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Defect CFT in the 6d (2,0) theory from M2 brane dynamics in $\mathrm{AdS}_7 \times S^4$

Nadav Drukker,[a] **Simone Giombi,**[b] **Arkady A. Tseytlin**[c,1] **and Xinan Zhou**[d]

[a] *Department of Mathematics, King's College London, The Strand, London WC2R 2LS, U.K.*
[b] *Department of Physics, Princeton University, Princeton, NJ 08544, U.S.A.*
[c] *The Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[d] *Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544, U.S.A.*

E-mail: nadav.drukker@kcl.ac.uk, sgiombi@princeton.edu, tseytlin@imperial.ac.uk, xinanz@princeton.edu

Abstract: Surface operators in the 6d (2,0) theory at large N have a holographic description in terms of M2 branes probing the $\mathrm{AdS}_7 \times S^4$ M-theory background. The most symmetric, 1/2-BPS, operator is defined over a planar or spherical surface, and it preserves a 2d superconformal group. This includes, in particular, an $\mathrm{SO}(2,2)$ subgroup of 2d conformal transformations, so that the surface operator may be viewed as a conformal defect in the 6d theory. The dual M2 brane has an AdS_3 induced geometry, reflecting the 2d conformal symmetry. Here we use the holographic description to extract the defect CFT data associated to the surface operator. The spectrum of transverse fluctuations of the M2 brane is found to be in one-to-one correspondence with a protected multiplet of operator insertions on the surface, which includes the displacement operator. We compute the one-loop determinants of fluctuations of the M2 brane, and extract the conformal anomaly coefficient of the spherical surface to order N^0. We also briefly discuss the RG flow from the non-supersymmetric to the 1/2-BPS defect operator, and its consistency with a "b-theorem" for the defect CFT. Starting with the M2 brane action, we then use AdS_3 Witten diagrams to compute the 4-point functions of the elementary bosonic insertions on the surface operator, and extract some of the defect CFT data from the OPE. The 4-point function is shown to satisfy superconformal Ward identities, and we discuss a related subsector of "twisted" scalar insertions, whose correlation functions are constrained by the residual superconformal symmetry.

Keywords: AdS-CFT Correspondence, Anomalies in Field and String Theories, Conformal Field Theory

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[1] Also at Lebedev Institute and ITMP, Moscow State University, Moscow, Russia.

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1 Introduction and summary

Non-local operators are an important class of observables in conformal field theories in various dimensions. When they are defined over planar or spherical submanifolds, they may preserve a subgroup of the conformal symmetry of the "bulk" CFT, and are often referred to as conformal defects. Using the AdS/CFT duality, one may develop a strong-coupling perturbation theory approach to the computation of their expectation values and correlation functions of local operators inserted on them. The most familiar example is that of a fundamental string ending along a curve on the boundary of AdS_5 within type IIB string theory, dual to the Wilson loop operator in $\mathcal{N}=4$ SYM theory [1]. When the curve is a circle or infinite straight line, the Wilson loop is 1/2-BPS and it preserves a 1d conformal symmetry,[1] corresponding to a string worldsheet with AdS_2 induced geometry [2]. The strong coupling expansion for the expectation value of the Wilson line and correlation

[1]The full symmetry group is $OSp(4^*|4)\supset SL(2,\mathbb{R})\times SO(3)\times SO(5)$.

functions of operators inserted along it is then controlled by the fluctuations [3] of the fundamental superstring near the static configuration (see, e.g., [4, 5] and refs. therein).

One can generalize this to other branes in different AdS background ending along different dimensional submanifolds on the boundary (for example, D3-brane and D5-branes probes in AdS_5 describing line operators, surfaces and domain walls, see, e.g., [6, 7]). In the most symmetric cases these branes have the world-volume metric of $AdS_{p+1} \times S^k$ with appropriate p and k.

In this paper we study the simplest such example within M-theory: an M2-brane probe ending along a surface on the boundary of AdS_7. The most symmetric configuration, which preserves half the supersymmetries of the bulk theory, is when the 3d world-volume of the M2-brane ends on a plane (or sphere) at the boundary. As the M-theory in the $AdS_7 \times S^4$ vacuum is a dual description of the (2,0) conformal theory, this configuration should be representing a supersymmetric surface defect operator in this 6d CFT (for a recent discussion and refs. see [8]). Our aim is to study this system beyond the classical brane limit by calculating its one loop fluctuation determinant and performing the holographic computation of 4-point correlators of the simplest local insertions into the surface operator. This is the direct analog of the calculation of insertions into the Wilson loops, captured by string fluctuations [5].

Let us recall the case of insertions into Wilson loops in $\mathcal{N} = 4$ SYM. The 1/2-BPS Maldacena-Wilson line along the x^1 direction and coupling to the scalar Φ_6 is $W = \mathrm{Tr}\,\mathcal{P}e^{\int dx^1(iA_1+\Phi_6)}$. We can insert any adjoint valued operators into the loop, the most natural being the remaining five scalars Φ_a and the combination of field strength and scalar $\mathrm{D}_i = \mathbb{F}_{ti} \equiv iF_{ti} + D_i\Phi_6$. The latter, known as the displacement operator, represents small geometric deformations of the line. In the defect CFT (dCFT), the scalars Φ_a have dimension one and the displacement dimension two. This translates in AdS to fluctuation modes of the AdS_2 worldsheet with $m^2 = 0$ and $m^2 = 2$. Their 4-point functions were studied in [5] by expanding the string action to quartic order and performing AdS_2 Witten-diagram calculations on the worldsheet. This allowed to deduce the spectrum of some of the operators appearing in their OPE, providing further details on the strong coupling Wilson loop dCFT. For instance, the scaling dimension of the singlet scalar insertion which is dual in AdS_2 to a two-particle "bound state" of string fluctuations along S^5 was found to be $\Delta = 2 - \frac{5}{\sqrt{\lambda}} + \ldots$. Recently, this result was confirmed by integrability techniques in [9], which also obtained several more orders in the strong coupling expansion.

For the case at hand, the (2,0) supersymmetric 6d CFT describing multiple M5-branes may be thought of as a $SU(N)$ generalization of a free (2,0) tensor multiplet containing the B_{mn}-field with self-dual strength H_{mnl}, 5 real scalars Φ^I and 4 symplectic Majorana fermions. In this abelian theory the locally-supersymmetric surface operator analogous to the Wilson loop operator of [1] may be defined as [8, 10][2]

$$
\begin{aligned}
V &= \exp\left(\int d^2\vec{x}\left[i\,\frac{1}{2}\epsilon^{\mu\nu}\partial_\mu X^m \partial_\nu X^n\, B_{mn}(X) + \sqrt{g(X)}\,\Phi_5(X)\right]\right) \\
&\to \exp\left(\int d^2\vec{x}\left[i\,B_{12}(X) + \Phi_5(X)\right]\right),
\end{aligned}
\tag{1.1}
$$

where Φ_5 is one of the 5 scalars of the (2,0) tensor multiplet, $X^m(x)$ are the 6d coordinates describing the surface and we specified to the case when the defect is a plane in the $(1,2)$ directions.[3] The surface operator breaks the $\mathrm{OSp}(8^*|4)$ supersymmetry of the 6d theory to $[\mathrm{OSp}(4^*|2)]^2$ with the bosonic subgroup $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4) = [\mathrm{SO}(2,1)\times\mathrm{SU}(2)\times\mathrm{SU}(2)]^2$. Here $\mathrm{SO}(2,2)$ corresponds to the 2d conformal symmetry, one $\mathrm{SO}(4)$ to rotations in the transverse directions to the surface, and the second $\mathrm{SO}(4)\subset\mathrm{SO}(5)$ is the remaining R-symmetry that rotates the four scalars that do not couple to the operator. As natural in defect CFT, one can consider correlation functions of operators inserted on the defect surface: the basic short multiplet includes four transverse scalars Φ_a $(a=1,\cdots,4)$ with dimension $\Delta=2$, four displacement operators[4] $\mathrm{D}_i = \mathbb{H}_{12i} \equiv iH_{12i} + \partial_i\Phi_5$ $(i=1,\cdots,4)$ with $\Delta=3$ and eight fermions with $\Delta=5/2$.

In the dual description this 1/2-BPS surface operator is represented by a probe M2-brane with worldvolume ending on a plane at the $\mathbb{R}^6$ boundary, stretched along z of AdS_7 and localized at a point in S^4. The M2-brane probe is described by a κ-symmetric generalization of the Dirac-Nambu action (see, e.g., [14, 15]). The induced 3-geometry in the static gauge is then AdS_3 and as in [3, 16] one finds that the transverse fluctuations of the M2-brane surface are represented by: 4 scalars y^a (S^4 fluctuations) with $m^2=0$, 4 scalars x^i (AdS_7 fluctuations transverse to the 3-surface) with $m^2=3$ and 8 fermions with $m^2=\frac{9}{4}$. The correlators of these "transverse" membrane fluctuations (and more generally their composites) should then define a 2d dCFT associated to the surface defect. Via $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence the dual boundary operators should have dimensions $\Delta=2,3$ and $\frac{5}{2}$ matching those of the scalars, displacement operator and fermions on the defect.[5]

Below we compute the correlators of the bosonic fluctuations $X^I=(\mathrm{x}^i,y^a)$ as defined by the M2-brane action in the inverse effective membrane tension $\mathrm{T}_2 = a^3T_2 = \frac{2}{\pi}N$ expansion (a is the radius of AdS_7). They should define the large N limit of the corresponding 6d correlators of the operators $\mathcal{O}_I = (\mathbb{H}_{12i},\Phi_a)$ inserted on the planar ($\vec{x}=(x^1,x^2)$) defect

$$
\langle\!\langle \mathcal{O}(\vec{x}_1)\cdots\mathcal{O}(\vec{x}_n)\rangle\!\rangle = \langle X(\vec{x}_1)\cdots X(\vec{x}_n)\rangle_{\mathrm{AdS}_3}\,. \tag{1.2}
$$

[2]The introduction of a surface operator with coupling to B-field [1, 2, 11] is natural by analogy with strings ending on D3-branes case, i.e. in the picture where the dynamics of M5-branes is described in terms of M2-branes [12] ending on strings coupled to B-field.

[3]Due to conformal invariance one can consider the defect with either planar or spherical (S^2) geometry.

[4]The displacement operator describes transverse deformations of the defect (see for instance [13] for a general discussion). For a defect with co-dimension $6-p$, the displacement operator D^i may be defined via $\partial_\mu T^{\mu i} = \delta^{(6-p)}(x_\perp)\mathrm{D}^i$ where T is the "bulk" stress tensor (the stress tensor of the 6d CFT), with $\Delta=6$. For the surface defect ($p=2$) the dimension of the displacement $\mathrm{D}^i(i=1,\cdots,4)$ is then $\Delta=3$. In general, for CFT_d with a co-dimension $d-p$ defect, $\Delta(\mathrm{D}^i)=p+1$.

[5]In general, in $\mathrm{AdS}_{p+1}/\mathrm{CFT}_p$ case we have $\Delta(\Delta-p)=m^2$ for scalars and $\Delta=m+p/2$ for the fermions. In the string (Wilson loop) case $p=1$, while here $p=2$.

A novel feature of the present M2-brane in $\mathrm{AdS}_7 \times S^4$ case compared to the string in $\mathrm{AdS}_5 \times S^5$ case in [5] is the presence of the WZ term in the action that contributes non-trivially to the 4-point correlator of the scalars y^a. This term $\sim \mathrm{T}_2 \int_4 \epsilon^{ABCDE} Y_A dY_B \wedge dY_C \wedge dY_D \wedge dY_E \to \frac{iN}{32\pi} \int d^3x\, \epsilon^{\mu\nu\lambda} e_{abcd}\, y^a \partial_\mu y^b \partial_\nu y^c \partial_\lambda y^d + O(y^5)$ originates from the coupling of the M2-brane to the potential C_3 of the magnetic 4-form flux of the $\mathrm{AdS}_7 \times S^4$ background [17].[6] Being intimately related to the underlying supersymmetry, the contribution of this term is important for the resulting 4-point function satisfying the constraints imposed by the residual superconformal symmetry. In contrast to the Wilson loop in $\mathcal{N} = 4$ SYM case where one can also directly compute a weak-coupling limit of the corresponding correlators on the gauge theory side, it is not clear how to do this in the (2,0) 6d theory that currently lacks an intrinsic definition.[7] It would be interesting to make contact with the results of this paper by bootstrap methods, as was done in [20] for the case of the Wilson line dCFT.[8]

The contents of this paper are as follows. Our starting point in section 2 is the expansion of the M2-brane action in $\mathrm{AdS}_7 \times S^4$ near the minimal 3-surface ending on a 2-plane or a 2-sphere at the boundary (and localized at a point in S^4). The value of the classical M2-brane action on this surface is proportional to the volume of AdS_3 . In the case of spherical boundary, the volume of AdS_3 is logarithmically divergent with the IR cutoff R. This is in contrast to the string in the AdS_5 case, where the classical value of the string action proportional to the volume of AdS_2 is finite (after subtraction), and matches the strong-coupling limit of the expectation value of the circular Wilson loop. Here instead the coefficient of $\log \mathrm{R}$ term may be interpreted as one of the conformal anomaly coefficients in the defect CFT.[9]

In section 3 we compute the 1-loop correction to the logarithm of the partition function of the M2-brane ending on a spherical surface. This gives a correction of order N^0 to the leading result coming from the classical action of the surface, which is of order $\mathrm{T}_2 = \frac{2}{\pi} N$. In a choice of normalization that will be explained below, we find for the anomaly coefficient of the spherical surface $b = 12N - 9 + \mathcal{O}(N^{-1})$. These first two terms match the prediction $b = 3(N-1)(4 + N^{-1})$ following from [27, 28]. In section 3.2 we also comment on the holographic description of the non-supersymmetric surface defect operator which does not couple to the scalar fields, following the analogy with the standard Wilson loop case in [29–32]. In this case the M2-brane surface should be delocalized in S^4, i.e. the scalars y^a should satisfy the Neumann boundary condition. Adding a boundary perturbation to the M2-brane action leads to a 2d RG flow between the UV (non-supersymmetric) and IR

[6] A similar term is present, e.g., in the D3-brane probe action in $\mathrm{AdS}_5 \times S^5$ [18].

[7] One can still mimic such computation by starting with the abelian 6d tensor multiplet theory and consider correlators of the fields with the defect (1.1). In particular, ref. [19] computed the 2-point function of the displacement operator by considering the second order in the "wavy surface" approximation. Its form $\langle \mathrm{D}^i(\vec{x}_1) \mathrm{D}^j(\vec{x}_2) \rangle \sim \frac{1}{|x_{12}|^6}$ is dictated by the associated dimension $\Delta = 3$.

[8] Among possible generalizations one may consider a BPS configuration of a M5 brane probe intersecting M5 branes over a line and wrapped on $S^3 \subset S^4$ so that the resulting M5 brane world volume geometry is $\mathrm{AdS}_3 \times S^3$ (cf. [21–25]). This should correspond to the case when the surface defect is in a large representation of $\mathrm{SU}(N)$.

[9] Similar logarithmic UV divergence appears in the log of expectation value of the surface operator (1.1) in the abelian (2,0) theory [8, 10]. For the dual M2-probe discussion see also [26].

