Consensus and Polarization in Competing Complex Contagion Processes

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In many situations, the rate of adoption of new information depends on reinforcement from multiple sources in a way that cannot be described by simple contagion processes. In such cases, contagion is said to be complex. This has been found in the diffusion of human behaviors, innovations, and knowledge. Based on that evidence, we propose a new model considering multiple, potentially asymmetric, and competing contagion processes and analyze its respective population-wide complex contagion dynamics. We show that the model spans a dynamical space in which the population exhibits patterns of polarization, consensus, and dominance, a richer dynamical environment that contrasts with single simple contagion processes. We find that these patterns are present for different population structures. We show that structured interactions increase the range of the dominance regime by reducing that of polarization. Finally, we show that external agents designing seeding strategies, to optimize social influence, can dramatically change the coordination threshold for opinion dominance, while being rather ineffective in the remaining dynamical regions.

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The study of how information – opinions, diseases, innovations, norms, attitudes, habits, or behaviors – spread throughout social systems has occupied the physical and social sciences for decades [1–7]. In that context, the propagation of information has traditionally been assumed to happen through simple-contagion – a contact process in which information spreads through pairwise interactions [8–18]. However, recent empirical evidence suggests that different types of information spread differently [19–24]. In particular, the acquisition of information that is either *risky*, *controversial*, or *costly* seems to require reinforcement from multiple contact sources [6]. Contrary to simple-contagion, these processes result in propagation impediments to, and through, isolated regions of social networks [25] and were coined as complex contagion. Although widely studied in the context of cascading phenomena [26], complex contagion has received, to the best of our knowledge, little attention in the literature of population dynamics [27], a scenario involving the competition between opposing information elements. Here, we provide a characterization of the spread of competing information under complex contagion, exploring the implications for the design of social interventions that are aimed at, for instance, optimizing the spread of social influence.

Let us start by considering a finite population of Z individuals who hold alternatively one of two opinions, say A or B. Thus, at each moment, there are k individuals with opinion A and $Z-k$ with opinion B. The number of contacts an individual i has defines its degree, z_i. Individuals revise their opinion by taking into consideration the opinion composition in their neighborhood. We assume that such events are unilateral and that the likelihood of individuals successfully updating their opinion is reinforced by multiple contact sources [6]. Hence, an individual i with opinion X and n_i^Y contacts with opinion $Y \neq X$ changes to opinion Y with probability

$$p_i^{X\to Y} = \left(\frac{n_i^Y}{z_i}\right)^{\alpha_{XY}}, \quad (1)$$

where X and Y can take the values A or B. Equation 1 allows us to interpolate between scenarios where opinions require few to many reinforcement sources to propagate (notice that with $\alpha_{XY}=1$ the probability of changing strategy is that of a voter model). We say that α_{XY} accounts for the complexity of opinion Y when learned by an individual that holds opinion X. Simpler opinions require less reinforcement from peers, while complex ones require more reinforcement. When $\alpha_{XY} \neq \alpha_{YX}$, we say the population evolves under asymmetric complexities. It is noteworthy that our approach contrasts with the fractional thresholds models, common in the literature of complex contagion [28, 29]. A fractional threshold implies that there is a well-defined threshold of neighbors above which new information is adopted by an individual and below which it is not. Dynamically, such a definition results in a deterministic process that either percolates or that becomes contained to a few elements of the system [30–33].

Our model contains the key elements to study complex contagion of competing non-overlapping processes, though alternative forms could be used[27]. We provide a more general approach in the Supplemental Material [34]. A limiting case happens when we consider well-mixed populations ($z_i = Z-1$), where all possible configurations of the system can be defined by k, the number of individuals with opinion A, and the dynamics are fully described by assessing the transition probabilities

