ represents the argument of the exponential in the integral (D9).[2] The result of the steepest-descent calculation matches the asymptotic expansion of ${\rm Ai}(x)+i{\rm Bi}(x)$ to leading order, and the coefficient implies

$$A_{\rm in}=i\pi\lambda^2 f(r_{\rm res})e^{-\alpha^3/3}\,. \tag{D11}$$

APPENDIX E: NUMERICALLY COMPUTING THE WAVE AMPLITUDE

E1 Computing the wave amplitude using the formal solution

When the wave amplitude is not too small, it can be easily obtained using the formal solution given in §2.5. First, we compute $y_{1,R}$ by integrating the homogeneous first order system in ξ and δK[3] inward, using an initial condition at $r_{\rm max}$ that satisfies the outer boundary condition. Then, we find another homogeneous solution that is linearly independent of $y_{1,R}$ by starting from the middle of the disk with arbitrary initial condition and integrating inwards and outwards. $y_{1,-}$ needs to be a linear combination of $y_{1,R}$ and this second homogeneous solution. We find the coefficients for this linear relation by minimizing $\int(d^2y_{1,-}/dr^2)^2$. ($y_{1,-}$ is often singular at 0 and $r_{\rm max}$; in practice, we minimize $\int(d^2y_{1,-}/dr^2)^2$ for $0.2r_{\rm max}<r<0.8r_{\rm max}$ to avoid these singularities.) Then, with $y_{1,-}$ and $y_{1,R}$, we can compute the amplitude using (17).

[2] There are two roots, $q_0=-i\alpha\pm\sqrt{-x}$, but only one of these is close to the positive real axis if $x\ll-1$, and neither if $x\gg+1$.
[3] We integrate the first-order system in order to avoid divergence of parameters at $r_{\rm max}$.

E2 Computing the wave amplitude using a nonwave solution

The method sketched in the previous subsection has one minor disadvantage: The integration (17) involves integrating a fast oscillating function, which makes the numerical error nontrivial once the amplitude becomes small. Here we introduce another method for numerically computing the wave amplitude, which helps to reduce the numerical error.

The solution that satisfies both boundary conditions can be written as

$$\mathbf{y} = \mathbf{y}_{\rm nw} + A_{\rm in}\mathbf{y}_{-}. \tag{E1}$$

Here $\mathbf{y}_{\rm nw}$ is a solution to the inhomogeneous problem whose angular momentum flux vanishes at $r \to 0$ (we call it a nonwave solution), and $A_{\rm in}$ is determined by the outer boundary condition. For $r \to 0$, we can write $\mathbf{y}_{\rm nw}$ as a power series in r (note that $F \sim y_1 y_2$, so $F \to 0$ requires $y_1 y_2 \to 0$), whose coefficients can be easily obtained from the equations of motion. We can then use this as the initial condition at some small r and integrate the system outwards to get $\mathbf{y}_{\rm nw}$. Meanwhile, $\mathbf{y}_{-}$ can still be computed using the method in the previous subsection. We can then determine $A_{\rm in}$ using the outer boundary condition. In most scenarios, $\mathbf{y}_{\rm nw}$ and $\mathbf{y}_{-}$ both diverge at $r_{\rm max}$. In this case, we determine $A_{\rm in}$ by matching their divergence for $r \to r_{\rm max}$, i,e, $A_{\rm in} \approx y_{i,{\rm nw}}/y_{i,-}$ for $r \to r_{\rm max}$.

The result obtained using this method agrees with that of the previous method when amplitude is relatively large (e.g. dimensionless amplitude $A_{\rm in} \gtrsim 10^{-6}$). For smaller amplitudes, the numerical uncertainty of the previous method often becomes large compared to the amplitude, whereas the method of this section converges well even for amplitude as small as $A_{\rm in} \sim 10^{-12}$.