(supersymmetric) fixed points with the resulting values of the boundary conformal anomaly coefficients consistent with the b-theorem for 2d defects [33–35].

In section 4 we compute the 4-point correlation functions for the scalar fluctuations y^a and x^i near the BPS surface, in the leading tree-level approximation. We find the expressions following from the Dirac-Nambu part of the action for the general dimension p of the brane, with the $p=1$ case reproducing the string-theory results of [5]. We observe that the $\langle yyyy\rangle$ 4-point function satisfies simple superconformal Ward identities that, as turns out, essentially determine its form. We also discuss the Mellin representation for the resulting AdS_3 correlators. In section 4.3 we perform an OPE analysis of the correlator $\langle yyyy\rangle$ extracting the leading $1/N$ terms in the anomalous dimensions of composite $y\partial^n y$ operators appearing in different channels.

In section 5 we discuss constraints on correlators imposed by a residual superconformal symmetry. In section 5.1 we follow the analogy with the SYM case [36] and consider a special twisted combination $\mathcal{Y}=\mathrm{t}^a(\vec{x})\,y^a(\vec{x})$ of scalar operator y^a whose correlators are constrained by residual supersymmetry. The 4-point correlator of the twisted fields has a very simple form given in section 5.2 and surprisingly has a very similar structure to that of the strong-coupling limit of the reduced correlator of 1/2-BPS scalar operators in the $\mathcal{N}=4$ SYM theory. In section 5.3 we show that the form of the $\langle yyyy\rangle$ correlator found in section 4.1 is essentially constrained by the superconformal symmetry and crossing up to an overall constant factor. A similar observation in the case of the scalar correlators on the BPS Wilson loop in $\mathcal{N}=4$ SYM is made in appendix B.

2 Membrane action in $\mathrm{AdS}_7\times$ S^4

We are interested in studying the fluctuations of an M2-brane in the $\mathrm{AdS}_7\times S^4$ background which is the near horizon geometry of N M5-branes

$$ds^2=a^2\big[ds^2_{\mathrm{AdS}_7}+\mathrm{r}^2 ds^2_{S^4}\big]\,,\qquad a^3=8\pi N\ell_p^3\,,\qquad \mathrm{r}=\frac{1}{2}\,, \tag{2.1}$$

$$F_4=\pi^2a^3\Omega_4\,,\qquad \int_{S^4}\Omega_4=1\,,\qquad \mathrm{vol}(S^4)=\frac{8\pi^2}{3}\,. \tag{2.2}$$

Here a is the radius of AdS_7, and ds_{AdS_7} and ds_{S^4} are the line elements on unit radius AdS_7 and S^4. Ω_4 in (2.2) denotes the normalized volume form of S^4, ℓ_p is defined via $2\kappa_{11}^2=(2\pi)^8\ell_p^9$ and $F_4=dC_3$. We use Euclidean signature throughout.

The (bosonic part of) the κ-symmetric action of an M2-brane probe [14] contains two terms: the standard Dirac-Nambu type term S_1 (the volume of the 3-surface in induced metric) and a WZ-type term S_2 of coupling to the 3-form C_3

$$S=S_1+S_2\ :\qquad S_1=T_2\int d^3x\,\sqrt{\det h_{\mu\nu}}\,,\qquad h_{\mu\nu}=\partial_\mu X^M\partial_\nu X^N G_{MN}(X) \tag{2.3}$$

$$S_2=-iT_2\int d^3x\,\frac{1}{3!}\epsilon^{\mu\nu\lambda}C_{MNK}(X)\,\partial_\mu X^M\partial_\nu X^N\partial_\lambda X^K\,, \tag{2.4}$$

where the fundamental M2-brane tension T_2 is

$$T_2=(2\pi)^{2/3}(2\kappa_{11}^2)^{-1/3}=\frac{1}{(2\pi)^2\ell_p^3}\,. \tag{2.5}$$

The world-volume on an M2 brane ending on the plane (or sphere) at the 6-boundary of AdS_7 has the classical solution with AdS_3 induced metric.[10] In the case of the planar surface, this AdS_3 subspace is spanned by the plane and the z coordinate in (2.1).

The effective membrane tension (i.e. the analog of the familiar fundamental string tension $\mathrm{T}_1 = \frac{a^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$ in the $\mathrm{AdS}_5 \times S^5$ case) is then given by the product of the fundamental M2-brane tension T_2 and the cube of the AdS radius, i.e. (see, e.g., [38, 39])[11]

$$\mathrm{T}_2 = a^3 T_2 = \frac{2}{\pi} N\,. \tag{2.6}$$

We proceed now to expand the probe brane action about this classical solution. For generality, let us consider a p-brane in AdS_{d+1} with world volume ending along a p-plane at the boundary and also stretched along z of AdS_{d+1}. Following [5] where the case of $p = 1$ and $d = 4$ was discussed, let us choose the following "AdS_{p+1}-adapted" parametrization of AdS_{d+1} (with radius 1)

$$ds^2_{d+1} = \frac{(1+\frac{1}{4}\mathrm{x}^2)^2}{(1-\frac{1}{4}\mathrm{x}^2)^2} ds^2_{p+1} + \frac{d\mathrm{x}^i d\mathrm{x}^i}{(1-\frac{1}{4}\mathrm{x}^2)^2}\,, \qquad ds^2_{p+1} = \frac{1}{\mathrm{z}^2}(d\mathrm{z}^2 + d\mathrm{x}^v d\mathrm{x}^v)\,, \tag{2.7}$$

where the indices of the boundary coordinates of AdS_{d+1} are split into $v = 1, \cdots, p$ and $i = 1, \cdots, d-p$. The minimal surface ending on a p-plane at the boundary is

$$\mathrm{x}^v = x^v\,, \qquad \mathrm{z} = z\,, \qquad \mathrm{x}^i = 0\,, \qquad ds^2_{p+1} = \frac{1}{z^2}(dz^2 + dx^v dx^v) \equiv g_{\mu\nu}(x) dx^\mu dx^\nu\,, \tag{2.8}$$

so that the corresponding induced metric is AdS_{p+1}. Choosing a static gauge in the p-brane action in $\mathrm{AdS}_{d+1} \times S^n$ we get for the S_1 part of its action in (2.3)

$$S_1 = \mathrm{T}_p \int d^{p+1}x \sqrt{\det\left[\frac{(1+\frac{1}{4}\mathrm{x}^2)^2}{(1-\frac{1}{4}\mathrm{x}^2)^2} g_{\mu\nu} + \frac{\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i}{(1-\frac{1}{4}\mathrm{x}^2)^2} + \frac{\partial_\mu y^a \partial_\nu y^a}{(1+\frac{1}{4\mathrm{r}^2} y^2)^2}\right]} \equiv \mathrm{T}_p \int d^{p+1}x \sqrt{g}\, L\,, \tag{2.9}$$

where y^a are coordinates of S^n and r is its radius in units of the radius of AdS_{p+1} (which is absorbed into the dimensionless effective tension T_p).

[10] The existence of such static M2 brane solution is related to the fact that M2 brane intersecting with a stack of M5 branes over a plane is a 1/2-BPS configuration. This can be easily seen, e.g., from the absence of force on a static M2 brane in this case [37] (C_3 here is purely magnetic).

[11] In the notation of [38, 39] $2\pi\ell_p^3 = \ell_{11}^3$.

Expanding (2.9) in powers of the fluctuations x^i and y^a we get

$$\begin{aligned} L &= L_2 + L_{4\mathrm{x}} + L_{2\mathrm{x},2y} + L_{4y} + \dots\,, \\ L_2 &= \frac{1}{2}\big[g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i + (p+1)\,\mathrm{x}^i\mathrm{x}^i\big] + \frac{1}{2}g^{\mu\nu}\partial_\mu y^a\partial_\nu y^a\,, \qquad (2.10)\\ L_{4\mathrm{x}} &= \frac{1}{8}(g^{\mu\nu}\partial_\mu \mathrm{x}^i\partial_\nu \mathrm{x}^i)^2 - \frac{1}{4}(g^{\mu\nu}\partial_\mu \mathrm{x}^i\partial_\nu \mathrm{x}^j)\,(g^{\rho\kappa}\partial_\rho \mathrm{x}^i\partial_\kappa \mathrm{x}^j) \\ &\quad + \frac{1}{4}p\,\mathrm{x}^i\mathrm{x}^i\,g^{\mu\nu}\partial_\mu \mathrm{x}^j\partial_\nu \mathrm{x}^j + \frac{1}{8}(p+1)^2\mathrm{x}^i\mathrm{x}^i\,\mathrm{x}^j\mathrm{x}^j\,, \qquad (2.11)\\ L_{2\mathrm{x},2y} &= \frac{1}{4}(g^{\mu\nu}\partial_\mu \mathrm{x}^i\partial_\nu \mathrm{x}^i)\,(g^{\rho\kappa}\partial_\rho y^a\partial_\kappa y^a) - \frac{1}{2}(g^{\mu\nu}\partial_\mu \mathrm{x}^i\partial_\nu y^a)\,(g^{\rho\kappa}\partial_\rho \mathrm{x}^i\partial_\kappa y^a) \\ &\quad + \frac{1}{4}(p-1)\mathrm{x}^i\mathrm{x}^i\,g^{\mu\nu}\partial_\mu y^a\partial_\nu y^a\,, \qquad (2.12)\\ L_{4y} &= -\frac{1}{4\mathrm{r}^2}y^b y^b\;g^{\mu\nu}\partial_\mu y^a\partial_\nu y^a + \frac{1}{8}(g^{\mu\nu}\partial_\mu y^a\partial_\nu y^a)^2 \\ &\quad - \frac{1}{4}(g^{\mu\nu}\partial_\mu y^a\partial_\nu y^b)\,(g^{\rho\kappa}\partial_\rho y^a\partial_\kappa y^b)\,. \qquad (2.13) \end{aligned}$$

The string in $\mathrm{AdS}_5\times S^5$ case considered in [5] corresponds to $p=1,\ d=4,\ n=5,\ \mathrm{r}=1$ while in the present M2-brane in $\mathrm{AdS}_7\times S^4$ case we have $p=2,\ d=6,\ n=4,\ \mathrm{r}=\frac{1}{2}$. In the latter case we get 4 transverse AdS_7 fluctuation fields x^i having $m^2=3$ and 4 massless S^4 fields y^a propagating in induced AdS_3 geometry. One may also include the fermionic terms coming from the corresponding $\mathrm{AdS}_7\times S^4$ supermembrane action as discussed in [16] getting (after fixing κ-symmetry gauge) eight 3d fermions with $m=3/2$.

To find the explicit expression for the WZ term (2.4) in the M2-brane action let us note that the normalized volume form Ω_4 of a unit-radius S^4 in (2.2) may be expressed in terms of a unit 5-vector Y^A as[12]

$$\Omega_4 = \frac{1}{64\pi^2}\epsilon_{ABCDE}Y^A dY^B\wedge dY^C\wedge dY^D\wedge dY^E\,, \qquad Y^AY^A=1\,, \quad A=1,\cdots,5\,. \qquad (2.14)$$

Using the expression (2.6) for the effective M2-brane tension the WZ term in (2.4) takes the form

$$S_2 = -iT_2\int C_3 = -iT_2\int F_4 = -\frac{iN}{32\pi}\int d^4x\;\epsilon_{ABCDE}\,\epsilon^{\mu\nu\lambda\rho}\,Y^A\partial_\mu Y^B\partial_\nu Y^C\partial_\lambda Y^D\partial_\rho Y^E\,. \qquad (2.15)$$

Like in the case of a similar WZ term in the D3-brane case [18] a manifestly SO(5) invariant form of the WZ term is non-local — given by an integral over a 4-surface that has the world-volume as its boundary. The normalization of (2.15) is checked by observing that if the 4-surface is S^4 the integral in (2.15) becomes $-2\pi iN$, i.e. $e^{-S_2}=1$.[13]

[12] In general, one has $\int_{S^d}\Omega_d = 1$, $\Omega_d = \frac{1}{\mathrm{vol}(S^d)\,d!}\epsilon_{d+1}Y(\wedge dY)^d$, $\mathrm{vol}(S^d)=\frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})}$. Ω_d appears in the expression for the Hopf index of the map $S^d\to S^d$. The associated topological current is $J^\lambda = \frac{1}{\mathrm{vol}(S^d)\,d!}\epsilon^{\lambda\mu_1\cdots\mu_d}\epsilon_{AB_1\cdots B_d}Y^A\partial_{\mu_1}Y^{B_1}\cdots\partial_{\mu_d}Y^{B_d}$, $\int_{S^d}J^0 = N$=integer.

[13] Being topological this WZ term should not be renormalized and should be derivable as in [18, 40, 41] from the 1-loop fermionic determinant in the dual 6d theory in the presence of a defect represented by the surface operator.

Setting

$$Y^5 = \frac{1 - \frac{1}{4\mathrm{r}^2} y^2}{1 + \frac{1}{4\mathrm{r}^2} y^2}\,, \qquad Y^a = \frac{\frac{1}{\mathrm{r}} y^a}{1 + \frac{1}{4\mathrm{r}^2} y^2}\,, \qquad Y^A Y^A = 1\,, \tag{2.16}$$

where we rescaled y^a by r to conform with (2.9), we find that the expansion of (2.15) in powers of y^a starts with the y^4 term (as $\mathrm{r} = \frac{1}{2}$ we have $32\pi \mathrm{r}^4 = 2\pi$)

$$S_2 = -\frac{iN}{2\pi} \int d^3x\, \epsilon^{\mu\nu\lambda}\, \epsilon_{abcd}\, y^a \partial_\mu y^b \partial_\nu y^c \partial_\lambda y^d + \mathcal{O}(y^5)\,. \tag{2.17}$$

The explicit normalization of the kinetic term for y^a in (2.9) is (using (2.6))

$$S_1 = \frac{N}{\pi} \int d^3x\, \sqrt{g}\, g^{\mu\nu} \partial_\mu y^a \partial_\nu y^a + \dots\,. \tag{2.18}$$

3 One-loop partition function: defect conformal anomaly

In this section we calculate the fluctuation determinants about the AdS_3 classical M2-brane solution. The more complicated problem of deriving the 1-loop quadratic fluctuation for 2 parallel planes was discussed in [16]. The fluctuation spectrum presented in the preceding section indeed matches their spectrum in the limit of large separation. The discussion is parallel to the one in the string case in [3, 42].

To recall, our spectrum has 4 bosons with $m^2 = 3$ plus 4 bosons with $m = 0$ and 8 fermions with $m = 3/2$.[14] The resulting partition function Z is then given by

$$\mathrm{F}_{1-\mathrm{loop}} = -\log Z = \frac{1}{2}\big[4 \log\det(-\nabla^2 + 3) + 4 \log\det(-\nabla^2) - 8 \log\det \Delta_{1/2}\big]\,. \tag{3.1}$$

To evaluate the determinants we may follow the approach used in the $\mathrm{AdS}_5 \times S^5$ string case in [3, 43], i.e. use the results for heat kernels of operators in AdS space from [44].

