Paper Money

By Christopher A. Sims

Drastic changes in central bank operations and monetary institutions in recent years have made previously standard approaches to explaining the determination of the price level obsolete. Recent expansions of central bank balance sheets and of the levels of rich-country sovereign debt, as well as the evolving political economy of the European Monetary Union, have made it clear that fiscal policy and monetary policy are intertwined. Our thinking and teaching about inflation, monetary policy, and fiscal policy should be based on models that recognize fiscal-monetary policy interactions. (JEL E31, E52, E58, E62, H63)

I. Introduction

Central banks since 2008 in many countries have greatly expanded their balance sheets, rapidly creating large amounts of what used to be called “high powered money,” without creating inflation. The European Central Bank’s policies and proposals for Europe-wide bank supervision are at the center of hot disputes because of their fiscal policy implications. The United States and Japan are accumulating debt at rates that are unprecedented in peacetime, which some worry may eventually generate inflation. Most central banks now pay interest on reserve deposits, making those deposits part of the government’s interest-bearing debt. These developments make it clear that monetary and fiscal policy are tied together, and that conventional macro models with non-interest-bearing high-powered money, a “money multiplier,” and a tight relation between the price level and the quantity of “money” are inadequate as a framework for current policy discussions.

The literature on the fiscal theory of the price level (FTPL) integrates discussion of monetary and fiscal policy, recognizing that fiscal policy can be a determinant, or even the sole determinant, of the price level. The first papers in the area may have seemed technical—they showed that when the government budget constraint

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† Many of the insights of the FTPL literature are implicit in Neil Wallace’s earlier paper (1981). Wallace presented a model in which conventional monetary policy actions had no effect on the price level so long as fiscal policy were held constant. Of course this implied that in his model fiscal policy determined the price level, though Wallace did not put it that way.
is properly taken into account, conditions for existence and uniqueness of the price level in dynamic general equilibrium were different from what they seemed in conventional models. They may also have seemed esoteric—they questioned conventional analysis most strongly in policy configurations that in the 1990s seemed unlike any observed in rich economies in recent history.\(^2\)

This paper tries to bring FTPL down to earth. It begins by citing some results from FTPL analysis and showing how they apply to current policy discussions. It then presents a couple of simple models illustrating FTPL, using them to make clear how the results used earlier in the paper arise and to refute some of the objections to and fallacies concerning FTPL that still circulate among economists.

**II. Insights from FTPL and Their Application**

**A. Monetary Policy Actions, to be Effective, Must Induce a Fiscal Policy Response**

This is easiest to understand in high-inflation, high nominal debt economies where fiscal policy is frozen by political deadlock or chicanery. In such an economy, the interest rate will be high, and with a high level of debt, the interest expense component of the budget is substantial, possibly even dominant. If inflation rises still higher, the usual monetary policy prescription would be for the policy interest rate to increase, by even more than the rise in the inflation rate.\(^3\) But if the legislature in such an economy is gridlocked, the central bank may realize that an interest rate increase will pass right through the government budget, with an increased rate of issue of nominal debt the only fiscal effect of the interest rate rise. If this is indeed the situation, and private sector bond buyers understand the situation, the interest rate rise will have no contractionary effect. Indeed, it will increase the rate of inflation rather than decrease it. The central bank, understanding this, may then forgo following the conventional policy prescription. In doing so, it is not accelerating the inflation, it is damping it.\(^4\)

In the 1990s, this may have seemed an analysis that applied at most to some mismanaged Latin American economies, but consider the reasoning many economists (including me) have used to argue that the great expansion of the balance sheet of the US Federal Reserve system in the last four years need not generate inflationary pressure. The Fed now has the authority to pay interest on reserve balances. While the rates are now low (though still in excess of rates on short term US Treasury Bills), the Fed is free to raise them if inflationary pressures arise. Even if it undertakes no open market operations to change the amount of reserves, raising the rates on reserves would have a powerful contractionary effect. Banks would have little incentive to expand their lending if perfectly safe reserve deposits paid interest at a rate comparable to loan rates.

\(^2\)Some of the important papers in this area are Leeper (1991), Woodford (1995), Cochrane (1999), and Davig and Leeper (2007).

\(^3\)This is sometimes called the “Taylor principle,” that monetary policy should respond to inflation changes strongly enough to raise real rates when inflation increases, and lower them when inflation decreases.

\(^4\)Loyo (1999) discusses a period in Brazil where this analysis applies.
But this story, like any story about the effects of monetary policy, has a fiscal policy backstory. Because reserve deposits and Treasury securities are close substitutes, raising the rate on reserve deposits would also raise rates on government debt generally. The level of US government debt relative to GDP is at unprecedented levels. If debt were at 100 percent of GDP, a rise in interest rates to 6 percent from its current level of about 2 percent would bring interest expense, now less than 10 percent of total Federal government expenditure, to 30 percent of government expenditure, increasing the conventionally measured deficit drastically if there were no response of fiscal policy. Would there be a response? Some years ago the answer might have been, “Surely yes.” But the increase in the conventional deficit would be so large that the response would have to involve substantial increases in tax revenue. With recent repeated congressional games of chicken over the debt limit and inability to bargain to a resolution of long-term budget problems, the answer may now be in some doubt.

With the central bank keeping interest rates stable in the face of inflation fluctuations and fiscal authorities not increasing primary surpluses in response to increased real debt, the price level is still likely to be determinate. But the main determinant of inflation becomes the fiscal deficit, rather than changes in the usual instruments of monetary policy.

B. Paper Money Requires Fiscal Backing

It is easy to construct models of economies in which unbacked paper money can have value, but in such models it is generally also possible for money to be valueless, or to dwindle rapidly in value so that the economy approaches a barter equilibrium. In such models, introducing taxation either to pay interest on government liabilities or to contract the supply of non-interest-bearing liabilities (and thus, via deflation, create a real return) tends to resolve the indeterminacy and provide a uniquely determined price level. The first two examples in Section III below show quite different models in which this pattern of results hold.