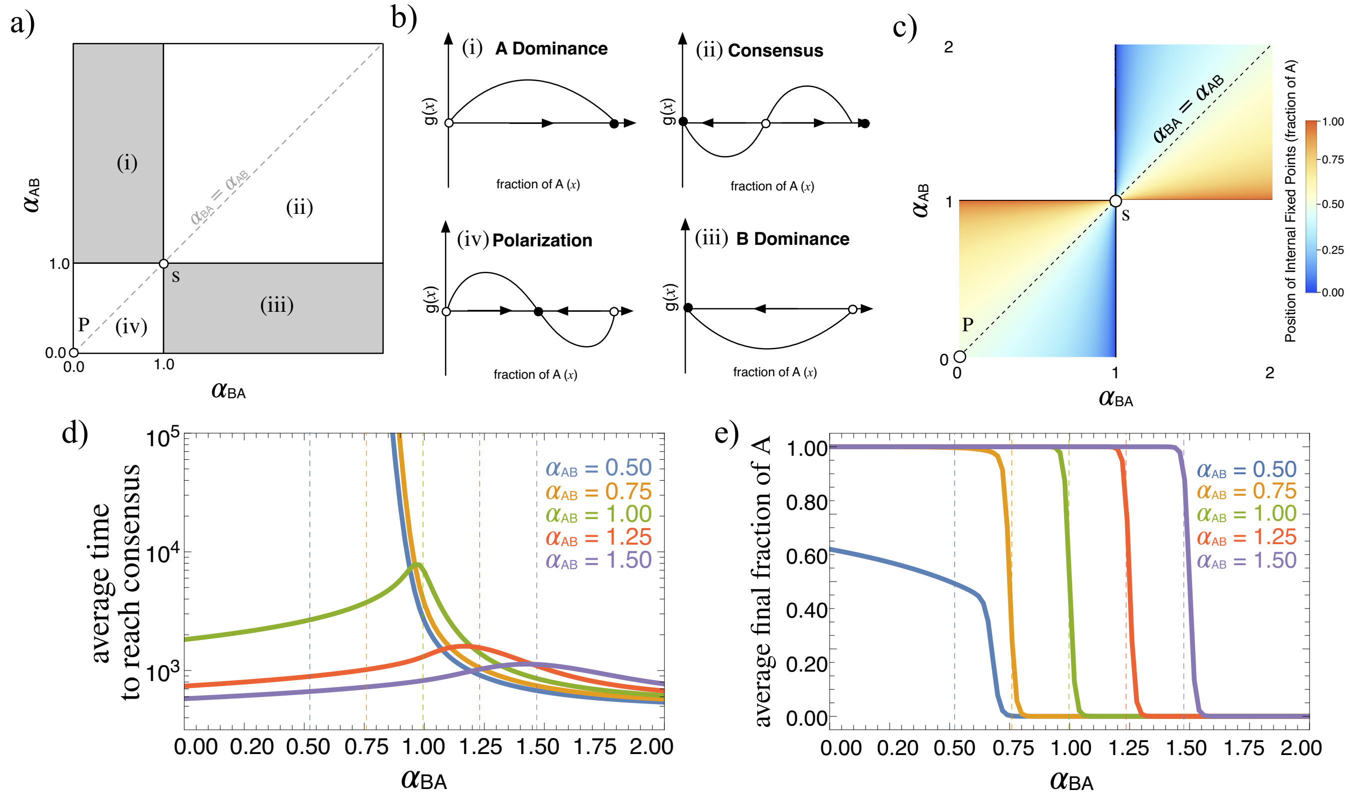

Figure 1. Opinion dynamics under complex contagion in well-mixed populations. **a**) the four different dynamical regions mapping into the parameter space $(\alpha_{AB}, \alpha_{BA})$. **b**) the four possible dynamical patterns (*i.e.*, shapes of $g(x)$) obtained, which correspond to Dominance of B (*i*) or A (*iii*), Polarization (*iv*), and Consensus (*ii*). **c**) Location of the internal fixed point in regions *ii* and *iv*. **d**) average fixation times to reach consensus, when starting from a configuration with equal prevalence of both opinions. **e**) average fraction of As in the equilibrium. In **d**) and **e**) vertical dashed lines indicate $\alpha_{AB} = \alpha_{BA}$. Other parameters: $Z = 100$.

between configurations. Assuming the probability that two changes of opinions occur in a small time interval, τ, to be $O(\tau^{\beta})$, with $\beta > 1$, we can reduce the problem to a one-step process [35] and simply compute the probabilities of increasing or decreasing k by one ($0 \leq k \leq Z$), respectively,

$$T_k^+ = \frac{Z-k}{Z} p^{B\to A} \text{ and } T_k^- = \frac{k}{Z} p^{A\to B}. \quad (2)$$