Cubic UV divergence cancels out due to the equal number of bosons and fermions. The linear divergences of $\log\det(-\nabla^2 + X)$ in 3d is proportional to $b_2 = \mathrm{tr}(\frac{1}{6} R^{(3)} - X)$ so here $b_{2\,\mathrm{tot}} = -6 = R^{(3)}$ and thus, as in string case, is proportional to the Euler number (assuming boundary terms are taken into account, cf. footnote 30 in [3]). In any case, such divergences are absent in an analytic regularization like ζ-function one and may be ignored.

The meaningful part of (3.1) is its logarithmically divergent piece, where we can rely on the results in section 3 of [45] (see also [46]), which studied the determinant of higher

[14] The bosonic and fermionic operator are essentially universal for the straight line or two parallel line configurations: what changes is just the induced geometry. The same was in the case of a string in $\mathrm{AdS}_5 \times S^5$: there one had [3, 42]: 2 bosons with $m^2 = 2/a^2$; 1 boson with $m^2 = 4/a^2 + R^{(2)}$; 5 massless S^5 bosons; 8 fermions with $m = 1/a$ or squared operator $-\nabla^2 + \frac{1}{4} R^{(2)} + 1/a^2$. In the straight-line or circular line surface the induced geometry was AdS_2 so $R^{(2)} = -2/a^2$. It is remarkable that the structure of the partition function in the string and M2 cases is very similar. This has to do, in particular, with the universal form of the Nambu-type term in the p-brane action and also the fact that in a natural κ-symmetry gauge the fermionic kinetic term comes from the supergravity covariant derivative projected to the world volume that contains the F-flux term that gets contribution from the sphere magnetic part that is not sensitive to the details of surface in the AdS space.

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spin theory on AdS_3. For a scalar of mass m ($\Delta(\Delta-2)=m^2$), the contribution to F is[15]

$$\mathrm{F}_0^{(\Delta)} = \frac{1}{2}\log\det(-\nabla^2+m^2) = -\frac{1}{12\pi}(\Delta-1)^3\,\mathrm{vol}(\mathrm{AdS}_3)\,. \tag{3.2}$$

For spin 1/2 fermion with $\Delta = p/2+m = 1+m$ we get

$$\mathrm{F}_{1/2}^{(\Delta)} = \frac{1}{2}\log\det\left(-\nabla^2+\frac{1}{4}R+m^2\right) = -\frac{1}{12\pi}(\Delta-1)\left[\Delta(\Delta-2)+\frac{1}{4}\right]\mathrm{vol}(\mathrm{AdS}_3)\,. \tag{3.3}$$

Introducing R as the AdS_3 IR cutoff regularizing the AdS_3 volume (e.g. the radius of the boundary S^2) we have (for the unit-radius AdS_3)[16]

$$\mathrm{vol}(\mathrm{AdS}_3) = -2\pi\log\mathrm{R}\,. \tag{3.4}$$

As a result, we get for (3.1)

$$\mathrm{F}_{1-\mathrm{loop}} = 4\mathrm{F}_0^{(\Delta=3)} + 4\mathrm{F}_0^{(\Delta=2)} - 8\mathrm{F}_{1/2}^{(\Delta=5/2)} = 3\log\mathrm{R}\,. \tag{3.5}$$

3.1 Interpretation of the result

Equation (3.5) is the 1-loop correction to the tree level contribution given by the value of the M2-brane action which is just the M2-brane tension (2.6) times the regularized volume of the induced AdS_3 metric (cf. [2])

$$\mathrm{F}_{\mathrm{tree}} = \mathrm{T}_2\mathrm{vol}(\mathrm{AdS}_3) = -2\pi\mathrm{T}_2\log\mathrm{R}\,. \tag{3.6}$$

The coefficient of $\log\mathrm{R}$ in equation (3.6) and in (3.5) has the interpretation of a conformal anomaly coefficient in the defect CFT. Surface operators have three anomaly coefficients,[17] each multiplying a particular conformally invariant integral on the surface related to its topology, extrinsic curvature and background Weyl tensor (see [49] for details). Since our calculation is focused on the single surface geometry of the sphere (the plane has trivial anomaly), our result captures one particular combination of the anomaly coefficients which we denote as b. Thus F can be expressed as

$$\mathrm{F} \equiv -\frac{1}{3}b\,\log\mathrm{R}\,, \qquad b = -3(-2\pi\mathrm{T}_2+3)+\ldots = 12N-9+\ldots\,, \tag{3.7}$$

where dots stand for possible higher-loop $1/\mathrm{T}_2 \sim 1/N$ terms.

The leading order at large N of the other two anomaly coefficients were calculated holographically by Graham and Witten [50] by considering M2-branes ending on arbitrary surfaces. They can also be inferred in other ways: the coefficient related to extrinsic

[15]The general formulas for the AdS_{d+1} spectral density for bosonic totally symmetric rank-s and fermionic in the $[s,1/2,\cdots 1/2]$ representation in general boundary dimension d (see [44]) are presented in [46] (see eqs. (3.20) and (3.22)). For $d=2$, they happen to coincide as a function of s.

[16]In general, the regularized volume of global AdS_{p+1} space with S^p as its boundary for even p is, discarding power-law divergences (see, e.g., [47]): $\mathrm{vol}(\mathrm{AdS}_{p+1}) = \frac{2(-\pi)^{p/2}}{\Gamma\left(1+\frac{p}{2}\right)}\log\mathrm{R}\,.$

[17]A fourth can be defined for nontrivial coupling to the scalar fields, see [8]. By adjusting the scalar coupling, the total anomaly of some BPS surface operators, different from the sphere studied here, vanishes [48].

curvature is proportional to the normalization of the displacement operator, as mentioned in section 4 below eq. (4.6). The remaining one was conjectured to also be fixed by the same normalization constant in theories with enough supersymmetry [51] (based on [52, 53]) as is indeed verified in [54].

Going back to our expression for the coefficient b in (3.7), we observe that it happens to be consistent with the result for the corresponding anomaly coefficient found in [27, 28] from the entanglement entropy for the "bubbling" M5-M2 geometry with M2-branes corresponding to a 1/2-BPS surface defect operator in (2,0) theory in a $su(N)$ representation with the Young tableau with a large number of boxes.[18] In the notation of [27] we have

$$b = 24(\rho, \lambda) + 3(\lambda, \lambda)\,, \tag{3.8}$$

where ρ is the Weyl vector of $su(N)$ and λ is the highest weight of a particular $su(N)$ representation. If we formally assume that this expression should be valid not just for large representations but also for the ones with finite number of boxes then in the present case of a single M2-brane corresponding to the surface operator in the fundamental representation one finds $(\rho, \lambda) = \frac{N-1}{2}$, $(\lambda, \lambda) = \frac{N-1}{N}$ and thus

$$b = 3(N-1)(4 + N^{-1}) = 12N - 9 - 3N^{-1}\,. \tag{3.9}$$

Remarkably, this is in agreement with (3.7) and suggests that the perturbative expansion in (3.7) may terminate after the 2-loop $\frac{1}{N}$ term.[19] It would be very interesting to compute this term directly from the 2-loop supermembrane Witten diagrams in AdS_3.[20]

3.2 Non-supersymmetric surface defect and 2d RG flow

The above discussion applied to the case of the dual description of the 1/2-BPS surface operator which (at least in the abelian case) should be represented by an analog of the Wilson-Maldacena exponent (1.1) with one of the 5 scalars of the (2,0) tensor multiplet coupled to the induced metric as in [8, 10]. This breaks SO(5) R-symmetry to SO(4) and corresponds, in the M2-theory in $AdS_7 \times S^4$, to an expansion near a point of S^4 with 4 S^4 massless scalars subject to the Dirichlet boundary condition in AdS_3.

By analogy with the Wilson loop case [29–32] we may also consider the dual description of non-supersymmetric surface operator without scalar coupling [11] preserving SO(5) symmetry. In this case the classical M2-brane minimal surface is the same AdS_3 but one is to impose the Neumann boundary condition on S^4 fluctuations (and average over an expansion point in the sphere) to preserve the SO(5) symmetry.

[18] An exact expression for another anomaly coefficient is derived in [55] from the computation of the associated superconformal index.

[19] If one assumes that the series in (3.7) terminates at $1/N$ order then the coefficient of this term can be of course fixed by requiring that the full expression should vanish for $N = 1$.

[20] For comparison, let us recall the expressions for the conformal anomaly coefficients of the "bulk" theory — the $(2,0)$ theory describing N coincident M5 branes (see, e.g., [56] for a review): $a = -\frac{1}{4\times 288}\,(16N^3 - 9\,N - 7) = -\frac{1}{288}\,(N-1)\big[(2N+1)^2 + \frac{3}{4}\big]$, $c = -\frac{1}{288}\,(4N^3 - 3N - 1) = -\frac{1}{288}\,(N-1)(2N+1)^2$. The leading N^3 terms follow [57] from the classical supergravity action, the order N terms originate from the R^4 corrections in 11d action [39] and order N^0 terms are from the 1-loop 11d supergravity corrections [58, 59]. The exact expressions follow also from non-perturbative approaches based on supersymmetry constraints [60, 61].

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As in the case of the surface for the circular Wilson loop in $AdS_5 \times S^5$ the 1-loop contributions of the 4 massive AdS_7 scalars and 8 fermions in (3.1) remain the same as in (3.5) while to find the contribution of the 4 massless S^4 fluctuations with alternative b.c. (i.e. with $\Delta = \Delta_- = 0 + \mathcal{O}(\frac{1}{N})$) we may use, e.g., the relation [47, 62, 63] between the AdS_{d+1} bulk field and S^d boundary conformal field partition functions: $Z^{(\Delta_-)}/Z^{(\Delta_+)} = Z_{\rm conf}$. For a scalar in AdS_{d+1} one has $\Delta_\pm = \frac{d}{2} \pm \nu, \quad \nu = \sqrt{\frac{d^2}{4} + m^2}$ so that the boundary conformal (source) field with canonical dimension $\Delta_- = d - \Delta_+$ has the kinetic term $\int d^d x\, \varphi(-\nabla^2)^\nu \varphi$.

In the present case $d = 2, \ \Delta_+ = 2, \ \Delta_- = 0, \ \nu = 1$ so that the induced boundary CFT_2 has the standard kinetic operator $-\nabla^2$ on S^2, i.e. the difference of the scalar free energies is

$$F_0^{(\Delta_-)} - F_0^{(\Delta_+)} = \frac{1}{2} \log \det{}'(-\nabla^2) = -\frac{1}{3} \log R + \dots \, . \tag{3.10}$$

where R plays the role of a UV cutoff in 2d. The positivity of the difference is in agreement with the expected "defect b-theorem" [33, 34], viewing, by analogy with the circular Wilson loop case [30, 31], the non-BPS surface operator as the UV limit deformed by the relevant operator $Y_5 \sim \Phi_5$ to flow to the BPS surface operator, i.e. $b_{\rm UV} - b_{\rm IR} = +1 > 0$.

As a result, taking into account the multiplicity 4 of S^4 scalars in (3.1), we conclude that in the non-supersymmetric (non-BPS) defect case we should get instead of the $b = b_{\rm susy}$ in (3.7)[21]

$$b_{\rm non-susy} = b_{\rm susy} + 4 = 12N - 5 + \dots \, . \tag{3.11}$$

One may attempt to understand the RG flow between the non-supersymmetric and supersymmetric cases by using the same approach as in the string theory description of the circular Wilson loop [30, 31]. Starting with the (super)membrane action[22] (2.3), (2.4) one may perturb it by a 2d boundary term (here we concentrate only on the S^4 fluctuations, see (2.16), (2.18))

$$\begin{aligned} S_1 &= \frac{1}{2} T_2 \int d^3x \, (\sqrt{g}\, g^{\mu\nu} \partial_\mu y^a \partial_\nu y^a + \dots) - \kappa\, T_2 \int d^2\vec{x}\, \sqrt{g_2}\, Y_5 \, , \\ Y_5 &= \sqrt{1 - Y_a Y_a} = 1 - 2 y_a y_a + \dots \, . \end{aligned} \tag{3.12}$$

Here $\sqrt{g_2} = \frac{1}{z^2}\big|_{z\to 0}$ and κ is a new coupling which will run between the UV and IR fixed points (see section 4.2 in [31] for details). κY_5 term should correspond to a similar scalar Φ_5 term in the exponent in the surface operator (cf. (1.1)) with coefficient running between 0 and 1. The variation of (3.12) implies that y^a should satisfy the massless wave equation in AdS_3 with the metric $ds^2 = \frac{1}{z^2}(dz^2 + d\vec{x} d\vec{x})$ subject to the Robin boundary condition $(\partial_n = n^\mu \partial_\mu, \quad n^\mu = (-z, 0, 0))$

$$(\partial_n - 4\kappa) y^a \Big|_{z\to 0} = 0 \, , \qquad \partial_n = -z \partial_z \, . \tag{3.13}$$

[21] Compared to the non-supersymmetric circular Wilson loop case in [31] here the S^4 zero mode contribution $\sim 4 \log N$ appears only in the finite part of F, i.e. is not relevant for the 1-loop conformal anomaly.

[22] One may expect that the standard first-derivative supermembrane action in $AdS_7 \times S^4$ is not renormalized (i.e. tension is not renormalized): it contains fermionic and bosonic WZ terms that can not be renormalized, and they are related by κ-symmetry to the rest of the terms (this is analogous, e.g., to non-renormalization of 11d supergravity action). Loop corrections may induce higher-derivative terms but presumably they should not be relevant for the discussion below.

The parameter $0 \le \kappa \le \infty$ thus interpolates between the Neumann ($\kappa = 0$) and Dirichlet ($\kappa \to \infty$) boundary conditions corresponding to $y^a = z^{\Delta_-} v^a + \mathcal{O}(z^2) = v^a + \mathcal{O}(z^2)$ and $y^a = z^{\Delta_+} u^a + \mathcal{O}(z^2) = z^2 \, u^a + \mathcal{O}(z^2)$ respectively. κ will be running with UV scale Λ of the 3d theory. Integrating y^a out we get at leading 1-loop order the following boundary divergence (ignoring power divergent terms)

$$\Gamma_\infty = 4 \times \frac{1}{2} \log\det(-\nabla^2)\Big|_\infty = -4A_3 \log\Lambda + \dots\,, \qquad A_3 = \frac{2}{\pi}\int d^2\vec{x}\,\sqrt{g_2}\,\kappa^2 + \dots\,, \tag{3.14}$$

where A_3 is the relevant Seeley coefficient (see, e.g., eq. (5.32) in [64] where the Robin parameter S is equal to 4κ here). Adding (3.14) to the bare action (3.12) and taking into account that κ has canonical dimension 2 we conclude that the renormalized κ should run according to (cf. [31])

$$\beta_\kappa = \mu\frac{d\kappa}{d\mu} = -2\kappa - \frac{8}{\pi \mathrm{T}_2}\kappa^2 + \dots = -2\kappa - \frac{4}{N}\kappa^2 + \dots\,. \tag{3.15}$$

Here we used (2.6). As a result, we get the UV fixed point at $\kappa = 0$ and also a possible IR fixed point at $\kappa = -\frac{1}{2}N$. Assuming the latter can be trusted in the large N expansion it should represent the Dirichlet limit of the Robin b.c. (3.13). Since the derivative of the β-function gives anomalous dimension at a fixed point the total dimension $(2 + \beta')$ of the perturbing operator $Y_5 = 1 - 2y^a y^a + \dots$ in (3.12) is then

$$\begin{aligned} \kappa_{\rm UV} = 0: &\qquad \Delta_{\rm UV} = 2 - 2 + \mathcal{O}\Big(\frac{1}{N}\Big) = \mathcal{O}\Big(\frac{1}{N}\Big)\,; \\ \kappa_{\rm IR} = -\frac{1}{2}N: &\qquad \Delta_{\rm IR} = 2 + 2 + \mathcal{O}\Big(\frac{1}{N}\Big) = 4 + \mathcal{O}\Big(\frac{1}{N}\Big)\,. \end{aligned} \tag{3.16}$$

Since y^a corresponds to $\Delta = 2$, the value of $\Delta_+ = 4 + \dots$ is consistent with the leading-order dimension of the composite $y^a y^a$ operator. To go beyond the leading order one would need to include higher order terms in the 3d action (2.13).