Depending on the institutional setup, the fiscal backing can be apparent in equilibrium, as with taxation to service a stable volume of nominal debt, or it can be implicit, invoked only under unusual circumstances, as with a commitment to treasury transfers to the central bank if the central bank balance sheet deteriorates. But in evaluating monetary and fiscal institutions, the question of the nature of fiscal backing for the price level is a useful starting point. It led me (Sims 2004; 1999) to think about where fiscal backing could come from in the European Monetary Union (EMU) and what kinds of conditions might force the EMU to confront the need for fiscal backing. Those two papers speculated about policy dilemmas that at the time might have been seen as obscure and unlikely, but now seem practically relevant. The policy discussion in the EMU during this recent crisis period has focused on fiscal transfers that will arise as partial default on Greek and possibly other euro area sovereign debt occurs. While resolving the allocation of these losses that have already occurred is important, controversy over them has hindered discussion of ways to provide clear fiscal backing for the euro, which is in many ways an easier problem. I discuss these issues in more detail in Sims (2012).
C. Central Bank Balance Sheets Matter Because They Connect Monetary and Fiscal Policy

Formerly, there were monetary economists who argued that the central bank balance sheet is an accounting fiction, of no substantive interest. It is true that the implications of having negative net worth at current market values are different for a central bank and an ordinary firm or private bank. A firm with negative net worth is likely to find its creditors demanding payment, and is unlikely to be able to pay them all. A central bank can “print money”—offer deposits as payment for its bills. It will not be subject to the usual sort of run, then, in which creditors fear not being paid and hence demand immediate payment. Its liabilities are denominated in government paper, which it can produce at will.

On top of this, most economists have thought of central banks as part of the government, with the only balance sheet that really matters being that of the government as a whole.

But both of these arguments come apart when the central bank aims at controlling the price level and fiscal and monetary policy are not set jointly. Traditional contractionary open market operations require selling assets to shrink the amount of reserve deposits and currency. If the central bank is in the red, an aggressively contractionary policy may not be possible, because people will see that it would require selling more assets than the central bank actually has. If the negative-net-worth central bank tries to contract by raising interest rates on reserves, yet wants to avoid expanding reserves, it is likely to need to sell assets to finance the interest on reserves, again putting it on an unsustainable path. Of course, if the treasury stands ready to back up the central bank—providing additional assets to the central bank in the form of interest-bearing securities whenever necessary—then the central bank balance sheet is indeed irrelevant.

In thinking about central bank policy when fiscal backing from the treasury is absent or uncertain, it helps to consider what “fiscal backing” the central bank can provide on its own, without assistance from the treasury. Of course central banks cannot impose explicit taxes, but they do have access to an implicit tax: seignorage. Even if its balance sheet shows negative net worth at current market values, a central bank can maintain a uniquely determined price level by using its seignorage revenues to restore its balance sheet. But seignorage revenue depends on the inflation rate, generally increasing with the rate of inflation except at extremely high inflation rates. A central bank with a severely enough impaired balance sheet may not be able to pin down the price level without treasury assistance, but modestly negative net worth can generally be “worked off” by seignorage. Of course most central banks see their task as maintaining low inflation, so balance sheet problems, by requiring inflation to generate seignorage, can be an obstacle to the central bank’s achieving its policy objectives. Even a central bank with positive net worth may be inhibited in taking some policy actions by fear of the consequences of negative net worth. Lender of last resort operations, for example, even when they have positive expected return, generally pose some risk of losses. A central bank with uncertain

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5 Sims (2005) showed that this insight generalizes to Taylor rules, where negative net worth puts an upper bound on the coefficient on inflation in the monetary policy rule.
fiscal backing may hesitate to undertake such operations out of fear of their balance sheet consequences.

A simple model of balance sheet dynamics for a central bank with negative net worth and no fiscal backing appears in my earlier paper, Sims (2005). However, that paper assumes the central bank has non-interest-bearing liabilities and short-term treasury bonds as assets. The US Federal Reserve system pays interest on its reserve liabilities and holds substantial amounts of long-maturity debt. It has been argued (by me, among others) that the Fed can take contractionary action by raising the rate paid on reserves, without necessarily selling its assets. But this argument assumes fiscal backing. Without it, high interest on reserves, while interest on long-maturity debt remains low, can create negative seignorage, even in the presence of inflation.

In the United States and the euro area today, it is not certain that fiscal repair of central bank balance sheets would emerge. In the euro area, there is a formal “capital key,” specifying in what proportions governments in the EMU should provide capital when the European Central Bank (ECB) calls for a capital infusion. But if the capital called for were substantial, and the call came in the wake of ECB policy actions that were politically unpopular in some countries, the provision of capital might not be automatic. Perhaps equally important is that, foreseeing the risk of a capital call and its implicit fiscal transfers, the ECB’s governing board might refuse to authorize market-stabilizing actions by the ECB that an ordinary central bank would have undertaken.

In the United States, the risk is that the need for capital infusion would most likely arise in the wake of stringent monetary policy tightening that caused capital losses on the Fed’s long term debt holdings and required an increased stream of interest payments on reserves. These actions would be restraining growth and forcing Congress to confront increased deficits arising from increased interest expense. In an environment where Congress cannot agree to let the debt limit increase to accommodate its own spending and revenue legislation, it is not hard to imagine Congress blaming the Fed for the painful decisions it faces and in the process casting doubt on its commitment to recapitalize the Fed.

D. Nominal Debt and Real Debt are Very Different

Real sovereign debt promises future payments of something the government may not have available—gold, under the gold standard, euros for individual country members of the EMU, and dollars for developing countries that borrow mainly in foreign currency. Nominal sovereign debt promises only future payments of government paper, which is always available. Both types of debt must satisfy the equilibrium condition that the real value of the country’s debt is the discounted present value of future primary surpluses—revenues in excess of expenditures other than interest payments. But if an adverse fiscal development increases debt, the increased real debt will require increased future primary surpluses, whereas with nominal debt there are two other ways to restore balance—inflation, which directly reduces the real value of future commitments, and changes in the nominal interest rate, which will change the current market value of long term debt.

Obviously outright default on nominal debt is much less likely than default on real debt. So long as the country is capable of generating any positive stream of primary
surpluses, its debt will have non-zero real value. But if debt is real and the country finds itself unable to maintain primary surpluses above its predetermined real debt service commitment, it must default, even if in absolute terms it is running substantial primary surpluses.