APPENDIX F: WAVE EXCITATION IN THE POLYTROPIC DISKS: ANALYTIC RESULTS

When ILR lies outside the disk, the integrand of (17) (and hence the torque density on the disk) oscillates rapidly. Such integrals tend to be exponentially small when the envelope of the oscillation tapers slowly and smoothly to zero at both limits of integration. The integration (17) ends abruptly at $r_{\rm max}$, however, and furthermore has a branch point there if $n = (\gamma-1)^{-1}$ is not an integer, because $\mathcal{W}^{-1}(r) \propto \Sigma_0(r) \propto\sim (r_{\rm max}-r)^n$. This raises the possibility that the disk edge makes a contribution to eq. (17) that decreases as some power of $\mathcal{M}_0$ rather than exponentially. To address this possibility, we analyse eq. (13) in a simpler local approximation.

When written in terms of $\tilde{K} \equiv K + W_m$ as the dependent variable instead of y_1, eq. (13) becomes

$$\frac{d^2}{dr^2}\tilde{K} + \tilde{p}\frac{d}{dr}\tilde{K} + \tilde{k}^2\tilde{K} = \tilde{f}, \tag{F1}$$

with

$$\tilde{p}(r) = \frac{d}{dr}\ln\left(\frac{\Sigma_0 r}{|\sigma^2-\kappa^2|}\right), \tag{F2}$$

$$\tilde{k}^2(r) = -\frac{2m\Omega}{r\sigma}\left[\frac{d}{dr}\ln\left(\frac{\Sigma_0\Omega}{|\sigma^2-\kappa^2|}\right)\right] - \frac{m^2}{r^2} + \frac{\sigma^2-\kappa^2}{c^2}, \tag{F3}$$

$$\tilde{f}(r) = \frac{\sigma^2-\kappa^2}{c^2}W_m. \tag{F4}$$

At the outer edge of the disk, Σ_0 and c^2 both go to zero and $\tilde{p}, \tilde{k}^2, \tilde{f}$ all have simple poles. The residue of $\tilde{k}^2$ defines a length scale δ,

$$\delta^{-1} \equiv n\left[\frac{\sigma^2-\kappa^2}{\eta GM_1/r^2} + \frac{2m\Omega}{r\sigma}\right]_{r=r_{\rm max}}. \tag{F5}$$

Note that $\delta/r \sim \mathcal{O}(\eta) \sim \mathcal{O}(\mathcal{M}_0^{-2})$. So for the purpose of studying wave excitation at the disk edge when $\mathcal{M}_0 \gg 1$, it is reasonable to discard all but the leading-order behaviors (poles) of the functions (F2), (F3), and (F4), and to adopt a scaled independent variable

$$x \equiv \frac{r - r_{\rm max}}{\delta}. \tag{F6}$$

The resulting simplified equation is

$$x\frac{d^2\tilde{K}}{dx^2} + n\frac{d\tilde{K}}{dx} - \tilde{K} = \mathcal{F}_0 \tag{F7}$$

The constant $\mathcal{F}_0 = \delta n[r^2(\sigma^2-\kappa^2)W_m/\eta GM_1]_{r=r_{\rm max}}$ differs from $W_m(r_{\rm max})$ by a small $O(\eta)$ correction.

Equation (F7) has a regular singular point at $x = 0$. The change of variable $x = -z^2/4$ makes it

$$\frac{d^2\tilde{K}}{dz^2} + \frac{2n-1}{z}\frac{d\tilde{K}}{dz} + \tilde{K} = -\mathcal{F}_0\,. \tag{F8}$$

Clearly $n = 1/2$ is a particularly simple case because the homogeneous solutions are then sinusoids. The regular solution of eq. (F7) has a convergent power series in integral powers of x and hence even powers of z, so we take $\cos z$ as the "regular" solution of eq. (F8). The ingoing wave is[4] $\exp(-iz)$, and the coefficient of this wave as $z \to \infty$ is formally $-i\int_0^\infty \mathcal{F}_0 \cos z\, dz$ [compare eq. (17)]. As it stands, this integral is not convergent. But $\mathcal{F}_0$ is a proxy for $W_m(r)$, which (since $m \neq 0$) tends to zero as $r \to 0$, corresponding to $z \to \infty$. So we should think of $\mathcal{F}_0$ as a smooth slowly decreasing function $\mathcal{F}(z/\mathcal{M}_0)$; furthermore $\mathcal{F}(t) = \mathcal{F}(-t)$ because W_m is a regular function of $r - r_{\max} = -z^2\delta/4$. So the amplitude of the ingoing wave is $(-i/2)\mathcal{M}_0 \int_{-\infty}^{\infty} \mathcal{F}(t)\cos(\mathcal{M}_0 t)\, dt$, which is exponentially small at large $\mathcal{M}_0$ since $\mathcal{F}(t)$ is smooth.