4 Defect 4-point correlation functions at large N from M2-brane action

Here we follow the same strategy as in the case of the correlators on the BPS Wilson line in [5] and compute the tree-level (large N) 4-point functions of the bosonic fields $X^I = (\mathrm{x}^i, y^a)$ (representing the displacement operator and the 4 scalars other than the one coupled to the surface operator in the 6d theory) directly from the M2-brane action (2.9)–(2.13).

Let us first discuss the normalization of the two-point functions. Given a scalar action

$$S_0 = \frac{\mathrm{T}_p}{2}\int d^{p+1}x\,\sqrt{g}(\partial^\mu X \partial_\mu X + m^2 X^2)\,, \tag{4.1}$$

in AdS_{p+1} with the metric $ds^2 = \frac{1}{z^2}(dz^2 + d\vec{x}d\vec{x})$ (cf. (2.8)) the bulk-to-boundary propagator will be normalized as in [65] (here $x = (z, \vec{x})$), i.e.[23]

$$G^\Delta_{B\partial}(x, \vec{x}') = C_\Delta\left[\frac{z}{z^2 + (\vec{x} - \vec{x}')^2}\right]^\Delta, \qquad C_\Delta = \frac{\Gamma(\Delta)}{\pi^{\frac{p}{2}}\Gamma(\Delta - \frac{p}{2})}\,, \qquad m^2 = \Delta(\Delta - p)\,. \tag{4.2}$$

[23] Note that in [5, 32] a different normalization was used: $C_\Delta \to \mathcal{C}_\Delta = \frac{\Gamma(\Delta)}{2\pi^{\frac{p}{2}}\Gamma(\Delta - \frac{p}{2} + 1)}$.

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and the two-point function of the corresponding boundary operator $\mathcal{O}(\vec{x})$ will be defined as

$$\langle\!\langle \mathcal{O}(\vec{x}_1)\,\mathcal{O}(\vec{x}_2)\rangle\!\rangle \equiv \langle X(\vec{x}_1)\,X(\vec{x}_2)\rangle = \frac{\mathrm{C}_X}{|\vec{x}_{12}|^{2\Delta}}\,, \qquad \mathrm{C}_X = \mathrm{T}_p\,(2\Delta - p)\,C_\Delta\,. \tag{4.3}$$

In the case of the scalars x^i and y^a in (2.10) with masses $m_{\mathrm{x}}^2 = p+1$ and $m^2 = 0$ corresponding to $\Delta_{\mathrm{x}} = p+1$ and $\Delta_y = p$ respectively we thus find

$$\begin{aligned}
\langle \mathrm{x}^i(\vec{x}_1)\,\mathrm{x}^j(\vec{x}_2)\rangle &= \delta^{ij}\frac{\mathrm{C}_{\mathrm{x}}}{|\vec{x}_{12}|^{2p+2}}\,, \qquad & \mathrm{C}_{\mathrm{x}} &= 2\left(1+\frac{2}{p}\right)\mathrm{C}_y\,,\\
\langle y^a(\vec{x}_1)\,y^b(\vec{x}_2)\rangle &= \delta^{ab}\frac{\mathrm{C}_y}{|\vec{x}_{12}|^{2p}}\,, \qquad & \mathrm{C}_y &= \mathrm{T}_p\frac{\Gamma(p+1)}{\pi^{\frac{p}{2}}\Gamma(\frac{p}{2})}\,.
\end{aligned} \tag{4.4}$$

In particular, in the case of the string in $\mathrm{AdS}_5\times S^5$ ($\mathrm{T}_1 = \frac{\sqrt{\lambda}}{2\pi}$) we find $\mathrm{C}_{\mathrm{x}} = 6\mathrm{C}_y$, $\mathrm{C}_y = \frac{\sqrt{\lambda}}{2\pi^2}$ (in agreement with direct identification of x^i with displacement operator and y^a with "transverse" scalars and strong-coupling limit of exact prediction in [66, 67]). In the present case of $p=2$, $\mathrm{T}_2 = \frac{2N}{\pi}$ we find[24]

$$\mathrm{C}_{\mathrm{x}} = \frac{16N}{\pi^2} = 4\mathrm{C}_y\,, \qquad \mathrm{C}_y = \frac{4N}{\pi^2}\,. \tag{4.5}$$

Since the x^i fluctuations are dual to the displacement operator D^i, from the above results we can read off the normalization C_{D} of the D^i two-point function on the surface defect in the (2,0) theory

$$\langle\!\langle \mathrm{D}^i(\vec{x}_1)\mathrm{D}^j(\vec{x}_2)\rangle\!\rangle = \delta^{ij}\frac{\mathrm{C}_{\mathrm{D}}}{\vec{x}_{12}^{\,6}}\,, \qquad \mathrm{C}_{\mathrm{D}} = \frac{16N}{\pi^2} + \mathcal{O}(N^0)\,. \tag{4.6}$$

The normalization constant C_{D} also determines the anomaly coefficient associated to extrinsic curvature of the surface [68].

4.1 The $\langle yyyy\rangle$ correlator

Let us first compute the 4-point boundary correlator of the four S^4 fluctuations

$$G(\vec{x}_i, \mathrm{t}_i) = \langle y(\vec{x}_1;\mathrm{t}_1)\,y(\vec{x}_2;\mathrm{t}_2)\,y(\vec{x}_3;\mathrm{t}_3)\,y(\vec{x}_4;\mathrm{t}_4)\rangle\,, \tag{4.7}$$

where we have multiplied each $y^a(\vec{x}_i)$ with an auxiliary constant 4-vector t_i^a to remove the SO(4) R-symmetry indices

$$y(\vec{x};\mathrm{t}) \equiv \mathrm{t}^a\,y^a(\vec{x})\,. \tag{4.8}$$

Here $\vec{x}$ stands for 2 boundary coordinates of AdS_3. It is convenient to extract a kinematic factor so that the correlation function can be expressed in terms of the cross ratios

$$G = \frac{\mathrm{t}_{12}\mathrm{t}_{34}}{\vec{x}_{12}^{\,4}\vec{x}_{34}^{\,4}}\,\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\,, \tag{4.9}$$

[24] Note that here we start with (2.10), i.e. assume that the scalars y^a are normalized in the same way as x^i (after the rescaling y^a by $\mathrm{r} = \frac{1}{2}$, cf. (2.1)). If the corresponding scalar operator on the defect is identified with unrescaled y^a we get $\mathrm{C}_y \to \mathrm{r}^2\mathrm{C}_y = \frac{N}{2\pi^2}$ while C_{x} is the same. See also [54].

where $\mathrm{t}_{ij} \equiv \mathrm{t}_i \cdot \mathrm{t}_j$, and

$$\begin{aligned} U &= \frac{\vec{x}_{12}^2 \vec{x}_{34}^2}{\vec{x}_{13}^2 \vec{x}_{24}^2} = \chi\bar{\chi} \ , \quad V = \frac{\vec{x}_{14}^2 \vec{x}_{23}^2}{\vec{x}_{13}^2 \vec{x}_{24}^2} = (1-\chi)(1-\bar{\chi}) \ , \\ \sigma &= \frac{\mathrm{t}_{13}\mathrm{t}_{24}}{\mathrm{t}_{12}\mathrm{t}_{34}} = \alpha\bar{\alpha} \ , \qquad \tau = \frac{\mathrm{t}_{14}\mathrm{t}_{23}}{\mathrm{t}_{12}\mathrm{t}_{34}} = (1-\alpha)(1-\bar{\alpha}) \ . \end{aligned} \tag{4.10}$$

The holographic computation of the correlator leads to a $1/N$ expansion

$$\mathcal{G} = \mathcal{G}_{\text{disc}} + \mathcal{G}_{\text{tree}} + \dots \ , \tag{4.11}$$

where the leading order contribution is given by the disconnected diagrams

$$\mathcal{G}_{\text{disc}} = \frac{16N^2}{\pi^4}\left(1 + \sigma U^2 + \tau\frac{U^2}{V^2}\right) . \tag{4.12}$$

Note that this scales as $(\mathrm{C}_y)^2$ in agreement with (4.4), (4.5). The order N tree-level contributions to the 4-point function can be divided into two parts: $\mathcal{G}_1$ from the Dirac-Nambu type action (2.9), (2.13) which is parity-even, and $\mathcal{G}_2$ from WZ type action (2.15), (2.17) which is parity odd

$$\mathcal{G}_{\text{tree}} = \mathcal{G}_1 + \mathcal{G}_2 \ . \tag{4.13}$$

The first contribution can be straightforwardly computed from the action, and after some simplification reads (after using (2.6))

$$\begin{aligned} \mathcal{G}_1 = -\frac{96N}{\pi^4}\, U^2 \bigg(& \Big[(U-1-V)\bar{D}_{3333} - U\bar{D}_{3322} + \bar{D}_{2222}\Big] \\ & + \sigma\Big[(1-U-V)\bar{D}_{3333} - \bar{D}_{3232} + \bar{D}_{2222}\Big] \\ & + \tau\Big[(V-1-U)\bar{D}_{3333} - \bar{D}_{3223} + \bar{D}_{2222}\Big]\bigg) \ . \end{aligned} \tag{4.14}$$

Here $\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}$ are functions of cross-ratios, and are related to the D-functions defined by

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(\vec{x}_i) = \int \frac{dz\, d^p\vec{x}}{z^{p+1}} \prod_{i=1}^{4} \left(\frac{z}{z^2 + (\vec{x}-\vec{x}_i)^2}\right)^{\Delta_i} \ , \tag{4.15}$$

via

$$\frac{2\prod_{i=1}^4 \Gamma(\Delta_i)}{\pi^{\frac{d}{2}}\,\Gamma(\Sigma - \frac{1}{2}d)} D_{\Delta_1\Delta_2\Delta_3\Delta_4} = \frac{(\vec{x}_{14}^2)^{\Sigma-\Delta_1-\Delta_4}(\vec{x}_{34}^2)^{\Sigma-\Delta_3-\Delta_4}}{(\vec{x}_{13}^2)^{\Sigma-\Delta_4}(\vec{x}_{24}^2)^{\Delta_2}} \bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4} \ , \quad \Sigma = \frac{1}{2}\sum_{i=1}^4 \Delta_i \ . \tag{4.16}$$

The three-derivative contact Witten diagram corresponding to the WZ vertex (2.17) has already been computed in [69], and with the normalization of the WZ term in (2.17) we have

$$\mathcal{G}_2 = -\frac{72N}{\pi^4}\, U^2(\chi-\bar{\chi})(\alpha-\bar{\alpha})\bar{D}_{3333} \ . \tag{4.17}$$

Note that the combination (4.13) of the two contributions (4.14) and (4.17) is very special, in that the resulting correlator satisfies differential relations which resemble superconformal Ward identities (see, e.g., [70] for examples in SCFT$_d$ with $d > 2$)

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\alpha=1/\chi} = 0\,, \qquad \left(-\frac{1}{2}\bar{\chi}\partial_{\bar{\chi}} + \bar{\alpha}\partial_{\bar{\alpha}}\right)\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha})\Big|_{\bar{\alpha}=1/\bar{\chi}} = 0\,. \tag{4.18}$$

Here it is understood that we first act with the differential operators on the correlator, and then set the R-symmetry cross ratios to specific values. One can easily check that the disconnected correlator $\mathcal{G}_{\rm disc}$ in (4.12) also satisfies the identities above. We will show in section 5 below that (4.18) are indeed Ward identities following from the superconformal symmetry $[\mathrm{OSp}(4^*|2)]^2$.

We note that the same differential identities are satisfied by the correlators on the 1/2-BPS Wilson loop in the string theory case (see appendix B). The major difference, however, is that in the 1d Wilson loop case there is only a single conformal cross ratio χ, and both α and $\bar{\alpha}$ can be twisted with respect to χ. Here the parity odd term (4.17) has broken the symmetry of interchanging χ and $\bar{\chi}$ in the correlator: only the simultaneous interchange of $\chi \leftrightarrow \bar{\chi}$, $\alpha \leftrightarrow \bar{\alpha}$ remains a symmetry of the 4-point function. Therefore, we have two differential identities which separately pair up χ with α and $\bar{\chi}$ with $\bar{\alpha}$, and the chirality of these relations parallels the factorized structure of the superconformal group $\mathrm{OSp}(4^*|2) \times \mathrm{OSp}(4^*|2)$. The same structure of the superconformal Ward identities was observed in [69] for $\mathrm{PSU}(1,1|2) \times \mathrm{PSU}(1,1|2)$.

4.2 The ⟨xxxx⟩ and ⟨xxyy⟩ correlators and Mellin representation

The calculation for the ⟨xxxx⟩ and ⟨xxyy⟩ correlators is almost identical to that of the parity-even part of the ⟨$yyyy$⟩ correlator as they are not affected by the WZ term at tree level, and only the Dirac-Nambu term (2.9) contributes. We first present the results for generic defect dimension p and then specify to $p = 2$. For the string (Wilson loop) case of $p = 1$ the expressions below agree with the results of [5].