Nominal Debt is a Cushion, Like Equity.—In a deterministic steady state, investors will insist on nominal interest rates high enough to compensate for inflation’s effect on the real value of their debt holdings. Real returns on government debt will be the same whether it is nominal or real. But if nominal interest rates fall after the date of issue of long-term nominal debt with a fixed coupon rate, the market value of the debt will rise, providing the debt holder with an unanticipated higher return. If inflation occurs at a higher than expected rate, the real value of nominal debt, whatever its maturity, suffers an unanticipated decline. These mechanisms can cushion the impact of unexpected changes in the fiscal situation. We live in a stochastic world, and surprises in returns on government debt from these two mechanisms are substantial. Figure 1 shows a time series of surprise gains and losses on US government debt as a fraction of GDP. The surprise gains and losses relative to GDP have been of the same order of magnitude as year to year fluctuations in the

\[\text{Gain or loss over GDP} \]

\[\text{−0.02} \quad 0.00 \quad 0.02 \quad 0.04\]

\[\begin{array}{c}
1960 \quad 1970 \quad 1980 \quad 1990 \quad 2000 \quad 2010
\end{array}\]

\[\text{Figure 1. Surprise Gain or Loss to US Debt Holders as Proportion of GDP}\]

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6 The order of magnitude of the surprise gains and losses in the chart is robust to various ways of computing them, but the time path itself is not. Different ways of computing expected inflation and one-year returns on long debt change the results. The results in the chart treat the Federal Reserve system as outside the government, so its gains and losses are included. Since 2009, with the Fed’s expanded balance sheet and interest being paid on reserves, treatment of it as inside or outside the government matters a great deal, and proper treatment of its interest-bearing reserve liabilities is a challenge. See the data Appendix.
fiscal deficit. Were they displayed as fractions of the value of outstanding debt, so they became surprises in rates of return, they would be much larger. It is clearly not a good approximation to model the US economy as if debt were real, even though a considerable part of the literature on optimal fiscal policy does so.

The southern countries in the euro area are now reckoning with the consequences of their having, by joining the euro, made their sovereign debt real. The 2008–2009 crisis led to great expansion of their debts, and the nominal debt cushion is not available to them. Greece already has defaulted on its debt, and quite possibly before the crisis resolves other southern tier EMU members will as well.

When only distorting taxes are available, there is a benefit to keeping tax rates stable. A highly variable time path of tax rates produces higher deadweight loss than a more stable path that delivers the same cumulative revenue. With nominal debt, flexible prices, and costless inflation, it is optimal to keep revenue very stable, allowing inflation to absorb most of any fiscal shocks. While this result is well known, Siu (2004) and Schmidt-Grohé and Uribe (2001) have shown that when nominal rigidities are present, variation in inflation becomes costly, and that this leads to very little use of inflation to smooth tax rates except (as Siu shows) when fiscal disturbances are very large. The model in Section IIIC below argues that this conclusion is sensitive to those papers having allowed only for one-period debt. With longer debt, the costs of tax smoothing via surprise inflation can be much lower. In any case, the economic situation in the southern-tier European countries probably reflects the “very large fiscal shock” case.

Nominal Debt is (Almost) Non-Defaultable, Hence Important to the Lender of Last Resort.—By the usual indicators, the first two countries in Table 1 are not in noticeably worse shape than the last three. But the last three are selling their bonds at much lower interest rates. This reflects the fact that the last three issue mainly nominal debt, denominated in their own country currency, while the first two have issued real (i.e., euro) debt. There is a non-trivial probability that the first two will default in some form, while the latter three are quite unlikely to default, because their debt is nominal.

Economists and journalists sometimes treat inflation as a form of default, but it is not. Default is a situation where the contracted payments cannot be delivered, and the contract does not specify what happens in that eventuality. For private firms, this leads to renegotiation and/or court proceedings. There can be a long period in which investors cannot get access to their investments and the amount that will be returned to them remains unknown. Creditors holding different maturities or types of debt

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### Table 1—Debt to GDP Ratios and Government Bond Interest Rates for Five Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Debt/GDP</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>48</td>
<td>5.64</td>
</tr>
<tr>
<td>Italy</td>
<td>117</td>
<td>4.95</td>
</tr>
<tr>
<td>US</td>
<td>77</td>
<td>1.75</td>
</tr>
<tr>
<td>Japan</td>
<td>175</td>
<td>0.77</td>
</tr>
<tr>
<td>UK</td>
<td>83</td>
<td>1.76</td>
</tr>
</tbody>
</table>

Notes: Debt/GDP is for 2010 for all countries. Bond interest rate is for October, 2012 for all countries except Japan; for Japan it is September 2012.
may suffer different degrees of loss, and the allocation of losses across creditors may be uncertain. For example, a minor default may involve a modest delay in returning principal of a short term debt. Other creditors may be unaffected, or, if the holder of the short debt goes to court, all debtors may find themselves impaired. Similar, or perhaps more severe, uncertainties surround sovereign default.

Unanticipated high inflation does impose losses on investors in nominal sovereign debt, but it does not involve renegotiation or court proceedings. Contracted payments are made. The securities remain tradable. In the same state of uncertainty about future primary surpluses, therefore, investors are likely to be much more uncertain about the return on their own investment in sovereign debt when resolution of the fiscal imbalance has to come from default, rather than inflation and nominal interest rate changes.

In a financial panic, counterparty risk becomes pervasive among market participants and credit markets freeze up. An institution of unquestioned soundness and liquidity can remedy the situation by lending freely. While large private banks can and have historically sometimes acted as such a lender of last resort, any private institution that attempts it risks itself becoming subject to worries about liquidity. A central bank, backed by a treasury that can run primary surpluses and issue nominal debt, is an ideal lender of last resort. Because it can create reserve money, it need never default. If it takes capital losses, and it is not backed by a fiscal authority, it could be forced to run a high inflation to restore its balance sheet, but this will not be a problem if it has fiscal backing. Europe, in setting up its Monetary Union, did not contemplate the ECB’s taking on a lender of last resort role. Individual country central banks can no longer play the role, because they have no independent authority to create reserve money and their country treasuries issue only real debt. During the recent crisis the ECB has in fact played a lender of last resort role, though its effectiveness is limited because its actions and announcements in this role are regularly criticized by some northern-tier economic officials.

III. Models

A. Samuelson’s Pure Consumption Loan Model with Storage

This model is one where, without tax backing for debt or money, the price level is indeterminate. The model in that case has one stable price level, in which the real allocation is efficient, and a continuum of other possible initial price levels, each of which corresponds to an inefficient equilibrium in which the real value of government debt or money shrinks toward zero. If the government runs a primary surplus (revenues in excess of non-interest expenditures), private agents see the future taxes as reducing their spending power. They will therefore save (attempt to accumulate money or government debt), until the price level is low enough that the value of their government paper matches the present value of their future taxes. This mechanism eliminates the non-uniqueness, no matter how small the primary surplus, and for small levels of primary surplus, the real allocation is arbitrarily close to the efficient one.

There is an infinite sequence of periods, in each of which the same number of two-period-lived agents is born and endowed with one unit of the consumption good,
grain. The grain can be stored, but decays in storage by a factor $\theta$. There is also
government debt, denominated in dollars. Its amount at the initial date $t = 1$ is $B_0$, and it is held by the initial old, who redeem it with the government, receiving in return new one-period debt in the amount $B_1 = R_0 B_0$. Since this new government paper is worthless to the initial old, they attempt to sell it to the initial young, for grain. The price level at date $t$ is the rate at which grain trades for newly issued government debt. This process repeats thereafter for $t = 1, \ldots, \infty$.