The rate of change in the average abundance of individuals with opinion A is given by the so-called gradient of selection $g(x)$ [36, 37], where $x \equiv k/Z$. The gradient of selection is equivalent to the drift term in the Fokker-Planck equation describing stochastic processes [35] and thus can be used to accurately characterize properties of finite population distributions [38]. In the limit of very large populations, $Z \to \infty$, the dynamics can be described by a non-linear differential equation of the form

$$\dot{x} \equiv g(x) = x(1-x)(x^{\alpha_{BA}-1} - (1-x)^{\alpha_{AB}-1}), \quad (3)$$

which, for $\alpha_{XY} \neq 1$, has two trivial solutions – at $x = 0$ and $x = 1$ – and an additional internal fixed point that can be inspected by solving the transcendental equation

$$1 - x = x^{\gamma}, \quad (4)$$

where $\gamma = (\alpha_{BA}-1)/(\alpha_{AB}-1)$. A detailed derivation of Eq. 3 and Eq. 4 can be found in [34]. Equation 3 holds a similar form to the Replicator Equation from Evolutionary Game Theory [39], where $x^{\alpha_{BA}-1}$ and $(1-x)^{\alpha_{AB}-1}$ play the roles of the fitness of individuals with opinion A and B, respectively. Indeed, the dynamical patterns of opinion Dominance, Polarization, and Consensus derived from Eq. 3 are identical to the Prisoner's Dilemma, Stag Hunt, and Snowdrift Game [40, 41] so often studied in that literature. This result provides another interpretation of competitive complex contagion processes: while here individuals change opinions unilaterally, this process is equivalent to a fitness-driven contact process.

Figure 1a shows how the different dynamical patterns map into the $\alpha_{AB} \times \alpha_{BA}$ domain, while Fig. 1b illustrates the different shapes of $g(x)$ that characterize the dynamics in each region. In region (*ii*), $g(x)$ is characterized by an unstable internal fixed point leading to a coordination-like dynamics towards a consensus, which depends only on the initial abundance of opinions. In region (iv), $g(x)$ has an internal stable fixed point that results in polarization of opinions, which is characterized by the sustained prevalence of both opinions. In both cases the specific location of the internal fixed point depends only on the relationship between the complexities of both opinions (see Fig. 1c). In regions (i) and (iii), $g(x)$ does not have any internal fixed point and the population will invariably be dominated by one of the two opinions. Two special points are also represented, S and P. In S, $\alpha_{AB} = \alpha_{BA} = 1$, every possible configuration of the system corresponds to a fixed point, and finite populations evolve under neutral drift. For P, $\alpha_{AB} = \alpha_{BA} = 0$, the dynamics on finite populations reduces to an Ornstein-Uhlenbeck process, with linear drift and constant diffusion.

A particularly interesting measure in finite populations is the time (τ_{k_0}) the population takes to reach a consensus when starting from configuration k_0. Although in finite populations the system always reaches a consensus, the time required to do so can be extremely large, in particular for region (iii). Indeed, for that region, even for small populations of 100 individuals, the time to consensus is of the order of 10^{13} generations, making it more likely to find the population in a polarized state, as we would expect from the existence of a stable fixed point. Figure 1d shows the time required for reaching consensus starting from a perfect mix of opinions (50-50) for different combinations of α_{AB} and α_{BA}. Figure 1e shows the evolutionary outcome under the same conditions.

Next, we turn our attention to the case in which individuals are only able to interact with a small subset of the population ($z_i << Z$). We model this by means of a complex network of social interactions, where nodes correspond to individuals and links to social interactions between pairs of individuals. We focus on three classes of networks: Homogeneous Random (**HRND**) [42]; Erdős–Rényi Random (**ER**) [43]; and Scale Free Barabási-Albert (**SF**) [44]. A detailed description of how these networks were generated is in [34].

Figure 2 compares the results of opinion dynamics under complex contagion in different populations structures along the four dynamical regions of interest: Consensus, A Dominance, Polarization, and B Dominance. With the exception of the Polarization region, the dynamical patterns observed in structured populations follow what was previously observed in well-mixed populations. However, population structure significantly shortens the domain where Polarization is observed (region iv). The mismatch is highlighted in Fig 2b. In [34] we further expand this analysis and show that, despite the differences in the Polarization, the dynamical outcomes are identical to the ones obtained in well-mixed populations. Moreover, we also point the larger time for fixation in structured populations in the consensus regime relative to well-mixed populations (see Fig 2a).