Using the action in (2.9)–(2.13) we find for the 4-point function of the transverse AdS$_{d+1}$ fluctuations

$$\begin{aligned}
\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)\mathrm{x}^k(\vec{x}_3)\mathrm{x}^l(\vec{x}_4)\rangle = -\frac{\mathrm{T}_p(1+p)^2\Gamma^4(1+p)}{\pi^{2p}\Gamma^4(1+\frac{p}{2})}\Big(\delta^{ij}\delta^{kl}\Big[(2+p)(4+5p)D_{p+1,p+1,p+1,p+1}\\
-4(2+p)(1+2p)\vec{x}_{34}^2D_{p+1,p+1,p+2,p+2}-4(1+p)^2(\vec{x}_{14}^2\vec{x}_{23}^2+\vec{x}_{13}^2\vec{x}_{24}^2-\vec{x}_{12}^2\vec{x}_{34}^2)D_{p+2,p+2,p+2,p+2}\Big]\\
+\delta^{ik}\delta^{jl}\Big[(2+p)(4+5p)D_{p+1,p+1,p+1,p+1}-4(2+p)(1+2p)\vec{x}_{24}^2D_{p+1,p+2,p+1,p+2}\\
-4(1+p)^2(\vec{x}_{14}^2\vec{x}_{23}^2+\vec{x}_{12}^2\vec{x}_{34}^2-\vec{x}_{13}^2\vec{x}_{24}^2)D_{p+2,p+2,p+2,p+2}\Big]\\
+\delta^{il}\delta^{jk}\Big[(2+p)(4+5p)D_{p+1,p+1,p+1,p+1}-4(2+p)(1+2p)\vec{x}_{23}^2D_{p+1,p+2,p+2,p+1}\\
-4(1+p)^2(\vec{x}_{12}^2\vec{x}_{34}^2+\vec{x}_{13}^2\vec{x}_{24}^2-\vec{x}_{14}^2\vec{x}_{23}^2)D_{p+2,p+2,p+2,p+2}\Big]\Big)\,,
\end{aligned} \tag{4.19}$$

where we have already used the relation $\Delta_{\mathrm{x}} = p+1$, and the D-function identities (summarized in, e.g., appendix D of [71]) to simplify the expression. For our present case,

$p = 2$, $\mathrm{T}_2 = \frac{2N}{\pi}$, this correlator may be written explicitly in terms of the $\bar{D}$-functions of cross ratios as (cf. (4.14))

$$\begin{aligned}
&\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)\mathrm{x}^k(\vec{x}_3)\mathrm{x}^l(\vec{x}_4)\rangle \\
&\quad = -\frac{182N}{\pi^4\vec{x}_{12}^6\vec{x}_{34}^6}U^3\Big(\delta^{ij}\delta^{kl}\Big[63\bar{D}_{3333} - 50\bar{D}_{3344} + 15(U-V-1)\bar{D}_{4444}\Big] \\
&\qquad\qquad + \delta^{ik}\delta^{jl}\Big[63\bar{D}_{3333} - 50\bar{D}_{3434} + 15(1-U-V)\bar{D}_{4444}\Big] \\
&\qquad\qquad + \delta^{il}\delta^{jk}\Big[63\bar{D}_{3333} - 50\bar{D}_{3443} + 15(V-1-U)\bar{D}_{4444}\Big]\Big)\ .
\end{aligned} \tag{4.20}$$

Similarly, the $\langle \mathrm{xx}yy\rangle$ correlator for generic p reads

$$\begin{aligned}
\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)y^a(\vec{x}_3)y^b(\vec{x}_4)\rangle &= -\frac{\mathrm{T}_p\, 4^{2p-1}\Gamma^4(\frac{1+p}{2})}{p^2\pi^{2p+2}}\delta^{ij}\delta^{ab}\Big(-(2+p+p^2)D_{1+p,1+p,p,p} \\
&-2p(3+p)\vec{x}_{34}^2D_{p+1,p+1,p+1,p+1} + 2(1+p)^2\Big[-\vec{x}_{12}^2D_{p+2,p+2,p,p} + \vec{x}_{13}^2D_{p+2,p+1,p+1,p} \\
&+\vec{x}_{14}^2D_{p+2,p+1,p,p+1} + \vec{x}_{23}^2D_{p+1,p+2,p+1,p} + \vec{x}_{24}^2D_{p+1,p+2,p,p+1} \\
&-2(\vec{x}_{14}^2\vec{x}_{23}^2 + \vec{x}_{13}^2\vec{x}_{24}^2 - \vec{x}_{12}^2\vec{x}_{34}^2)D_{p+2,p+2,p+1,p+1}\Big]\Big)\ ,
\end{aligned} \tag{4.21}$$

where we have used that $\Delta_{\mathrm{x}} = p+1$ and $\Delta_y = p$. For the $p = 2$ the correlator may be written in terms of functions of cross ratios as (after using again the D-function identities)

$$\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)y^a(\vec{x}_3)y^b(\vec{x}_4)\rangle = -\frac{96N}{\pi^4}\frac{\delta^{ij}\delta^{ab}}{\vec{x}_{12}^6\vec{x}_{34}^4}U^3\Big[5(U-1-V)\bar{D}_{4433} - 13\bar{D}_{3333} + 8\bar{D}_{3322}\Big]\ . \tag{4.22}$$

For comparison, let us also record the parity-even part of the $\langle yyyy\rangle$ correlator with general p and r following from (2.13) (generalizing the $p = 2$, $\mathrm{r} = \frac{1}{2}$ expression in (4.9), (4.13), (4.14))

$$\begin{aligned}
G_1(\vec{x}_i, \mathrm{t}_i) &= -\frac{\mathrm{T}_p\ p^2\Gamma^4(p)}{\pi^{2p}\Gamma^4(\frac{p}{2})} \\
&\quad\times\Big[\mathrm{t}_{12}\mathrm{t}_{34}G_1^{12;34}(\chi,\bar{\chi}) + \mathrm{t}_{13}\mathrm{t}_{24}G_1^{13;24}(\chi,\bar{\chi})) + \mathrm{t}_{14}\mathrm{t}_{23}G_1^{14;23}(\chi,\bar{\chi})\Big]\ , \qquad (4.23) \\
G_1^{12;34} &= -\frac{2}{\mathrm{r}^2}(D_{p,p,p,p} - 2\vec{x}_{12}^2D_{p+1,p+1,p,p}) + p^2(5D_{p,p,p,p} - 8\vec{x}_{12}^2D_{p+1,p+1,p,p}) \\
&\quad - 4p^2(\vec{x}_{14}^2\vec{x}_{23}^2 + \vec{x}_{13}^2\vec{x}_{24}^2 - \vec{x}_{12}^2\vec{x}_{34}^2)D_{p+1,p+1,p+1,p+1}\ , \\
G_1^{13;24} &= -\frac{2}{\mathrm{r}^2}(D_{p,p,p,p} - 2\vec{x}_{13}^2D_{p+1,p,p+1,p}) + p^2(5D_{p,p,p,p} - 8\vec{x}_{13}^2D_{p+1,p,p+1,p}) \\
&\quad - 4p^2(\vec{x}_{14}^2\vec{x}_{23}^2 + \vec{x}_{12}^2\vec{x}_{34}^2 - \vec{x}_{13}^2\vec{x}_{24}^2)D_{p+1,p+1,p+1,p+1}\ , \\
G_1^{14;23} &= -\frac{2}{\mathrm{r}^2}(D_{p,p,p,p} - 2\vec{x}_{14}^2D_{p+1,p,p,p+1}) + p^2(5D_{p,p,p,p} - 8\vec{x}_{14}^2D_{p+1,p,p,p+1}) \\
&\quad - 4p^2(\vec{x}_{12}^2\vec{x}_{34}^2 + \vec{x}_{13}^2\vec{x}_{24}^2 - \vec{x}_{14}^2\vec{x}_{23}^2)D_{p+1,p+1,p+1,p+1}\ ,
\end{aligned}$$

G_1 in (4.23) is related to $\mathcal{G}_1$ defined in (4.13) as in (4.9), i.e.

$$G_1(x_i, \mathrm{t}_i) = \frac{\mathrm{t}_{12}\mathrm{t}_{34}}{\vec{x}_{12}^{2p}\vec{x}_{34}^{2p}}\mathcal{G}_1(\chi,\bar{\chi};\alpha,\bar{\alpha})\ . \tag{4.24}$$

Note that the WZ term in (2.15), (2.17) is specific to $p=2$ case so that its contribution to the 4-point correlator (4.17) does not admit a generalization to an arbitrary p.

We can develop a better intuition about the above expressions for the correlators using the Mellin representation [72, 73]. It is straightforward to translate the D-functions into the Mellin space (see, e.g., appendix A of [74] for explicit expressions), and we find $(s+t+u=\sum_r \Delta_r)$

$$\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)\mathrm{x}^k(\vec{x}_3)\mathrm{x}^l(\vec{x}_4)\rangle = \frac{F_1(U,V)}{\vec{x}_{12}^{2p+2}\vec{x}_{34}^{2p+2}}, \tag{4.25}$$
$$F_1(U,V) = \int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t-2p-2}{2}}\,\mathcal{M}^{ijkl}(s,t)\,\Gamma^2\left(\frac{2p+2-s}{2}\right) \times\Gamma^2\left(\frac{2p+2-t}{2}\right)\Gamma^2\left(\frac{2p+2-u}{2}\right),\quad s+t+u=4p+4,$$

$$\langle \mathrm{x}^i(\vec{x}_1)\mathrm{x}^j(\vec{x}_2)y^a(\vec{x}_3)y^b(\vec{x}_4)\rangle = \frac{F_2(U,V)}{\vec{x}_{12}^{2p}\vec{x}_{34}^{2p+2}}, \tag{4.26}$$
$$F_2(U,V) = \int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t-2p-1}{2}}\,\mathcal{M}^{ij;ab}(s,t)\,\Gamma\left(\frac{2p-s}{2}\right)\Gamma\left(\frac{2p+2-s}{2}\right) \times\Gamma^2\left(\frac{2p+1-t}{2}\right)\Gamma^2\left(\frac{2p+1-u}{2}\right),\quad s+t+u=4p+2$$

where the Mellin amplitudes are given by

$$\mathcal{M}^{ijkl}(s,t) = \frac{\mathrm{T}_p\Gamma\left(\frac{3p}{2}+4\right)}{3\pi^{\frac{3p}{2}}(3p+4)\Gamma^4\left(\frac{p}{2}+1\right)}\Big[\delta^{ij}\delta^{kl}M(t,u)+\delta^{ik}\delta^{jl}M(s,u)+\delta^{il}\delta^{jk}M(s,t)\Big], \tag{4.27}$$
$$M(t,u) = -3(3p+4)tu+2(2p+1)(3p+4)(t+u)-4p(p+1)(4p+5)$$
$$\overset{p=2}{\to} -30tu+100(t+u)-312,$$
$$\mathcal{M}^{ij;ab}(s,t) = \frac{\mathrm{T}_p\,p^2\Gamma\left(\frac{3p}{2}+1\right)}{16\pi^{\frac{3p}{2}}\Gamma^4\left(\frac{p}{2}+1\right)}\,\delta^{ij}\delta^{ab}M'(t,u), \tag{4.28}$$
$$M'(t,u) = -(3p+2)(3p+4)tu+p(3p+2)(4p+5)(t+u)-16p^4-28p^3-5p^2+14p+8$$
$$\overset{p=2}{\to} -80tu+208(t+u)-464.$$

Similarly, the parity-even part of the $\langle yyyy\rangle$ correlator (4.23) admits the following Mellin representation

$$G_1(\vec{x}_i,\mathrm{t}_i) = \frac{1}{\vec{x}_{12}^{2p}\vec{x}_{34}^{2p}}\int_{-i\infty}^{i\infty}\frac{dsdt}{(4\pi i)^2}U^{\frac{s}{2}}V^{\frac{t-2p}{2}}\,\mathcal{M}_1(s,t;\mathrm{t}_i) \times\Gamma^2\left(\frac{2p+2-s}{2}\right)\Gamma^2\left(\frac{2p-t}{2}\right)\Gamma^2\left(\frac{2p-u}{2}\right),\quad s+t+u=4p, \tag{4.29}$$

$$\mathcal{M}_1(s,t;\mathrm{t}_i) = -\frac{\mathrm{T}_p\Gamma\left(\frac{3p}{2}+1\right)}{6\pi^{\frac{3p}{2}}\Gamma^4\left(\frac{p}{2}\right)}\Big[\mathrm{t}_{12}\mathrm{t}_{34}M''(t,u)+\mathrm{t}_{13}\mathrm{t}_{24}M''(s,u)+\mathrm{t}_{14}\mathrm{t}_{23}M''(s,t)\Big], \tag{4.30}$$

$$M''(t,u) = 3(2+3p)tu-6\left(2p^2-\frac{1}{\mathrm{r}^2}\right)(t+u)+4p\left(4p^2-3p-\frac{4}{\mathrm{r}^2}\right)$$
$$\overset{p=2,\,\mathrm{r}=\frac{1}{2}}{\to}\quad 6tu-6(t+u)-12\,. \tag{4.31}$$

As expected, the contact interactions with up to four derivatives (cf. (2.10)–(2.13)) give amplitudes that are quadratic polynomials in the Mellin-Mandelstam variables.

Let us also point out that the parity-odd part (4.17) of the $\langle yyyy\rangle$ correlator, which only exists for $p=2$ as it derives from the WZ action (2.15), does not admit a Mellin representation. Indeed, the parity-odd contribution (4.17) gains a minus sign under $\chi \leftrightarrow \bar{\chi}$ while the variables U and V (that appear in the standard definition of the Mellin representation) are invariant under this transformation (cf. their definition in (4.10)).

4.3 OPE analysis

In this subsection, we perform a preliminary OPE analysis to extract the CFT data for low-lying operators from the 4-point correlator $\langle yyyy\rangle$. Because of the chiral nature of the correlator, we have to use 2d conformal blocks which are not symmetrized with respect to χ and $\bar{\chi}$

$$g_{h,\bar{h}}(\chi,\bar{\chi}) = \chi^{\frac{h}{2}}\bar{\chi}^{\frac{\bar{h}}{2}}\, {}_2F_1\left(\frac{h}{2},\frac{h}{2},h;\chi\right)\, {}_2F_1\left(\frac{\bar{h}}{2},\frac{\bar{h}}{2},\bar{h};\bar{\chi}\right)\,, \qquad h-\bar{h}\in\mathbb{Z}\,. \tag{4.32}$$

This conformal block corresponds to an operator with holomorphic dimension h and anti-holomorphic dimension $\bar{h}$.[25] For the same reason, we should use SU(2) R-symmetry polynomials for $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R=\mathrm{SO}(4)$

$$R_{L,m}(\alpha)=P_m(1-2\alpha)\,,\qquad R_{R,m}(\bar{\alpha})=P_m(1-2\bar{\alpha})\,, \tag{4.33}$$

where $m=0,1,\cdots$ corresponds to the spin-$\frac{m}{2}$ representation of SU(2).

We can analyze the 4-point function in small χ (or $\bar{\chi}$) expansion,[26] which is dominated by operators with small h (or $\bar{h}$). By going to higher orders in the expansion, we can systematically read off CFT data for operators with increasing conformal twists. However, at higher conformal twists the mixing effect of operators become important, and analyzing the $\langle yyyy\rangle$ correlator alone gives only "averages" of anomalous dimensions over the operators appearing in the mixing (weighted by the corresponding OPE coefficients).

We will postpone the unmixing analysis (which involves $\langle \mathrm{xxxx}\rangle$ and $\langle \mathrm{xx}yy\rangle$ as well) for a future study and focus on the leading double-trace operators with $h=4$ where the mixing is absent. Since we are only interested in the leading operators in the OPE, we do not need superconformal blocks and bosonic conformal blocks are sufficient.

It is useful to decompose the correlator (4.11) into different R-symmetry channels

$$\begin{aligned}\mathcal{G}(\chi,\bar{\chi};\alpha,\bar{\alpha}) &= R_{L,0}(\alpha)R_{R,0}(\bar{\alpha})\,\mathcal{G}_{(\mathbf{1},\mathbf{1})}(\chi,\bar{\chi}) + R_{L,1}(\alpha)R_{R,0}(\bar{\alpha})\,\mathcal{G}_{(\mathbf{2},\mathbf{1})}(\chi,\bar{\chi})\\ &\quad + R_{L,0}(\alpha)R_{R,1}(\bar{\alpha})\,\mathcal{G}_{(\mathbf{1},\mathbf{2})}(\chi,\bar{\chi}) + R_{L,1}(\alpha)R_{R,1}(\bar{\alpha})\,\mathcal{G}_{(\mathbf{2},\mathbf{2})}(\chi,\bar{\chi})\,,\end{aligned} \tag{4.34}$$

[25] They are related to Δ and ℓ by $\min\{h,\bar{h}\}=\Delta-\ell$, $\max\{h,\bar{h}\}=\Delta+\ell$.