Formally, the generation born at $t$ maximizes its lifetime utility $U(C_{1t}, C_{2,t+1})$ subject to the constraints

\begin{align}
(1) \quad & C_{1t} + S_t + \frac{B_t}{P_t} = 1 \\
(2) \quad & C_{2,t+1} = \frac{R_t B_t}{P_{t+1}} + \theta S_t \\
(3) \quad & S_t \geq 0, \quad B_t \geq 0.
\end{align}

Because the government is doing nothing but rolling over the debt each period, the market clearing condition is simply $R_t B_t = B_{t+1}$. The government sets an arbitrary value for $R_t$ each period. The first-order conditions for an agent in generation $t$, assuming perfect foresight about next period’s $P$, are

\begin{align}
(4) \quad & \partial C_1: \quad D_1 U(C_{1t}, C_{2,t+1}) = \lambda_t \\
(5) \quad & \partial C_2: \quad D_2 U(C_{1t}, C_{2,t+1}) = \mu_{t+1} \\
(6) \quad & \partial B_t: \quad \frac{\lambda_t}{P_t} = \frac{R_t \mu_{t+1}}{P_{t+1}}, \quad \text{if } B_t > 0 \\
(7) \quad & \partial S_t: \quad \lambda_t = \theta \mu_{t+1}, \quad \text{if } S_t > 0.
\end{align}

The $B$ and $S$ first order conditions tell us, as we would expect, that if agents are storing grain and also buying debt, their returns must match, so that in that case

\begin{align}
(8) \quad & \frac{R_t P_t}{P_{t+1}} = \theta.
\end{align}

In order to get easily computed solutions that give us some insight into how the model works, we assume $R_t$ is constant and

\begin{align}
U(C_{1t}, C_{2,t+1}) = \log(C_{1t}) + \log(C_{2,t+1}).
\end{align}

Then the Lagrange multipliers can be solved out to deliver

\begin{align}
(9) \quad & \frac{R_t P_t}{P_{t+1}} = \frac{C_{2,t+1}}{C_t}, \quad \text{if } B_t > 0
\end{align}
\[ \theta_t = \frac{C_{2,t+1}}{C_t}, \quad \text{if } S_t > 0. \]

Let savings be represented by \( W_t = S_t + B_t/P_t \). Logarithmic utility makes solution easy because it implies that whatever the rate of return to savings, call it \( \rho_t \), we will have \( \rho_t = C_{2,t+1}/C_{1,t} \), and this in turn implies that

\[ C_{1,t} + W_t = 1 = C_{1,t} + \frac{C_{2,t+1}}{\rho_t} = 2C_{1,t}. \]

Thus savings is always half the endowment, i.e., 0.5.

This economy has an equilibrium in which there is no storage and nominal debt has value (i.e., \( P_t < \infty \)). With no storage, \( C_{1,t} = 0.5 \) and, since savings is all used to buy debt from the older generation, \( C_{2,t} = 0.5 \) also. This means \( \rho_t \equiv 1 \) and therefore \( R = P_{t+1}/P_t \), all \( t \). In other words, the price level grows at the gross interest rate and the real value of both newly issued and maturing debt is constant at 0.5. In order for this equilibrium to prevail, the initial price level \( P_1 \) must be \( 2B_1 \), i.e., \( 2RB_0 \).

The economy also has equilibria in which \( S_t > 0 \), however. In these equilibria, of course, \( \rho \equiv \theta = RP_t/P_{t+1} \). In other words, The price level grows not at the rate \( R \), but at the higher rate \( R/\theta \). The nominal debt still grows at the rate \( R \), however, so the real debt shrinks over time, with

\[ \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}}. \]

The economy can start with any \( B_1/P_1 < 0.5 \). Storage will then be \( S_1 = 0.5 - B_1/P_1 \). In subsequent periods, \( S_t \) increases toward 0.5 as the real value of savings in the form of nominal bonds shrinks toward zero.

In other words, every initial price level \( P_1 \) that exceeds \( 2RB_0 \), including \( P_1 = \infty \) (in which case bonds are valueless and all savings is in the form of storage), corresponds to a perfect-foresight equilibrium in this economy. This is an economy with an indeterminate price level.

Note that the economy’s resource constraint is \( C_{1,t} + C_{2,t} + S_t = 1 + \theta S_{t-1} \): consumption and storage by the young plus consumption by the old is endowment of the young plus the proceeds from storage by the old. Since in all the equilibria with positive storage \( S \) is either increasing or (when \( P_t = \infty \)) constant, and since \( C_{1,t} \equiv 0.5 \), \( C_{2,t} < 0.5 \) in all these equilibria with \( S_t > 0 \). That is, these equilibria with \( S_t > 0 \) are strictly worse than the one in which \( S_t = 0 \). It may be comforting to believe that somehow these worse equilibria would be avoided, but there is nothing in the structure of the model that should make the worse equilibria less likely.

When we use the “\( B \)” and “\( R \)” notation as here, it is perhaps unsurprising that we get an indeterminate equilibrium when the government issues debt without any backing from taxation. But if we replace \( B \) by \( M \) and set \( R = 1 \), this becomes Samuelson’s model of “money” and is sometimes taken as a useful metaphor to aid understanding of how fiat money can have value.
But back to thinking of it as debt. What if we do provide tax backing for the debt? Suppose everything is as before, but now the government imposes a lump sum tax \( \tau \) on the young each period. The government budget constraint is now

\[
\frac{B_t}{P_t} = \frac{RB_{t-1}}{P_t} - \tau. \tag{12}
\]

Suppose there were an equilibrium in which savings is in the form of both bonds and storage. Then both must have real gross rate of return \( \theta \). That makes the government budget constraint

\[
\frac{B_t}{P_t} = \theta \left( \frac{B_{t-1}}{P_{t-1}} - \tau \right). \tag{13}
\]

This is a stable difference equation in \( B_t/P_t \). If it starts operation at \( t = 0 \), we will have

\[
\frac{B_t}{P_t} = \sum_{s=0}^{t-1} - \tau \theta^s + \theta^t \frac{B_0}{P_0}. \tag{14}
\]

But notice that, since \( \theta < 1 \), the right-hand side of this expression eventually becomes negative, converging as \( t \to \infty \) to \(-\tau/(1-\theta)\). That is, if the economy started on a path satisfying this condition, eventually it would reach a point where the government is putting grain in the amount \( \tau \) on the market to exchange for mature debt, but no one would have any debt to exchange for it. Anyone foreseeing this would have a motive for holding on to some debt to exchange for grain at an extremely favorable price ratio when everyone else had run out. So these paths cannot be equilibria. By imposing the tax, no matter how small, the government has eliminated all those equilibria in which storage and debt coexist. It has also eliminated the equilibria in which debt is valueless \( (P_t = \infty) \) for the same reason: the government is trying to exchange \( \tau \) units of grain per capita for mature debt, so the mature debt is necessarily of some value.