Returning now to the general case, let $\nu \equiv n - 1$ and set $\tilde{K} = z^{-\nu} w(z)$, so that eq. (F8) becomes

$$\frac{d^2 w}{dz^2} + \frac{1}{z}\frac{dw}{dz} + \left(1 - \frac{\nu^2}{z^2}\right) w = -z^{\nu}\mathcal{F}(z/\mathcal{M}_0)\,. \tag{F9}$$

The homogeneous solutions of this last equation are Bessel functions of order ν. The solution corresponding to the regular homogeneous solution of (F7) is $J_\nu(z)$, the ingoing wave is $H_\nu^{(2)}(z) = J_\nu(z) - iY_\nu(z)$, and the amplitude of this wave in the particular solution as $z \to \infty$ is

$$A_{\rm in}(n) \approx \frac{i\pi}{2}\int_0^\infty z^{\nu+1} J_\nu(z)\mathcal{F}(z/\mathcal{M}_0)\, dz\,, \tag{F10}$$

which has made use of the Wronskian $\mathcal{W}[J_\nu,\, H_\nu^{(2)}] = -i\mathcal{W}[J_\nu,\, Y_\nu] = 2i/\pi z$. We write "$\approx$" rather than "$=$" because eq. (F10) is based on the local approximation (F7) rather than the exact LWE. This last integral is again exponentially small, which can be seen as follows. Set $\mathcal{F}(t) = g(t^2)$ (since $\mathcal{F}$ is even in its argument) and suppose that $g(u)$ has an inverse Laplace transform $\hat{g}(s)$, so that $\mathcal{F}(t) = \int_0^\infty \hat{g}(s)e^{-st^2}\, ds$. Putting this into eq. (F10), reversing the order of integration, and invoking Abramowitz & Stegun (1972, §11.4.29) yields

$$\begin{aligned} A_{\rm in}(n) &\approx \frac{i\pi}{2}\int_0^\infty \left(\frac{\mathcal{M}_0^2}{2s}\right)^{\nu+1} \exp\left(-\frac{\mathcal{M}_0^2}{4s}\right)\hat{g}(s)\, ds \\ &= \frac{i\pi}{2}\mathcal{M}_0^2 \int_0^\infty (2\sigma)^{-\nu-1} e^{-1/(4\sigma)}\hat{g}(\mathcal{M}_0^2\sigma)\, d\sigma. \end{aligned} \tag{F11}$$

Note finally that $\hat{g}(s)$ decreases faster than any power of s as $s \to \infty$ because $g(u) = \mathcal{F}(\sqrt{u})$ has a convergent Taylor series at $u = 0$ ($r = r_{\max}$), so that

$$\frac{d^k g}{du^k}(0) = (-1)^k \int_0^\infty \hat{g}(s)s^k\, ds$$

must exist. Therefore the integral (F11) decreases faster than any power of $\mathcal{M}_0^{-2}$ as $\mathcal{M}_0 \to \infty$.

To sum up, by inspecting the integral (17) we find that the excitation will be exponentially small except when the thin region near outer edge of the disk (with characteristic size $\delta \sim \eta$) makes a nontrivial contribution. Then, we illustrated (using the local model discussed above) that the contribution from this region decreases faster than any power of $\mathcal{M}_0^{-2}$ as $\mathcal{M}_0 \to \infty$. As the numerical result in §3.2 shows, this is indeed the case and the amplitude decreases exponentially as $\mathcal{M}_0$ increases.

REFERENCES

Abramowitz M., Stegun I. A., 1972, Handbook of Mathematical Functions. Dover
Flannery B. P., 1975, MNRAS, 170, 325
Goldreich P., Nicholson P. D., 1989, ApJ, 342, 1075
Goldreich P., Tremaine S., 1979, ApJ, 233, 857

[4] Interior to the ILR, negative radial group velocity corresponds to a phase that decreases inward from the disk edge, and therefore with increasing z.