[26] The analysis of small χ expansion is identical to that of the small $\bar{\chi}$ expansion, after interchanging α and $\bar{\alpha}$. Therefore, in the following we will only focus on the former.

where $\mathbf{R} = (\mathbf{1},\mathbf{1}),\ldots$ in $\mathcal{G}_{\mathbf{R}}(\chi,\bar\chi)$ labels the representation of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ that is exchanged. Each $\mathcal{G}_{\mathbf{R}}(\chi,\bar\chi)$ can be decomposed into the conformal blocks

$$
\begin{aligned}
\mathcal{G}_{\mathbf{R}}(\chi,\bar\chi) = &\underbrace{\sum_{h,\bar h} C^{(0),\mathbf{R}}_{h,\bar h}\, g_{h,\bar h}(\chi,\bar\chi)}_{\text{disconnected}} \\
&+ \underbrace{\sum_{h,\bar h} C^{(1),\mathbf{R}}_{h,\bar h}\, g_{h,\bar h}(\chi,\bar\chi) + \frac{1}{2}\gamma^{(1),\mathbf{R}}_{h,\bar h} C^{(0),\mathbf{R}}_{h,\bar h}(\partial_h+\partial_{\bar h}) g_{h,\bar h}(\chi,\bar\chi) + \ldots}_{\text{tree level}}\,,
\end{aligned}
\tag{4.35}
$$

where $\gamma^{(1),\mathbf{R}}_{h,\bar h}$ are anomalous dimensions associated with $\log(z\bar z)$ divergences in the correlator, i.e.,

$$
h_{\text{exact}} = h + \gamma^{(1),\mathbf{R}}_{h,\bar h} + \ldots\,, \qquad \bar h_{\text{exact}} = \bar h + \gamma^{(1),\mathbf{R}}_{h,\bar h} + \ldots\,, \tag{4.36}
$$

The $(\mathbf{1},\mathbf{1})$ channel. From the χ^2 coefficient in the small χ expansion of the disconnected part of the correlator in (4.12) comparing to (4.35) we find that

$$
C^{(0),(\mathbf{1},\mathbf{1})}_{4,\bar h} = \frac{2^{4-\bar h}(\bar h-2)\Gamma(\frac{\bar h}{2}+1)}{\pi^{7/2}\Gamma(\frac{\bar h-1}{2})} N^2\,, \qquad \bar h \in 4\mathbb{Z}_+\,. \tag{4.37}
$$

From the $\chi^2\log(\chi\bar\chi)$ coefficient of the tree-level correlator (4.13) we can read off $\gamma^{(1),(\mathbf{1},\mathbf{1})}_{4,\bar h} C^{(0),(\mathbf{1},\mathbf{1})}_{4,\bar h}$. We find contributions only from $\bar h = 4, 8$, which correspond to spin 0 and spin 2 operators respectively. As a result,

$$
\gamma^{(1),(\mathbf{1},\mathbf{1})}_{4,4} = -\frac{24}{5N}\,, \qquad \gamma^{(1),(\mathbf{1},\mathbf{1})}_{4,8} = -\frac{48}{35N}\,. \tag{4.38}
$$

The fact that the anomalous dimensions have a finite support on spins is expected because the tree-level correlator is computed as a finite sum of contact diagrams, and each contact diagram has a finite support on spins [75].

The $(\mathbf{1},\mathbf{2})$ channel. From the χ^2 expansion coefficient in the disconnected part (4.12) we find

$$
C^{(0),(\mathbf{1},\mathbf{2})}_{4,\bar h} = \frac{2^{4-\bar h}(\bar h-2)\Gamma(\frac{\bar h}{2}+1)}{\pi^{7/2}\Gamma(\frac{\bar h-1}{2})} N^2\,, \qquad \bar h \in 4\mathbb{Z}_+ + 2\,. \tag{4.39}
$$

From the $\chi^2\log(\chi\bar\chi)$ coefficient in the tree-level correlator (4.13) we can extract the corresponding anomalous dimension. We find that there is only one operator with $\bar h = 6$ contributing and its anomalous dimension is

$$
\gamma^{(1),(\mathbf{1},\mathbf{2})}_{4,6} = -\frac{24}{5N}\,. \tag{4.40}
$$

The $(\mathbf{2},\mathbf{1})$ channel. From the small χ expansion of the disconnected correlator we find again

$$
C^{(0),(\mathbf{2},\mathbf{1})}_{4,\bar h} = \frac{2^{4-\bar h}(\bar h-2)\Gamma(\frac{\bar h}{2}+1)}{\pi^{7/2}\Gamma(\frac{\bar h-1}{2})} N^2\,, \qquad \bar h \in 4\mathbb{Z}_+ + 2\,. \tag{4.41}
$$

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However, here there is no $\chi^2\log(\chi\bar{\chi})$ term in the tree-level correlator. The operator with $h=4$ and $\bar{h}=6$ receives only a correction to the OPE coefficient

$$C^{(1),(\mathbf{2},\mathbf{1})}_{4,6} = -\frac{4}{5\pi^4}\,N\,. \tag{4.42}$$

The first logarithmic singularity arises at χ^3 order which corresponds to $h=6$. There are two operators responsible for this logarithmic singularity, with the anti-holomorphic dimensions $\bar{h}=4,8$.

The $(\mathbf{2},\mathbf{2})$ channel. Here the zeroth order OPE coefficients read

$$C^{(0),(\mathbf{2},\mathbf{2})}_{4,\bar{h}} = \frac{2^{4-\bar{h}}(\bar{h}-2)\Gamma(\frac{\bar{h}}{2}+1)}{\pi^{7/2}\Gamma(\frac{\bar{h}-1}{2})}\,N^2\,, \qquad \bar{h}\in 4\mathbb{Z}_+\,. \tag{4.43}$$

As in the $(\mathbf{2},1)$ channel here we find no $\chi^2\log(\chi\bar{\chi})$ term in the tree-level correlator. This is consistent with the fact that the $(\mathbf{2},\mathbf{2})$ channel receives contribution from the 1/2-BPS operator with $h=\bar{h}=4$ that has no anomalous dimension. The tree-level correlator leads only to a correction to the OPE coefficient of this operator

$$C^{(1),(\mathbf{2},\mathbf{2})}_{4,4} = -\frac{4}{\pi^4}N\,. \tag{4.44}$$

5 Superconformal symmetry of holographic correlators

In section 4.1 we observed that the $\langle yyyy\rangle$ correlator satisfies intricate differential relations (4.18) which can be interpreted as superconformal Ward identities following from the global symmetry $[D(2,1|-\frac{1}{2})]^2=[\mathrm{OSp}(4^*|2)]^2$. We give a derivation of these identities in section 5.2 by studying the supersymmetry of "twisted" operators defined below. In fact, these superconformal Ward identities are so restrictive to essentially fix the form of the 4-point function. This is shown in section 5.3, where we "bootstrap" the $\langle yyyy\rangle$ tree-level correlator from the Ward identities and crossing symmetry without inputting the precise form of the AdS_3 "bulk" vertices in the M2 brane action, thus shortcutting the computation in section 4.1.

5.1 Twisted operators and supersymmetry

It is possible to construct a set of "twisted" combinations of the scalar fluctuations y^a, following a similar construction in [36].[27] We start by rewriting (4.8) in spinor notations ($\alpha=1,2$)

$$y(\vec{x};\mathrm{t}) = y(v,\bar{v};\mathrm{a},\bar{\mathrm{a}}) = y_{\alpha\dot{\alpha}}\,a^\alpha\bar{a}^{\dot{\alpha}}\,, \quad \mathrm{t}^a=\rho^a_{\alpha\dot{\alpha}}a^\alpha\bar{a}^{\dot{\alpha}}\,, \quad a^\alpha=(1,\mathrm{a})\,, \quad \bar{a}^{\dot{\alpha}}=(1,\bar{\mathrm{a}})\,. \tag{5.1}$$

We switched here to complex coordinates $v=x^1+ix^2$ and ρ^a are SO(4) gamma matrices ($\rho=(i\vec{\sigma},I)$). Explicitly

$$\mathrm{t}^a(\mathrm{a},\bar{\mathrm{a}}) = \{i(\mathrm{a}+\bar{\mathrm{a}}),\,\mathrm{a}-\bar{\mathrm{a}},\,i(1-|\mathrm{a}|^2),\,1+|\mathrm{a}|^2\}\,. \tag{5.2}$$

[27] This should not be confused with the chiral algebra twist of [60]. The construction of [60] requires twisting a subalgebra $\mathfrak{psu}(1,1|2)$ which does not fit into $D(2,1|-\frac{1}{2})$. Moreover, it will be clear that our twisted correlator has both χ and $\bar{\chi}$ dependence, rather than being chiral.

The inner product is now simply

$$\mathrm{t}^a(\mathrm{a}_1, \bar{\mathrm{a}}_1)\, \mathrm{t}^a(\mathrm{a}_2, \bar{\mathrm{a}}_2) = 2|\mathrm{a}_1 - \mathrm{a}_2|^2 \,. \tag{5.3}$$

We define the half-twisted operator by setting $\mathrm{a} = v$

$$\mathcal{Y}(v, \bar{v}; \bar{\mathrm{a}}) = y(v, \bar{v}; v, \bar{\mathrm{a}}) \,. \tag{5.4}$$

We can also define a half-twist in the opposite chirallity by setting $\bar{\mathrm{a}} = \bar{v}$, or twist both.

As y^a is a massless ($\Delta = 2$) field whose 2-point function is given by (4.4), eq. (5.3) implies that $\mathcal{Y}$ behaves like

$$\langle \mathcal{Y}(v_1, \bar{v}_1; \bar{\mathrm{a}}_1)\, \mathcal{Y}(v_2, \bar{v}_2; \bar{\mathrm{a}}_2) \rangle = \frac{2\mathrm{C}_y(\bar{\mathrm{a}}_1 - \bar{\mathrm{a}}_2)}{(v_1 - v_2)(\bar{v}_1 - \bar{v}_2)^2} \,. \tag{5.5}$$

This operator $\mathcal{Y}$ twists the left-moving SU(2) of SO(4) $\simeq$ SU(2) $\times$ SU(2) R-symmetry into the left-moving SO(2,1) of the SO(2,2) $\simeq$ SO(2,1) $\times$ SO(2,1) conformal group.

To see that, note that the doublets a^α are in the fundamental of the left-moving SU(2). We can implement then the R-symmetry transformations of y^a in terms of the following differential operators acting on $y(v, \bar{v}; \mathrm{a}, \bar{\mathrm{a}})$

$$R_v^- = \partial_\mathrm{a} \,, \qquad R_v^0 = \mathrm{a}\partial_\mathrm{a} - \frac{1}{2} \,, \qquad R_v^+ = -\mathrm{a}^2\partial_\mathrm{a} + \mathrm{a} \,. \tag{5.6}$$

The action of the conformal group on a field is implemented, as usual by

$$P_v = \partial_v \,, \qquad D_v = v\partial_v + \delta \,, \qquad K_v = v^2\partial_v + 2v\delta \,. \tag{5.7}$$

Note that the dimension δ for the field y is equal to 1 (the total conformal dimension is $\Delta = \delta + \bar{\delta} = 2$).

Now we can translate $\mathcal{Y}(v, \bar{v}; \bar{\mathrm{a}})$ via

$$\partial_v \mathcal{Y}(v, \bar{v}; \bar{\mathrm{a}}) = (\partial_v + \partial_\mathrm{a}) y(v, \bar{v}; \mathrm{a}, \bar{\mathrm{a}})\Big|_{\mathrm{a}=v} \,. \tag{5.8}$$

Hence $\mathcal{Y}$ transforms covariantly under the twisted generators

$$P_v + R_v^- \,, \qquad D_v + R_v^0 \,, \qquad K_v - R_v^+ \,. \tag{5.9}$$

Let us now examine the supersymmetry constraints on the twisted operators. The surface defect breaks the OSp($8^*|4$) of the $\mathcal{N} = (2,0)$ 6d theory to $[\mathrm{OSp}(4^*|2)]^2$. The OSp($8^*|4$) transformations can be parameterized by the spinor $\epsilon(x) = \epsilon^0 + \bar{\epsilon}^1 x^\mu \gamma_\mu$, where ϵ^0 corresponds to the super-Poincaré transformations and $\bar{\epsilon}^1$ to the superconformal ones. They are chiral and antichiral respectively, are in a spinor representation of the Sp(4) R-symmetry group and satisfy the symplectic Majorana condition $\bar{\epsilon} = -c\Omega\epsilon$, where c is the charge conjugation matrix and Ω the symplectic form.

Using γ_μ and ρ^a for space-time and R-symmetry gamma matrices, the surface in the (x^1, x^2) plane ($v = x^1 + ix^2$) imposes the condition

$$\epsilon(x)(1 + i\gamma_{12}\rho^5) = 0 \,. \tag{5.10}$$

Defining $\gamma_v = \frac{1}{2}(\gamma_1 - i\gamma_2)$ and likewise $\gamma_{\bar{v}}$, we have $i\gamma_{12} = \gamma_v\gamma_{\bar{v}} - \gamma_{\bar{v}}\gamma_v$. Then the above equation splits into

$$\epsilon_v(x)(1+\rho^5) = \epsilon_{\bar{v}}(x)(1-\rho^5) = 0\,, \tag{5.11}$$

where $\epsilon_v = \epsilon\gamma_v$ and $\epsilon_{\bar{v}} = \epsilon\gamma_{\bar{v}}$ are the generators of the two $D(2,1;-\frac{1}{2})$ superalgebras.

In addition to the generators in (5.6) and (5.7), each algebra has another SU(2) acting on the x^i fields (whose generators we denote by T) and supercharges Q and S. The algebra (see e.g. [76, 77]) is

$$\begin{aligned}
\{Q_v^{\alpha m}, Q_v^{\beta n}\} &= \varepsilon^{\alpha\beta}\varepsilon^{nm}P_v\,,\\
\{Q_v^{\alpha m}, S_v^{\beta n}\} &= -\varepsilon^{\alpha\beta}\varepsilon^{nm}D_v - 2\varepsilon^{mn}(\varepsilon\sigma^i)^{\alpha\beta}R_v^i + \varepsilon^{\alpha\beta}(\varepsilon\sigma^i)^{nm}T_v^i\,,\\
\{S_v^{\alpha m}, S_v^{\beta n}\} &= \varepsilon^{\alpha\beta}\varepsilon^{nm}K_v\,,
\end{aligned} \tag{5.12}$$

where $\varepsilon = i\sigma^2$.