If there is no storage, the rate of return on debt can be positive. The tax is recognized by individual agents as reducing their wealth, so first-period consumption is reduced. Formally, the private budget constraint in the first period is now

\[
C_{1t} + \frac{B_t}{P_t} + \tau = 1. \tag{15}
\]

We will still have, from the first-order conditions, \( RP_t/P_{t+1} = \rho_t = C_{2,t+1}/C_t \), where \( \rho_t \) is just notation for the real rate of return. Using these last two expressions to rewrite the first-period budget constraint, we have

\[
C_{1t} + \tau + \frac{C_{2,t+1}}{\rho_t} = 1 = C_{1t} + \frac{\rho_t C_{1t}}{\rho_t} + \tau. \tag{16}
\]
Thus,
\[ C_{1t} = \frac{1 - \tau}{2}, \]

with, as usual with log utility, first-period consumption being half of total wealth \(1 - \tau\). With no storage, \(C_{1t} + C_{2t} = 1\), which implies
\[ C_{2t} = \frac{1 + \tau}{2}. \]

Note that in this unique equilibrium, the utility of each generation is \(\log(1 - \tau) + \log(1 + \tau) + 2\log(1/2)\), which is less than the upper bound of \(2\log(1/2)\). Thus with \(\tau = 0\), the utility-maximizing equilibrium exists, but is not unique, while small positive values of \(\tau\) make equilibrium unique, and can approach the utility of the optimum for small \(\tau\).

The debt valuation equation holds in these equilibria. The gross real interest rate is \(\rho_t = C_{2t+1}/C_{1t} \equiv (1 + \tau)/(1 - \tau)\). From the government budget constraint, then, we see that
\[ \frac{B_t}{P_t} = \rho_{t-1} \frac{B_{t-1}}{P_{t-1}} - \tau, \]
and since \(B/P = C_2/\rho\) is constant in the equilibrium,
\[ \frac{B_t}{P_t} = \frac{\tau}{\rho - 1}. \]

Note that as \(\tau\) approaches zero, \(B/P\) does not approach zero, as this formula might suggest. \(\rho = C_2/C_1 = (1 + \tau)/(1 - \tau)\) in equilibrium and substituting this for \(\rho\) in (18) gives us
\[ \frac{B_t}{P_t} = \frac{1 - \tau}{2}. \]

Because \(\rho \to 1\) as \(\tau \to 0\), in other words, real debt converges to one half, its value in the utility-maximizing equilibrium, as \(\tau \to 0\), even though the debt valuation equation (18) continues to hold.

To see how the initial price level is determined, we look at the initial government budget constraint. \(R_{-1}\) and \(B_{-1}\) are given by history, so
\[ \frac{B_0}{P_0} = \frac{1 - \tau}{2} = \frac{R_{-1}B_{t-1}}{P_t} - \tau. \]

This equation can be solved for a unique, positive value of \(P_0\), so long as \(R_{t-1}B_{t-1} > 0\). The subsequent sequence of prices is determined by the sequence of policy choices for \(R_t\), with higher \(R_t\) values producing higher inflation.
What if initial $R_{t-1}B_{t-1} = 0$? So long as we maintain the constraint that $B_0 > 0$, fiscal policy cannot then at $t = 0$ be simply to set $\tau$ to its constant value. The old at time 0 in this case have no way to finance consumption. It is plausible then to suppose that the government imposes the tax $\tau$ on the young, issues new debt bought by the young, and uses the proceeds to provide a subsidy to the time-0 old. From that point on the equilibrium would be as we have calculated above.

**B. Fiscal Backup for a Taylor Rule**

Cochrane (2007) has argued against attempts to claim a determinate price level in models with Taylor-rule monetary policy by invoking “fiscal backing” that comes into play only off the equilibrium path. I don’t understand his reasoning, but in any case the simple model of this section shows that we can also justify uniqueness of the price level with a Taylor rule by invoking fiscal backing that is always in play, even in equilibrium, but is negligibly small in size. The equilibrium is then arbitrarily close to that of the model without fiscal backing. In this and the preceding model, we conclude that the existence of fiscal backing is important for stable prices, but that if market perception of the backing is there, the size of the backing can be quite small in equilibrium. Institutions like those in the EMU that make it unclear where the fiscal backing would come from, or even whether it exists, are destabilizing; yet in normal times, because large fiscal-backing interventions do not occur in equilibrium, it is easy for the importance of fiscal backing to be lost sight of.

The model is very simple, a continuous time extension of Leeper’s (1991) original framework. The monetary policy rule is

\[
\dot{r} = \gamma (\theta \dot{p} - (r - \rho)).
\]

This makes the nominal interest rate $r$ respond with a delay (larger $\gamma$ means less delay) to inflation $\dot{p}$. The “Taylor principle” that the interest rate should eventually respond more than one for one to inflation changes corresponds here to $\theta > 1$.

We assume a constant real rate $\rho$ and a no-risk-aversion Fisher equation connecting a constant real rate $\rho$ and the nominal rate:

\[
r = \rho + \hat{\dot{p}}.
\]

The $\hat{\dot{p}}$ notation represents the right time derivative of the expected path of the log of the price level. On a perfect foresight solution path, $\hat{\dot{p}} = \dot{p}$ at all dates after the initial date, but $p$ can move discontinuously at the initial date. These two equations (21) and (22) can be solved to yield a second order differential equation in $p$:

\[
\ddot{p} = \gamma (\theta - 1)\dot{p},
\]

which holds after the initial date $t = 0$ on any perfect foresight equilibrium path. With $\theta > 1$, this is an unstable differential equation, with solutions of the form $\dot{p}_0 = \dot{p}_0 e^{\gamma(\theta-1)t}$.