An increasingly relevant application of opinion dynamics concerns the design of seeding strategies to maximize the spread of social influence [45, 46]. However, past works have ignored the possibility of competition among multiple opinions with different complexities. In such cases, the problem of interest is not understanding the size of a cascade or the time to consensus, but how seeding strategies impact the dynamical properties of opinion diffusion. Here, we employ three simple seeding strategies: placing individuals with opinion A along the nodes of the network either 1) randomly, or preferentially in 2) higher degree nodes or 3) lower degree nodes. Since these strategies do not apply to **HRND** networks, we will limit our analysis to their impact on **SF** populations in the main text and on **ER** populations in [34].

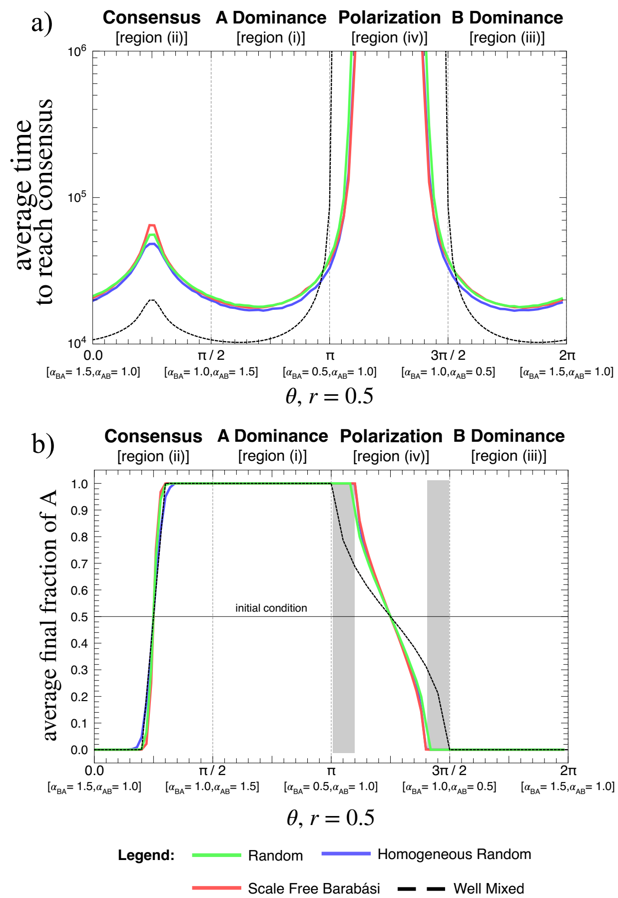

Figure 2. Opinion dynamic under complex contagion in structured populations. Panel **a**) shows the fixation times, measured in Monte Carlo steps, for different population structures. Panel **b**) shows the average final fraction of individuals with opinion A in the population for different population structures. Shaded areas indicate the mismatch between well-mixed and structured populations in the Polarization region (iv). The complexity parameters have been reparameterized as $\alpha_{BA} = 1 + r\cos(\theta)$ and $\alpha_{AB} = 1 + r\sin(\theta)$, with $r = 1/2$. Notice that $\gamma = \tan(\theta)$, which defines the position of the internal fixed point in Eq.(4), is independent of r. Other parameters: $Z = 10^3$ and average degree of 4.