The supersymmetry transformations of the y fields are

$$Q_v^{\alpha m} y^{\beta\dot{\beta}} = \varepsilon^{\alpha\beta}\psi^{m\dot{\beta}}\,. \tag{5.13}$$

For the twisted field at $z \neq 0$ we need to also include the S transformations

$$Q_v^{\alpha m}\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}}) = a^\alpha\psi^{m\dot{\beta}}\bar{a}_{\dot{\beta}}\Big|_{\mathrm{a}=v}\,, \qquad S_v^{\alpha m}\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}}) = a^\alpha v\psi^{m\dot{\beta}}\bar{a}_{\dot{\beta}}\Big|_{\mathrm{a}=v}\,. \tag{5.14}$$

Thus $\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}})$ is annihilated by the combinations $\mathcal{Q}_v^m = Q_v^{2m} - S_v^{1m}$.

Let us define a fermionic field which is the action of one of the supersymmetry generators on $\mathcal{Y}$

$$\Psi^n(v,\bar{v};\bar{\mathrm{a}}) = \frac{1}{v}Q_v^{2n}\mathcal{Y} = \psi^{n\dot{\beta}}\bar{a}_{\dot{\beta}} = \psi^{n\dot{1}} + \psi^{n\dot{2}}\bar{\mathrm{a}}\,. \tag{5.15}$$

We now examine the Ward identity for the supercharge $\mathcal{Q}_v^m$ for the correlation function of this fermion with any number of $\mathcal{Y}$ fields. Since $\mathcal{Q}_v^m$ annihilates $\mathcal{Y}$, we get

$$\begin{aligned}
0 &= \big\langle \mathcal{Q}^m[\Psi^n(v_1,\bar{v}_1;\bar{\mathrm{a}}_1)\mathcal{Y}(v_2,\bar{v}_2;\bar{\mathrm{a}}_2)\cdots\mathcal{Y}(v_p,\bar{v}_p;\bar{\mathrm{a}}_p)]\big\rangle\\
&= \big\langle [\mathcal{Q}^m\Psi^n(v_1,\bar{v}_1;\bar{\mathrm{a}}_1)]\,\mathcal{Y}(v_2,\bar{v}_2;\bar{\mathrm{a}}_2)\cdots\mathcal{Y}(v_p,\bar{v}_p;\bar{\mathrm{a}}_p)\big\rangle\\
&= \frac{1}{v_1}\big\langle [\{\mathcal{Q}_v^m, Q_v^{2n}\}\mathcal{Y}(v_1,\bar{v}_1;\bar{\mathrm{a}}_1)]\,\mathcal{Y}(v_2,\bar{v}_2;\bar{\mathrm{a}}_2)\cdots\mathcal{Y}(v_p,\bar{v}_p;\bar{\mathrm{a}}_p)\big\rangle\,,
\end{aligned} \tag{5.16}$$

where in the last step we have used again that $\mathcal{Q}_v^m$ annihilates $\mathcal{Y}$, leaving the anticommutator of $\mathcal{Q}_v^m$ and Q_v^{2n}. Using (5.12), this anticommutator evaluates to

$$\{\mathcal{Q}_v^m, Q_v^{2n}\} = \varepsilon^{nm}D_v + 2\varepsilon^{nm}R_v^0\,, \tag{5.17}$$

and its action on $\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}})$ can be expressed as the differential operators (5.6), (5.7)

$$\big(D_v + 2R_v^0\big)\,\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}}) = (v\partial_v + 2\mathrm{a}\partial_{\mathrm{a}})y(v,\bar{v},\mathrm{a},\bar{\mathrm{a}})\big|_{\mathrm{a}=v}\,. \tag{5.18}$$

Let us apply this relation to the 4-point function (4.7), (4.9) of the $\mathcal{Y}(v,\bar{v};\bar{\mathrm{a}})$ operators. Examining the definitions of conformal cross-ratios in (4.10), we conclude that in this case

$\alpha = 1/\chi$, i.e.

$$\begin{aligned} G_{\text{left-twist}}(v_i, \bar{v}_i; \bar{\mathrm{a}}_i) &= \langle \mathcal{Y}(v_1, \bar{v}_1; \bar{\mathrm{a}}_1) \mathcal{Y}(v_2, \bar{v}_2; \bar{\mathrm{a}}_2) \mathcal{Y}(v_3, \bar{v}_3; \bar{\mathrm{a}}_3) \mathcal{Y}(v_4, \bar{v}_4; \bar{\mathrm{a}}_4) \rangle \\ &= \frac{4|\mathrm{a}_1 - \mathrm{a}_2|^2 |\mathrm{a}_3 - \mathrm{a}_4|^2}{|v_1 - v_2|^4 |v_3 - v_4|^4} \mathcal{G}(\chi, \bar{\chi}; \alpha, \bar{\alpha}) \Big|_{\mathrm{a}_i = v_i} \\ &= \frac{4(\bar{\mathrm{a}}_1 - \bar{\mathrm{a}}_2)(\bar{\mathrm{a}}_3 - \bar{\mathrm{a}}_4)}{(v_1 - v_2)(v_3 - v_4)(\bar{v}_1 - \bar{v}_2)^2 (\bar{v}_3 - \bar{v}_4)^2} \mathcal{G}(\chi, \bar{\chi}; 1/\chi, \bar{\alpha}) \,. \end{aligned} \tag{5.19}$$

Noting that the differential operator (5.18) annihilates the prefactor in the second line above, we can translate the Ward identity (5.16), (5.18) to act on the function of cross ratios $\mathcal{G}$. Moreover, if we fix $v_1 = 0$, $v_3 = 1$ and $v_4 \to \infty$, we find from (4.10) that $\chi = v_2$, and since the twisting (5.4) fixes $\mathrm{a}_2 = v_2$, then $\alpha = 1/\mathrm{a}_2$, and we reproduce precisely the first relation in (4.18).

Note that our construction has only used the left-moving copy of $D(2, 1; -\frac{1}{2})$. We can repeat the entire analysis by twisting instead the right-moving $D(2, 1; -\frac{1}{2})$, which leads to the second identity in (4.18). Thanks to the factorization property, our derivation above implies that the superconformal Ward identity

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{F}(\chi; \alpha)\Big|_{\alpha=1/\chi} = 0 \,, \tag{5.20}$$

applies as well to the 4-point functions of 1/2-BPS insertions on a line defect in 4d $\mathcal{N} = 2$ theories preserving half of the supersymmetry. This system has the same $D(2, 1; -\frac{1}{2}) = \mathrm{OSp}(4^*|2)$ symmetry [78]. Here χ is the conformal cross ratio on a straight line, and α is the cross ratio for the SO(3) R-symmetry.

Although we derived (4.18) for correlators of fields with conformal dimensions $(\delta, \bar{\delta}) = (1, 1)$ and R-symmetry charges $(q, \bar{q}) = (\frac{1}{2}, \frac{1}{2})$, the same superconformal Ward identities (4.18) apply to 4-point correlators of dCFT operators with general R-symmetry charges $(q, \bar{q})$ and $(\delta, \bar{\delta}) = (2q, 2\bar{q})$. That q and $\bar{q}$ do not need to be equal is a consequence of the factorized form of the superconformal algebra $[D(2, 1; -\frac{1}{2})]^2$. We can understand the extension to general 1/2-BPS insertions by realizing that we can construct higher-weight 1/2-BPS operators by taking products of the ones with $\delta = 2q = 1$ or $\bar{\delta} = 2\bar{q} = 1$. The n-point correlators of operators with lowest weights satisfy the constraint (5.16), which becomes (4.18) when regrouping them into four composite operators.

5.2 Twisted 4-point correlator and a curious relation to special 4-point function in $\mathcal{N} = 4$ SYM

We can twist both $D(2, 1; -\frac{1}{2})$ algebras with $\mathrm{a} = v$ and $\bar{\mathrm{a}} = \bar{v}$. This gives a dimension-one non-chiral scalar operator

$$\langle y(v_1, \bar{v}_1; v_1, \bar{v}_1)\, y(v_2, \bar{v}_2; v_2, \bar{v}_2) \rangle = \frac{2\mathrm{C}_y}{|v_1 - v_2|^2} \,. \tag{5.21}$$

Now take the 4-point function of such double-twisted operators

$$G_{\text{twist}}(v_i, \bar{v}_i; \mathrm{a}_i, \bar{\mathrm{a}}_i) = G(v_i, \bar{v}_i; v_i, \bar{v}_i) \,. \tag{5.22}$$

Using (4.14), (4.17) we find that the correlator (5.19) has a remarkably simple expression proportional to just one D-function

$$G_{\text{twist}} = -\frac{96N}{\pi^4}\vec{x}_{12}^2\vec{x}_{23}^2\vec{x}_{24}^2 D_{2422}\ . \tag{5.23}$$

Surprisingly, the same function arises in a totally different setting, namely, in the 4-point function of the stress tensor multiplet of $\mathcal{N}=4$ SYM theory at strong coupling computed from the $\text{AdS}_5\times S^5$ IIB supergravity.

Indeed, let us consider the 4-point function of the 1/2-BPS operator $\mathcal{O}_2(\vec{x};\text{t}) = \text{t}_I\text{t}_J\ \text{tr}(\Phi^I\Phi^J)(\vec{x})$ where Φ^I $(I=1,\cdots,6)$ are the 6 real scalars of $\mathcal{N}=4$ SYM.[28] This operator has protected conformal dimension $\Delta=2$, and transforms in the rank-2 symmetric traceless representation of SO(6) R-symmetry. We contracted the indices with a null vector t_I satisfying $\text{t}^2=0$, which automatically performs the projection to the symmetric traceless representation. Thanks to superconformal symmetry, the 4-point function has a "partially non-renormalized" structure [80, 81]

$$G_{\text{SYM}}(\vec{x}_i;\text{t}_i) = \langle\mathcal{O}_2(\vec{x}_1;\text{t}_1)\mathcal{O}_2(\vec{x}_2;\text{t}_2)\mathcal{O}_2(\vec{x}_3;\text{t}_3)\mathcal{O}_2(\vec{x}_4;\text{t}_4)\rangle\ , \tag{5.24}$$

$$G_{\text{SYM}}(\vec{x}_i;\text{t}_i) = G_{\text{free}}(\vec{x}_i;\text{t}_i) + R(\vec{x}_i;\text{t}_i)\, H(\vec{x}_i)\ . \tag{5.25}$$

Here $G_{\text{free}}(\vec{x}_i;\text{t}_i)$ is the correlator in the free SYM theory

$$G_{\text{free}}(\vec{x}_i;\text{t}_i) = \frac{\text{t}_{12}^2\text{t}_{34}^2}{\vec{x}_{12}^4\vec{x}_{34}^4}\left[\left(1+\sigma^2U^2+\tau^2\frac{U^2}{V^2}\right)+\frac{1}{c}\left(\sigma U+\tau\frac{U}{V}+\sigma\tau\frac{U^2}{V}\right)\right]\ , \tag{5.26}$$

where we assumed the canonical normalization $\langle\mathcal{O}_2(\vec{x}_1;\text{t}_1)\mathcal{O}_2(\vec{x}_2;\text{t}_2)\rangle = \frac{\text{t}_{12}^2}{\vec{x}_{12}^4}$. Note that the free correlator is exact in $1/c$, where $c=\frac{1}{4}(N^2-1)$ is the "central charge" of the SU(N) SYM theory. R in (5.25) is a kinematical factor fully determined by the superconformal symmetry

$$R = \text{t}_{12}^2\text{t}_{34}^2\,\vec{x}_{13}^4\vec{x}_{24}^4(1-\chi\alpha)(1-\chi\bar{\alpha})(1-\bar{\chi}\alpha)(1-\bar{\chi}\bar{\alpha})\ , \tag{5.27}$$

and $H(\vec{x}_i)$ is the *reduced correlator* which encodes all the dynamical information. We can compute $H(\vec{x}_i)$ in $1/c$ expansion at strong coupling, using the dual bulk description of IIB supergravity on $\text{AdS}_5\times S^5$

$$H(\vec{x}_i) = H_{\text{tree}}(\vec{x}_i) + H_{\text{1-loop}}(\vec{x}_i) + \dots\ . \tag{5.28}$$

The tree-level reduced correlator reads [82]

$$H_{\text{tree}}(\vec{x}_i) = -\frac{6}{\pi^2 c}\frac{D_{2422}}{\vec{x}_{13}^2\vec{x}_{34}^2\vec{x}_{14}^2}\ . \tag{5.29}$$

Comparing this to the twisted correlator in (5.23)[29] we conclude that they match up to an overall constant and a factor that can be interpreted as a "tetrahedron" contraction of

[28] We will be brief in the following about the superconformal kinematics of 1/2-BPS 4-point functions in $\mathcal{N}=4$ SYM, and refer the interested reader to section 2 of [79] for a more detailed review.

[29] The reader might be concerned that we are comparing results in different spacetime dimensions. However, a nice feature of D-functions defined for AdS_{d+1} is that the d-dependence only appears in the overall normalization. The functional dependence on $\vec{x}_{ij}^2$ is the same for all d. Moreover, for four points we can always use a conformal transformation to restrict them on a two-dimensional plane.

generalized free fields[30]

$$\mathcal{T} = \prod_{1\le i<j\le 4} \frac{1}{\vec{x}_{ij}^2} \,. \tag{5.30}$$

The observed relation[31] between the twisted correlator on BPS surface defect in 6d (2, 0) theory, and the BPS correlator in strongly coupled $\mathcal{N} = 4$ SYM theory is quite curious and we hope to shed light on its meaning in the future.

5.3 Fixing the $\langle yyyy\rangle$ correlator from the superconformal Ward identities

In this subsubsection, we provide an alternative perspective on the tree-level holographic 4-point function $\langle yyyy\rangle$. We show that the holographic correlator (4.13) can be "bootstrapped" by imposing superconformal constraints captured by the relations (4.18), without using any precise information about the coefficients of the bulk vertices. Similar techniques have already been implemented in a number of maximally supersymmetric AdS backgrounds, and lead to unique answers for tree level 4-point functions in theories with no defects [84–87].

We start from an ansatz for a local AdS_3 bulk action which consists of all possible contact interactions of y^a fields with up to four derivatives. The structure of the corresponding Witten diagrams translates into the ansatz for the following tree-level 4-point function

$$G = G_1 + G_2 \,, \tag{5.31}$$

where G_1 is the parity-even part given by a linear combination of contributions of all possible 0-, 2-, and 4-derivative contact diagrams and G_2 is the parity odd part coming from the 3-derivative contact interaction. The precise combination of these Witten diagrams can be fixed as in section 4.1 using the explicit form of the M2 brane Lagrangian (2.13), but here we will leave them arbitrary and to be determined by symmetries.

A convenient parameterization for G_1 is given by the linear combination of all possible D-functions that can show up at this order with the coefficients that are any possible parity-even R-symmetry structures

$$G_1 = \sum_i (\mu_{1,i}\, \mathrm{t}_{12}\mathrm{t}_{34} + \mu_{2,i}\, \mathrm{t}_{13}\mathrm{t}_{24} + \mu_{3,i}\, \mathrm{t}_{14}\mathrm{t}_{23})\, W_i \,, \tag{5.32}$$

$$\{W_i\} = \{D_{2222};\ \vec{x}_{12}^2 D_{3322},\ \vec{x}_{13}^2 D_{3232}, \cdots ;\ \vec{x}_{12}^2\vec{x}_{34}^2 D_{3333},\ \vec{x}_{13}^2\vec{x}_{24}^2 D_{3333}, \cdots \}\,. \tag{5.33}$$

[30] Here we do not want to absorb $\mathcal{T}$ into the definition of the reduced correlator H_{tree}, because it is important that H_{tree} has conformal dimension 4 to exhibit the ten-dimensional hidden conformal symmetry [83]. Replacing the argument $\vec{x}_{ij}^2$ in $H_{\text{tree}}(\vec{x}_{ij}^2)$ with $\vec{x}_{ij}^2 + \mathrm{t}_{ij}$ gives a generating function for the reduced correlators of higher Kaluza-Klein modes (see section 2.3 of [79] for a discussion of this point).