Leeper assumed that such explosive paths for the price level were not equilibrium paths and focused on the one stable solution to the equation, $\dot{p} \equiv 0$. On such a path
\( r = \rho \) from (22). From (21) this implies also that \( \dot{r} = 0 \). The policy equation (21), since it holds in actual (not expected-right) derivatives, implies that the time path of \( r - \gamma \theta p \) is differentiable, even at the initial date \( t = 0 \), but this leaves it possible that both \( p \) and \( r \) jump discontinuously at \( t = 0 \), so long as the jumps satisfy \( \Delta r = \gamma \theta \Delta p \). This makes the initial price level determinate. Using \( r_0^-, p_0^- \) to indicate the left limits of these variables at time 0 (i.e., their pre-jump values), we have

\[
\Delta r_0 = \rho - r_0^- = \gamma \theta (p_0 - p_0^-) = \gamma \theta \Delta p_0^-. 
\]

This equation can be solved for a unique value of \( p_0^- \) (right limit of the log of the price level at time 0) as a function of \( \rho, r_0^- \), and \( p_0^- \).

If the initial price level should be below this level, \( r \) and thus \( \dot{p} \) would also be lower, which implies inflation tends to \(-\infty\) at an exponential rate. The policy rule (21) cannot possibly be maintained on such a path, as it would require pushing \( r \) to negative values. It is natural to suppose that there would be a shift in the rule at very low inflation rates, with fiscal policy ruling out such a path. If the initial price level and inflation rate are above the steady state level, the inflation rate rises at an exponential rate. The opportunity costs of holding non-interest-bearing money balances become arbitrarily high. If real balances are essential (utility is driven to \(-\infty\) as \( M/P \to 0 \)), these explosive paths may be viable equilibria. If not, there may be an upper bound on the interest rate above which real balances become zero. Paths on which real balances shrink to zero in finite time may also be viable equilibria. Here again, though, we can postulate a shift in policy at very high inflation rates that eliminates these unstable paths, while leaving the stationary \( \dot{p} = 0 \) equilibrium viable.

Cochrane finds these hypothetical policy shifts at high and low inflation rates, which then never are observed in equilibrium, implausible. But suppose we add a government budget constraint and fiscal policy, as did Leeper. The budget constraint, with real debt (not log of real debt) denoted as \( b \) and primary surplus denoted as \( \tau \), is

\[
\dot{b} = (r - \dot{p})b - \tau. \tag{25}
\]

A version of what Leeper calls a passive fiscal policy is

\[
\tau = -\phi_0 + \phi_1 b. \tag{26}
\]

Along a perfect foresight path, where \( r = \rho + \dot{p} \), this gives us

\[
\dot{b} = \rho b + \phi_0 - \phi_1 b. \tag{27}
\]

With \( \phi_1 > \rho \), this is a stable equation in \( b \). No matter what the equilibrium time path of \( p \), real debt converges to \( \phi_0 / (\phi_1 - \rho) \). This equation can therefore play no role in determining the price level, and thus cannot resolve the indeterminacy.

But what if we add to the right-hand side of (26) a positive response of the primary surplus to inflation, i.e., replace (26) with

\[
\tau = -\phi_0 + \phi_1 b + \phi_2 \pi? \tag{28}
\]
Then (27) becomes

\[ \dot{b} = (\rho - \phi_1) b + \phi_0 - \phi_2 \pi. \]  

The three-equation differential equation system formed by (21), (22), and (29) is recursive, since \( b \) appears only in the last equation. That means that the solution paths that make \( \pi \) explode up or down that we observed when considering the first two equations alone are still mathematical solution paths for the three-equation system. But now notice what happens to \( b \) along a path on which \( \dot{p} \to \infty \). From the debt equation (29) we see that on such a path \( b \) eventually becomes negative and more negative over time. This implies that \( b \) goes to zero in finite time. From the point of view of private agents in the economy, since we assume they can’t borrow from the government, this means that their future tax obligations exceed their wealth in the form of government debt, and thus that they cannot finance their planned consumption with the income and wealth they have. They will therefore reduce consumption and try to save. If they truly have perfect foresight, this would instantly, at the initial date \( t = 0 \), bring \( p_0 \) back to the level consistent with stability. If it takes agents some time to realize what kind of a path they are on, the adjustment might come with a delay, still producing the same reversion to the stable solution. Unstable paths with accelerating deflation can also be ruled out. On such paths, real debt would rise without bound, while primary surpluses shrank and eventually became negative. Now people would see their wealth in the form of government debt growing without bound, with no offsetting increase in future tax obligations. They would therefore spend, raising prices, bringing the economy back to its stable path.

These arguments do not depend on the size of \( \phi_2 \), so long as it is positive. In equilibrium, \( \pi \) will be zero or (if people are imperfectly foresighted, or if we add random disturbances to the system) fluctuate in a narrow range. If \( \phi_2 \) is small enough, its presence might be difficult to detect from data. In any case its presence would have no effect on the first two equations of the system or on the equilibrium time path of prices and interest rates, except for its elimination of the unstable solutions as equilibria of the economy.

C. Debt as a Fiscal Cushion

Barro (1979) showed in a simple, stylized model that in the presence of distorting taxation it is not optimal to rapidly pay off public debt, because the deadweight losses from heavy initial taxation to reduce the debt are not offset by the present value of lower future deadweight losses after the debt is reduced. Instead, in his model, debt and tax revenue optimally follow a martingale process, with \( E_t b_{t+1} = b_t, E_t \tau_{t+1} = \tau_t \). (Here \( \tau \) is total tax revenue.) In his framework, \( \tau \) increases with increases in \( b \). Lucas and Stokey (1983) showed that when the government can issue contingent liabilities, it is actually optimal for taxes to be set without reference to the current level of debt. Chari, Christiano, and Kehoe (1994) showed that monetary policy, by determining inflation, can create appropriate contingencies in the return to debt. I showed (Sims 2001) that if these insights are brought back to Barro’s (1979) stylized framework, we get a simple and stark conclusion—\( \tau \) should be constant,
with $b$ brought in line with stochastically fluctuating future government spending by surprise inflation and deflation.

However, these results all depend on surprise inflation and deflation being costless. In a Keynesian model with sticky prices or wages, or in a model with incomplete markets and borrowing and lending via standard debt contracts in nominal terms, surprise inflation has a substantial cost. Schmidt-Grohé and Uribe (2001) showed that in a New Keynesian model with one-period government debt, optimal policy is much closer to Barro’s (1979) initial prescription than to a constant-$\tau$ policy. It makes a great deal of difference, though, whether government debt is long or short term. When debt is short, as in Schmitt-Grohé and Uribe’s setup, inflation or deflation is the only way to change its market value in response to government spending surprises. But if the debt is long term, large changes in the value of the debt can be produced by changes in the nominal interest rate, with much smaller changes in inflation. Interest rates fluctuate widely and their fluctuations are not thought of as very costly, while price fluctuations may generate inefficient output and employment fluctuations. The model of this section revisits Barro’s (1979) framework, adding endogenous price determination and allowing for short or long debt. It concludes that substantial use of the nominal debt fiscal cushion to limit tax fluctuations may be optimal if debt maturity is long.