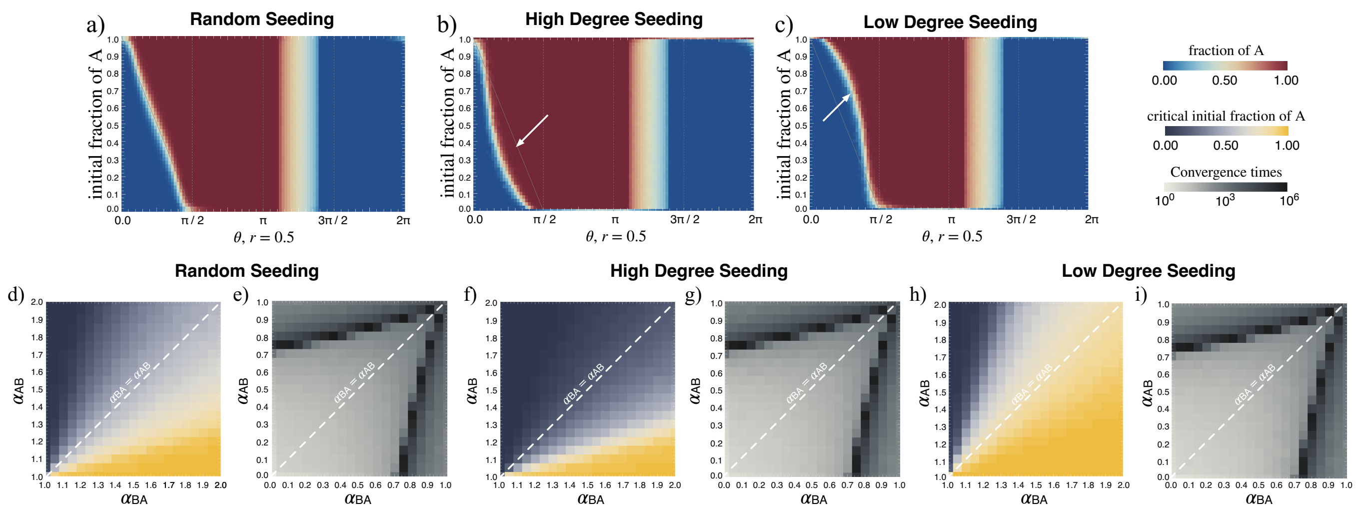

Figure 3. The impact of seeding strategies – Random (a, d, and e), High Degree (b, f, and g), and Low Degree (c, h, and i) – in Scale Free Networks. Panels a–c show how the different seeding strategies impact the dominance of opinions for different initial fractions of seeded As along the four dynamical regions. Arrows in panels b) and c) point to the deviations from the random seeding strategy. Panels d, f, and h show the critical initial fraction of seeded As needed to obtain a dominance of As in the consensus region in more than 50% of the simulations. Panels e, g, and i show the average time of convergence towards the polarized state starting from a configuration with 20% As in the population in the polarization region. Averages taken over 2500 independent simulations and 2.4 million steps. Other parameters: $Z = 10^3$ and average degree of 4.

Figures 3a–c show how the different seeding strategies impact the dominance of opinion A for different initial fractions of seeded As along the four dynamical regions. Interestingly, while seeding strategies have an impact in the consensus region (i), they do not alter the dynamical properties of the remaining regions. These results suggest that seeding strategies operate by changing the coordination threshold between opinions, being ineffective in regions where such coordination is absent.

Figures 3d,f,h show the critical initial fraction of seeded As that are necessary, according to each strategy, to promote an A Dominance regime [47]. Note that, following the findings on Fig. 3a–c, we limit our analysis to the consensus region (ii). We find that seeding preferentially towards higher degree nodes has a major effect in increasing the effectiveness of seeding, in the sense that it is possible to achieve the full dominance of As by seeding a rather small number of individuals, widening the range of scenarios where A is dominant to regions where it is more complex than B. Conversely, seeding low degree nodes is shown to be an ineffective strategy. In Fig. 3e,g,i, we restrict our analysis to the polarization region (iv) and compare the first passage time in the equilibrium for the different seeding strategies. Finally, seeding preferentially towards higher degree nodes speeds up the convergence times towards equilibrium in the polarization regime. Similar qualitative results are obtained in **ER** networks [34].

In this letter we presented a new model of competitive complex contagion dynamics that contains dynamical patterns of Dominance, Polarization, and Consensus, depending only on the relative complexity of the diffusing information. These patterns are in many ways equivalent to the ones obtained in other contexts, namely in Evolutionary Game Theory, which deals with problems as diverse as the spin-flips [48], selection of gut biome [49], management of common and public goods [50, 51], and socioecological resilience [52]. Our work raises important questions in terms of feasibility of assessing empirically which mechanisms are at play. Are empirical patterns the result of game-theoretical reasoning of agents that influence strategy adoption or the result of the spreading of information with different levels of complexity? This equivalence, however, occurs when considering the processes are dependent on the fraction of the neighboring types, as structured populations are otherwise known to change the macroscopic nature of the evolutionary dynamics [53].

Social planners or other network intervenients who aim at understanding the macroscopic behavior of the population can derive important conclusions from our work. In fact, we show that polarization results from low acquisition complexity and, in that case, targeting of individuals might have a small impact on the final outcome. Conversely, interventions based in seeding strategies are only effective in domains characterized by a coordination-like dynamics (consensus), where both propagating processes have rather high levels of complexity. Alternatively, planners can act in the system by modifying the complexity of what is being spread to improve the chances of getting a dominant behavior. This, however, can lead to an *arms race* and drive the system to a polarization trap.

Finally, open questions have been left for future research as, for instance, expanding the model to scenarios that involve more than two competing opinions and the exploration of optimal seeding strategies in competitive scenarios under complex contagion.

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