[31] It might also be instructive to view the relation from the Mellin perspective. For the twisted correlator (5.23), we have a factorized polynomial Mellin amplitude

$$\mathcal{M}_{\text{twisted}} \sim (s-2)(t-2)(u-2)\,, \quad s+t+u=4\,,$$

and it has an interpretation of contact interactions with up to six derivatives. For the supergravity case (5.29), the Mellin amplitude is the inverse of the above expression

$$\mathcal{M}_{\text{supergravity}} \sim \frac{1}{(s-2)(t-2)(u-2)}\,, \quad s+t+u=4\,.$$

We also require that the parity-even part G_1 (as coming from a local bulk action) should be crossing symmetric.

The only parity-odd 4-vertex allowed by symmetries is the one in (2.17), i.e. $\sim \epsilon_{abcd}\epsilon^{\mu\nu\lambda}y^a\partial_\mu y^b\partial_\nu y^c\partial_\lambda y^d$ and thus G_2 is proportional to the 3-derivative contact Witten diagram in AdS$_3$ discussed in [69] (cf. (4.2))

$$G_2 \sim \int \frac{d^3x}{z^3} z^3\, \epsilon^{\mu\nu\rho}\partial_\mu G^{\Delta_1}_{B\partial}(x,\vec{x}_1)\partial_\nu G^{\Delta_3}_{B\partial}(x,\vec{x}_3)\partial_\rho G^{\Delta_4}_{B\partial}(x,\vec{x}_4)G^{\Delta_2}_{B\partial}(x,\vec{x}_2)\,. \tag{5.34}$$

It is easy to check by integration by parts that this contribution is antisymmetric with respect to all four points $\vec{x}_i$. In order for G_2 to be crossing symmetric, the R-symmetry factor has to be anti-symmetric and can only be

$$\mathrm{t}_{12}\mathrm{t}_{34}(\alpha-\bar{\alpha})\,. \tag{5.35}$$

Using the result of [69], G_2 can be written as (cf. (4.17))

$$G_2 = \lambda\, \frac{\mathrm{t}_{12}\mathrm{t}_{34}}{\vec{x}^4_{12}\vec{x}^4_{34}} U^2(\chi-\bar{\chi})(\alpha-\bar{\alpha})\bar{D}_{3333}\,, \tag{5.36}$$

where λ is an undetermined coefficient.

Imposing the superconformal Ward identities (4.18), we find that all the coefficients μ_k in the ansatz (5.31) can be fixed except for an overall scaling factor.[32] The overall normalization is not fixed by (4.18) because these relations are homogenous. The solution is proportional to $\mathcal{G}_{\rm tree}$ we obtained above (4.13) (4.14), (4.17) by the direct computation from the M2-brane action.

Thus the superconformal symmetry is effectively determining the relative coefficients in the underlying bulk action. Note that even though we included the zero-derivative contact interactions in the ansatz for G_1 in (5.32), such contributions are absent in the final solution fixed by superconformal symmetry. This is consistent with the fact that the are no such terms in the M2-brane action for y^a in (2.13).

We can also apply a similar bootstrap approach to the case of 1/2-BPS Wilson loop. The corresponding 4-point function for the dimension 1 operators can be uniquely fixed by the superconformal Ward identity, up to an overall constant which can be determined using supersymmetric localization [5]. We will give the details of this calculation in appendix B.

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[32] To implement the superconformal Ward identities, we used the algorithm developed in [85] (see section 5 of the reference for notation and details). We decompose all the $\bar{D}$-functions into the basis spanned by 1, $\log U$, $\log V$ and the 1-loop scalar box diagram $\Phi(U,V)$, by using differential recursion relations of $\Phi(U,V)$. The superconformal Ward identities are expanded into this basis, with rational functions as coefficient functions. Requiring the coefficient functions to vanish gives linear equations for the unfixed coefficients in the ansatz.

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A $\bar{D}$-functions

For reader's convenience, we collect some useful properties of the $\bar{D}$-functions in (4.15), (4.16)) (see, e.g., [71]), which can be used to obtain the explicit form of correlators as functions of cross ratios U and V or χ and $\bar{\chi}$ in (4.10).

The simplest $\bar{D}$-function has $\Delta_i = 1$, and is just the scalar one-loop box digram in four dimensions

$$\bar{D}_{1111} = \Phi(\chi, \bar{\chi}) \,, \tag{A.1}$$

$$\Phi(\chi, \bar{\chi}) = \frac{1}{\chi - \bar{\chi}} \left[\log(\chi\bar{\chi}) \log\left(\frac{1-\chi}{1-\bar{\chi}}\right) + 2\mathrm{Li}_2(\chi) - 2\mathrm{Li}_2(\bar{\chi}) \right] . \tag{A.2}$$

To obtain $\bar{D}$-functions with higher weights, we can use the following differential operators

$$\begin{aligned}
\bar{D}_{\Delta_1+1,\Delta_2+1,\Delta_3,\Delta_4} &= -\partial_U \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \,, \\
\bar{D}_{\Delta_1,\Delta_2,\Delta_3+1,\Delta_4+1} &= (\Delta_3 + \Delta_4 - \Sigma - U\partial_U) \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \,, \\
\bar{D}_{\Delta_1,\Delta_2+1,\Delta_3+1,\Delta_4} &= -\partial_V \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \,, \\
\bar{D}_{\Delta_1+1,\Delta_2,\Delta_3,\Delta_4+1} &= (\Delta_1 + \Delta_4 - \Sigma - V\partial_V) \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \,, \\
\bar{D}_{\Delta_1,\Delta_2+1,\Delta_3,\Delta_4+1} &= (\Delta_2 + U\partial_U + V\partial_V) \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} \,, \\
\bar{D}_{\Delta_1+1,\Delta_2,\Delta_3+1,\Delta_4} &= (\Sigma - \Delta_4 + U\partial_U + V\partial_V) \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}
\end{aligned} \tag{A.3}$$

where $\Sigma = \frac{1}{2}(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)$.

Note that the function $\Phi(\chi, \bar{\chi})$ satisfies the following differential recursion relations

$$\begin{aligned}
\partial_\chi \Phi &= -\frac{1}{\chi - \bar{\chi}} \Phi - \frac{1}{\chi(\chi - \bar{\chi})} \log(1-\chi)(1-\bar{\chi}) + \frac{1}{(-1+\chi)(\chi - \bar{\chi})} \log(\chi\bar{\chi}) \,, \\
\partial_{\bar{\chi}} \Phi &= \frac{1}{\chi - \bar{\chi}} \Phi + \frac{1}{\bar{\chi}(\chi - \bar{\chi})} \log(1-\chi)(1-\bar{\chi}) - \frac{1}{(-1+\bar{\chi})(\chi - \bar{\chi})} \log(\chi\bar{\chi}) \,.
\end{aligned} \tag{A.4}$$

We can therefore recursively decompose $\bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}$ into a basis spanned by 1, $\log U$, $\log V$, $\Phi(\chi, \bar{\chi})$ with coefficients being rational functions of χ, $\bar{\chi}$.

B 4-point correlator on 1/2-BPS Wilson line from superconformal invariance

In this appendix, we implement the techniques of section 5.3 to determine the tree-level contribution to the 4-point function of insertions in the 1/2-BPS Wilson loop from the

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superconformal Ward identity and crossing. This reproduces the expression derived in [5] from the fundamental string action in $AdS_5 \times S^5$.

Recall that the Wilson loop has an $OSp(4^*|4)$ superconformal symmetry. The insertions Φ^a with $a = 1, \cdots, 5$ have conformal dimension 1, and transforms as a vector under the $SO(5) = Sp(4)$ R-symmetry. Holographically, they corresponds to the S^5 fluctuation y^a. As in (4.8), we contract the R-symmetry index of Φ^a with a constant auxiliary vector t_a

$$\Phi(w; \mathrm{t}) = \mathrm{t}_a \, \Phi^a(w) \; , \tag{B.1}$$

so that its correlators depend on the coordinates $w_i \in \mathbb{R}^1$ parametrizing the straight Wilson line and on the internal "coordinates" t_i.

For simplicity, we will fix the normalization of the two-point function so that

$$\langle\!\langle \Phi(w_1; \mathrm{t}_1)\Phi(w_2; \mathrm{t}_2)\rangle\!\rangle = \frac{\mathrm{t}_{12}}{|w_{12}|^2} \, , \tag{B.2}$$

where $\mathrm{t}_{ij} = \mathrm{t}_i \cdot \mathrm{t}_j$, $w_{ij} = w_i - w_j$. The 4-point function can be written in terms of a function $\mathcal{F}$ of cross ratios as

$$A = \langle\!\langle \Phi(w_1; \mathrm{t}_1)\Phi(w_2; \mathrm{t}_2)\Phi(w_3; \mathrm{t}_3)\Phi(w_4; \mathrm{t}_4)\rangle\!\rangle = \frac{\mathrm{t}_{12}\mathrm{t}_{34}}{|w_{12}|^2|w_{34}|^2} \, \mathcal{F}(\chi; \alpha, \bar{\alpha}) \, , \tag{B.3}$$

where

$$\chi = \frac{w_{12}w_{34}}{w_{13}w_{24}} \, , \qquad \sigma = \frac{\mathrm{t}_{13}\mathrm{t}_{24}}{\mathrm{t}_{12}\mathrm{t}_{34}} = \alpha\bar{\alpha} \, , \qquad \tau = \frac{\mathrm{t}_{14}\mathrm{t}_{23}}{\mathrm{t}_{12}\mathrm{t}_{34}} = (1-\alpha)(1-\bar{\alpha}) \; . \tag{B.4}$$

Note that unlike the $SO(4)$ case of the M2-brane theory in the main text, here the R-symmetry cross ratios α, $\bar{\alpha}$ should appear symmetrically in $\mathcal{F}(\chi; \alpha, \bar{\alpha})$ (we cannot have $\det(\mathrm{t}_{ij}) \propto \mathrm{t}_{12}\mathrm{t}_{34}(\alpha - \bar{\alpha})$). In other words, $\mathcal{F}(\chi; \alpha, \bar{\alpha})$ is a linear function of σ and τ. The 4-point function also needs to satisfy the superconformal Ward identities [20] analogous to (4.18)

$$\left(-\frac{1}{2}\chi\partial_\chi + \alpha\partial_\alpha\right)\mathcal{F}(\chi; \alpha, \bar{\alpha})\Big|_{\alpha=1/\chi} = 0 \, , \qquad \left(-\frac{1}{2}\chi\partial_\chi + \bar{\alpha}\partial_{\bar{\alpha}}\right)\mathcal{F}(\chi; \alpha, \bar{\alpha})\Big|_{\bar{\alpha}=1/\chi} = 0 \; . \tag{B.5}$$

Because $\mathcal{F}(\chi; \alpha, \bar{\alpha})$ is symmetric under $\alpha \leftrightarrow \bar{\alpha}$, the second equation is redundant. Moreover, if we set $\alpha = \bar{\alpha} = 1/\chi$, (B.5) implies that the twisted correlator (the analog of (5.23)) is now topological

$$\partial_\chi \mathcal{F}(\chi; 1/\chi, 1/\chi) = 0 \; . \tag{B.6}$$

We take an ansatz for the tree-level contribution to the correlator A in (B.3) which is essentially the same as for the parity-even part G_1 in (5.31), (5.32) in section 5.3. It includes all D-functions that can appear in AdS_2 contact Witten diagrams with up to four derivative vertices

$$\{W_i\} = \{D_{1111}; \; w_{12}^2 D_{2211}, \; w_{13}^2 D_{2121}, \cdots ; \; w_{12}^2 w_{34}^2 D_{2222}, \; w_{13}^2 w_{24}^2 D_{2222} \; , \cdots \} \, . \tag{B.7}$$

The ansatz for A, the analog of (5.32), now has only coefficients that are linear in σ and τ

$$A = \mathrm{t}_{12}\mathrm{t}_{34} \sum_i (\mu_{1,i} + \mu_{2,i}\,\sigma + \mu_{3,i}\,\tau) \, W_i \, . \tag{B.8}$$

Note that the dimension $d = p$ dependence of the D-functions only comes as an overall factor and does not affect the dependence on the cross ratios (as is clear, for example, from the Mellin representation). Therefore, we can compute them for generic d, and set $\chi = \bar{\chi}$, $d = 1$ in the end.

We now require that A should be (i) crossing symmetric, and (ii) satisfy the superconformal Ward identity (B.5). Remarkably, this allows us to determine all the coefficients $\mu_{r,i}$ in the ansatz (B.8) up to an overall factor ν (cf. (B.3))

$$A = \nu \frac{\mathrm{t}_{12}\mathrm{t}_{34}}{|w_{12}|^2|w_{34}|^2}\mathcal{A}\,, \qquad \mathcal{F} = \nu\mathcal{A}\,, \tag{B.9}$$

$$\begin{aligned}\mathcal{A}(\chi;\alpha,\bar{\alpha}) = &\frac{\chi^2-2\chi+2}{\chi-1} - \sigma\frac{\chi\left(2\chi^2-2\chi+1\right)}{\chi-1} - \tau\frac{\chi\left(\chi^2+1\right)}{(\chi-1)^2} \\ &+\left[\frac{(\chi-1)\left(\chi^2+\chi+2\right)}{\chi} - \sigma(\chi-1)\left(2\chi^2+\chi+1\right) - \tau\left(\chi^2+1\right)\right]\log(1-\chi) \\ &+\left[-\frac{\left(\chi^2-2\chi+2\right)\chi^2}{(\chi-1)^2} + \sigma\frac{\left(2\chi^2-5\chi+4\right)\chi^3}{(\chi-1)^2} + \tau\frac{\left(\chi^2-3\chi+4\right)\chi^3}{(\chi-1)^3}\right]\log\chi\,.\end{aligned} \tag{B.10}$$

Note that as predicted in (B.6) the twisted correlator is a constant since

$$\mathcal{F}(\chi;1/\chi,1/\chi) = \nu\mathcal{A}(\chi;1/\chi,1/\chi) = -3\nu\,. \tag{B.11}$$

This twisted correlator, however, can be independently computed using supersymmetric localization [5],[33] so that at leading order in the inverse string tension we should have

$$\mathcal{F}(\chi;1/\chi,1/\chi) = -\frac{3}{\sqrt{\lambda}} + \mathcal{O}(\frac{1}{\lambda})\,. \tag{B.12}$$

This fixes the overall factor to be

$$\nu = \frac{1}{\sqrt{\lambda}}\,. \tag{B.13}$$

The resulting expression (B.9), (B.10) agrees with the one found in [5] directy from the string action.

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