Following Barro (1979), we model the government as wanting to minimize the deadweight loss from taxation, modeled as proportional to $\tau^2$, the square of total revenue. But we add to his specification a concern with wide swings in inflation, leading to the objective function

$$-\frac{1}{2}E\left[\sum_{t=0}^{\infty} \beta^t \left( \tau^2 + \theta \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 \right) \right].$$

It simplifies notation for us to use the single symbol $\nu_t = P_{t-1}/P_t$ to denote the inverse of the gross inflation rate from this point on. There is a constant real interest rate, and private sector behavior requires the real rate to match the expected nominal return on a bond:

$$R_t E_t \nu_{t+1} = \rho.$$  

The government budget constraint is

$$b_t = R_{t-1} \nu_t b_{t-1} - \tau_t + g_t,$$

where $b$ is real debt and $g_t$ is government spending, which we treat as an exogenous stochastic process. The government then maximizes (30) by choosing $R$, $P$, $b$, and $\tau$ subject to the two constraints (31) and (32). The first order conditions for an optimum are

$$\frac{\partial}{\partial \tau}: \quad \tau_t = \lambda_t$$

$$\frac{\partial}{\partial b}: \quad \lambda_t = \beta R_t E_t [\nu_{t+1} \lambda_{t+1}]$$
\[ \begin{align*}
\partial R &: \mu_t E_t \nu_{t+1} = \beta E_t [\nu_{t+1} \lambda_{t+1}] b_t \\
\partial \nu &: \theta \cdot (\nu_t - 1) = -\lambda_t R_{t-1} b_{t-1} + \mu_{t-1} R_{t-1} \rho.
\end{align*} \]

These first order conditions look complicated, but when \( \theta = 0 \), so that there is no cost to inflation, they collapse to a surprisingly simple solution. To keep things neat, we assume \( \beta \rho = 1 \). From the \( b \) and \( R \) FOCs and the Fisher equation (31) we can derive

\[ b_t \lambda_t = \mu_t \rho. \]

Substituting into the \( \nu \) first order condition, we arrive at

\[ \theta \cdot (\nu_t - 1) = (-\lambda_t + \lambda_{t-1}) R_{t-1} b_{t-1} = (\tau_t - \tau_{t-1}) R_{t-1} b_{t-1}. \]

If \( \theta = 0 \), this lets us conclude that \( \tau_t = \tau_{t-1} \), so long as \( R_{t-1} \) and \( b_{t-1} \) are both positive. With \( \tau_t \) constant and \( g_t \) exogenous and stochastic, (31) is an unstable equation. Feasibility (\( b > 0 \)) and transversality (\( b \to \infty \) while future \( \tau \)'s are constant cannot be optimal) imply that \( b \) must not explode. This implies that we can solve the budget constraint (32) forward to produce

\[ b_t = \frac{\tau_t}{\rho - 1} - E_t \left[ \sum_{s=1}^{\infty} \rho^{-s} g_{t+s} \right]. \]

In the special case where \( g \) is i.i.d., \( b \) is constant. \( b \) is maintained at these stability-consistent values by fluctuations in \( \nu_t \), the inverse inflation rate, that offset the effects of \( g \) on the real value of the debt.

So far, we have derived an analogue in this simple model of the Lucas-Stokey result, by setting \( \theta = 0 \). We can also consider \( \theta = \infty \), i.e., a case where the price level is kept constant and only real government debt exists. Then we drop the \( \nu \) first order condition, because \( \nu \) is no longer freely chosen, and use the fact that \( \nu_t \equiv 1 \). Then the \( b \) first order condition lets us conclude that \( E_t \tau_{t+1} = \tau_t \), and we are back to Barro’s (1979) conclusion (since we are now back to Barro’s model, which had only real debt).

The cases of most interest, though, are those with \( 0 < \theta < \infty \). For these cases, we want to contrast this version of the model with one in which government debt is not only nominal, but long term. We will consider the extreme case of consol debt, which pays a stream of one “dollar” per period forever, never returning principal. The number of consols held by the public is \( A_t \), and the price, in dollars, of a consol is \( Q_t \). (So \( 1/Q_t \) is the long term interest rate). Then the Fisher equation requires that the expected one-period yield on a consol be equal to \( \rho \), i.e.,

\[ E_t \left[ \frac{Q_{t+1} \nu_{t+1} + 1}{Q_t} \right] = \rho, \]
and the budget constraint becomes

\[
(41) \quad b_t = \frac{A_t Q_t}{P_t} = b_{t-1} \left( \frac{Q_t \nu_t + 1}{Q_{t-1}} \right) - \tau_t + g_t.
\]

Because these systems with non-trivial $\theta$ become difficult to handle analytically, we omit laying out the first order conditions for the consol-debt case, and we solve, numerically, locally linearized versions of both the short debt and long debt models.

We assume $g_t$ is independent across time, with constant mean $E g_t = \bar{g} = 1$. We set $\rho = \beta^{-1} = 1.1$, and $\tau = 2$ in the initial steady state. Because this makes $\tau - \bar{g} = 1$, and the net real rate $\rho - 1 = 0.1$, initial steady state real debt $b = 10$.

With real debt, as in Barro’s (1979) original framework (i.e., $\theta = \infty$), a unit increase in $g_t$ above its mean $\bar{g}$ requires $\tau$ and $b$ to move to new levels that could be sustained forever if future $g$ values reverted to $\bar{g}$. So with our parameter settings, $10/11$ of the $g$ shock goes into $b$, $1/11$ into $\tau$. The increased $\tau$ is exactly enough to service the increased debt at the 10 percent interest rate. The new values of $b$ and $\tau$ are sustained forever in the absence of new shocks. The interest rate remains constant.

At the opposite extreme, with perfectly flexible prices ($\theta = 0$) and nominal debt, optimal policy absorbs all of the fiscal surprise in surprise inflation. The inflation proportionally reduces the real value of maturing debt, which is 11, and must be sufficient to offset the unit increase in $g$, since $\tau$ will optimally not change at all. The result is an inflation of 10 percent, with $b$, $\tau$, and the interest rate all unchanged. The inflation is limited to the initial period, after that returning to zero. Here again, the interest rate remains constant.

With $\omega = 10$, optimal policy depends on whether we have one-year or consol debt. With one-year debt, optimal policy allows 43 percent of the $g$ shock to flow into $b$ and permanently adjusts $\tau$ by 4.3 percent, to cover the increased debt service. This leaves some of the $g$ shock unaccounted for, though, and that is absorbed in a one-time surprise inflation of 4.8 percent. And once again the interest rate remains constant.

With $\omega = 10$ and consol debt, only 6.9 percent of the $g$ shock passes into increased $b$, and $\tau$ increases by only 0.69 percent. Most of the shock is absorbed by simultaneous, permanent small increases in the nominal interest rate ($1/Q$) and the inflation rate. The interest rate increases by 0.84 percentage points and the inflation rate by 0.76 percentage points. These small changes in the interest rate and the inflation rate are enough to create a capital loss for consol holders that offsets most of the $g$ shock.

It may seem, since the interest rate increases by more than the inflation rate, that the real rate has increased, even though we have assumed constant $\rho$. However, this happens only because of the definition of the “nominal rate” as $1/Q$. This is a good approximation when there is no inflation, but when there is steady inflation, as in the wake of this shock, a consol’s constant stream of nominal payments is front-loaded in real terms, so that in fact the constant real rate is preserved by this combination of permanent changes in inflation and $1/Q$.

The response to the $g$ shock in the four cases we have discussed in the preceding paragraphs is displayed in Figure 2. Each column of plots shows the time path of changes in the four variables listed on the left side of the chart, in the case labeled at
the top of the column. Note that the 1-year debt case with $\omega = 10$ is about halfway between the pure real debt case of Barro (1979) and the flex-price case. The consol case is very close to the flex-price case for the time paths of the real variables $b$ and $\tau$, though its time path for inflation and interest rates is quite different.

The point of this comparison is not to claim that a combination of long debt and low response of taxes to fiscal shocks is optimal. The model is extremely stylized, and the costs of inflation have been calibrated only to a value that makes contrasts between cases easy to see. As is by now well understood, the first-order accurate solution obtained as here by local linearization does not allow us directly to compute expected welfare, even for the stylized objective function. Since both taxes and inflation vary much less in the consol solution, it seems likely that it delivers higher welfare, but because the budget constraint is nonlinear, we can’t be certain of this. The amount of shock absorption available from surprise changes in prices and interest rates depends on the size of the real debt, and the real debt is locally non-stationary in this solution.

However, the results with long and short debt contrast so sharply that this example does provide a reason for caution in interpreting the results of analyses like those of Schmitt-Grohé and Uribe (2001) and Siu (2004). These papers conclude that in normal (non-war) times, it is optimal to make very little use of surprise inflation in cushioning fiscal shocks, but both papers assume that all debt is one-period debt. It is likely that their conclusions are sensitive to this assumption.
IV. Conclusion

The kinds of models that have been the staple of undergraduate macroeconomics teaching, with price level determined by balance between “money supply” and “money demand,” and money supply described using the “money multiplier,” are obsolete and provide little insight into the policy issues facing fiscal and monetary authorities in the last few years. There are relatively simple models available, though, that could be taught in undergraduate and graduate courses and that would allow discussion of current policy issues using clearer analytic foundations.

APPENDIX

A. Surprise Gains and Losses on the Debt

The calculations for Figure 1 were done as follows. The unanticipated gains or losses were formed as

\[ B_t(1 - \tilde{\pi}_t) + S_t - (1 + r_{t-1})B_{t-1} \]

\( B_t \) is the market value of the marketable US debt, as calculated by the Federal Reserve Bank of Dallas. This series has recently been updated by them, and they sent me the updated version. The time unit for this equation is one year. \( S_t \) is the primary surplus, calculated from the US national income and product accounts, Table 3.2, as net lending or borrowing, line 39, plus interest payments, line 22. \( r_t \) is the one-year interest rate on treasury securities at the beginning of the year. \( \tilde{\pi}_t \) is the error in a forecast of inflation for year \( t \) made at the beginning of year \( t \). The logic is that holders of debt at the beginning of the year expected their holdings to grow to \((1 + r_{t-1})B_{t-1}\) at the end of the year. Some of the debt is retired, though, and that is accounted for by the \( S_t \) term. And the real value of the debt undergoes surprising change because of errors in predicting inflation. This formula is at best approximate for several reasons. One-year government debt carries some liquidity premium, and indeed Figure 1 shows that “surprises” in yield are on average positive, probably because of the liquidity premium. Some debt is of maturity less than one year, so as information about inflation accumulates during the year, interest rates on these components of the debt can compensate. Thus some gains and losses that were actually anticipated within the year are treated as unanticipated in this formula. The calculation of the market value of the debt involves some interpolation and approximation, though the quantity used here, total marketable debt, is the least affected by these considerations of the three concepts reported by the Dallas Fed.

Total marketable debt excludes “debt” of the government to itself in Treasury accounts, but it does include treasury securities held by the Federal Reserve system. Since the Fed is not a profit or utility maximizing agent, it would be better to include it as part of the government, and the Dallas Fed does report a concept that excludes Fed holdings. But if the Fed is part of the government, then its interest-bearing liabilities are part of the public debt and changes in its holdings of debt that do not correspond to changes in non-interest-bearing liabilities are part of the primary
surplus. These considerations are quantitatively important in recent years, with the expansion of the Fed’s balance sheet and its beginning to pay interest on reserves. Surprise inflation is not easy to quantify. During the late 1970s and early 1980s inflation underwent wide swings that were not well tracked by statistical models fit to historical data. Probably expectations about future inflation were diverse during 1973–1983, not well summarized by any single number. A Bayesian VAR model using the log of the chained PCE deflator and the one-year interest rate gives reasonable-looking results, but its time series of forecast errors is quite different from what is obtained by predicting inflation over year $t$ as simply equal to inflation over year $t - 1$, and the sum of squared forecast errors is nearly the same for the two forecasting methods. The simple $\hat{\pi}_t = \pi_{t-1}$ forecasts were used to produce the figure, but the figure would have looked noticeably different if the VAR forecasts had been used.

Hall and Sargent (2011) calculate ex post real and nominal returns on US government debt by a method that does not rely on the NIPA data to compute a primary surplus. It is possible to use their ex post real returns, together with the one-year interest rate series and the inflation forecast error series and a series for the level of the debt, to compute the same concept shown in Figure 1. I made those calculations, and they produce fluctuations of the same order of magnitude as in the figure, but again with a different time profile.

B. Sources for Table 1

These data were drawn from the international component of the FRED database provided by the Federal Reserve Bank of St. Louis. The interest rates are those labeled “government bond” interest rates. The debt to GDP ratios are provided in that form by FRED. They are the most recent available at the time of writing.

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