

A Cooperative Bayesian Nonparametric Framework for Primary User Activity Monitoring in Cognitive Radio Networks

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Abstract—This paper introduces a novel approach that enables a number of cognitive radio devices that are observing the availability pattern of a number of primary users (PUs), to cooperate and use Bayesian nonparametric techniques to estimate the distributions of the PUs’ activity pattern. To address this problem, a coalitional game is formulated between the cognitive devices and an algorithm for cooperative coalition formation is proposed. It is shown that the proposed coalition formation algorithm allows the cognitive nodes that are experiencing a similar behavior from some PUs to self-organize into disjoint, independent coalitions. Inside each coalition, the cooperative cognitive nodes use Bayesian nonparametric techniques so as to improve the accuracy of the estimated PUs’ activity distributions. Simulation results show that the proposed algorithm significantly improves the estimates of the PUs’ distributions.

I. INTRODUCTION

Cognitive radio is a novel communication paradigm that allows an efficient sharing of the under-utilized radio spectrum resources between licensed or primary users (PUs) and unlicensed or secondary users (SUs) [1], [2]. The main enablers for cognitive radio are smart SU devices that can intelligently monitor the spectrum, operating only when the PUs are inactive and making sure to vacate the spectrum whenever a PU starts its transmission. One of the key challenges of cognitive radio is to maintain a conflict-free coexistence between PUs and SUs [2–6].

For detecting the PUs’ activity, the SUs can sense and detect the unoccupied spectrum so as to transmit opportunistically, e.g., [4–6] (see [6] for a comprehensive review). Spectrum sensing is a key step for deploying robust cognitive radio networks [4–7]. In particular, advanced spectrum sensing techniques such as *cooperative sensing* have been proposed in [5], [7] so as to improve the SU’s detection capability. However, the sensing process is known to be time consuming and can affect the access performance of the SUs [2], [4]. To address this problem, recent research activities brought forward the idea of providing, using *control channels*, spectrum monitoring assistance to the SUs [8],

[9]. For example, recently, the Cognitive Pilot Channel (CPC) has been introduced [8], [9] as a control channel that can convey critical information (e.g., frequency or location data) to the SUs, allowing them to enhance their sensing and access decisions.

The use of CPCs and cooperative spectrum sensing have received considerable attention in the research community. However, on the one hand, most of the existing work on CPC deployment such as [8], [9] has focused on implementation aspects. On the other hand, existing cooperative sensing techniques such as in [7] often assume that the PU’s activity follows a known or assumed distribution. However, no work seems to have investigated how cognitive device such as CPC nodes can be used to provide information on the activity of the PUs in a practical cognitive network. To operate efficiently, the SUs must obtain a good overview of the activity of the PUs, so as to access the spectrum at the right time and for a suitable duration. Our goal is to leverage the use of the CPC in order to convey to the SUs accurate estimates of the distribution of the activity of the PUs, which is often unknown a priori.

The main contribution of this paper is to introduce a novel cooperative approach between cognitive devices such as CPC nodes that allows them to improve their estimations of the distributions of the PUs’ activity. Given a number of PUs whose availability is perceived differently by a number of CPC nodes, we propose a scheme that allows these nodes to cooperate in order to estimate the distributions of the PUs’ activity, assumed to be completely unknown. We formulate a coalitional game between the CPC nodes and we develop a suitable coalition formation algorithm. The proposed game allows the CPC nodes to decide, in a distributed manner, on whether to cooperate or not, based on a utility that captures the gain from cooperation, in terms of an improved estimate of the PUs’ distributions, and a cost for coordination. Each group of CPC nodes that decides to cooperate and form a coalition will subsequently use *Bayesian nonparametric techniques* to infer the perceived distributions of the PUs’ activity. We show that, by performing coalition formation, the CPC nodes self-organize into a network of disjoint and independent coalitions that form a Nash-stable partition.

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Simulation results show that the proposed cooperative approach yields a significant performance improvement.

The rest of this paper is organized as follows: Section II presents the system model. In Section III, we present the proposed cooperative Bayesian nonparametric scheme. In Section IV, we propose an algorithm for distributed coalition formation. Simulation results are analyzed in Section V and conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a network of N cognitive radio devices (e.g., SUs or CPC-carrying nodes) that are seeking to transmit, opportunistically, over K channels that represent a number of PUs. For brevity, we use the term CPC or CPC node to refer to any such cognitive node. The set of all CPCs is denoted by \mathcal{N} while the set of PUs is denoted by \mathcal{K} . At any point in time, from the perspective of any CPC $i \in \mathcal{N}$, every PU $k \in \mathcal{K}$ is considered to be active, i.e., its channel is occupied, with a probability θ_{ik} . For a given PU $k \in \mathcal{K}$, two distinct CPCs $i, j \in \mathcal{N}$, $i \neq j$ can see a *different value* of the probability that k is active, i.e., $\theta_{ik} \neq \theta_{jk}$, depending on various factors such as the distance to the PU or the wireless channel fading.

The PUs can change their pattern of activity depending on many random parameters, e.g., due to their type or capabilities. Hence, for a given PU k , the value of θ_{ik} from the perspective of any CPC $i \in \mathcal{N}$, is a random variable having a distribution $P_{ik}(\theta_{ik})$ which is a probability density function over the state space $\Theta = [0, 1]$ of θ_{ik} , $\forall i \in \mathcal{N}$, $k \in \mathcal{K}$. We consider that the CPCs in \mathcal{N} have *no* prior knowledge on the distribution of the PUs' activity. Thus, for *any* CPC $i \in \mathcal{N}$ and any PU $k \in \mathcal{K}$, the actual real distribution $P_{ik}(\theta_{ik})$ is *completely unknown* by the CPC. Hereinafter, for brevity, we use the term the expression *distribution of the PUs* or *PUs distribution* to refer to the distribution of the PUs' activity/availability.

Each CPC $i \in \mathcal{N}$ performs a limited number of L_{ik} observations $\mathcal{L}_{ik} = \{\theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}}\}$ for every PU channel $k \in \mathcal{K}$ so as to get an estimate of the distributions $P_{ik}(\theta_{ik})$. Each observation $\theta_{ik}^t \in \mathcal{L}_{ik}$ is a value for the probability θ_{ik} observed at a time period t . To obtain \mathcal{L}_{ik} for a channel k , a CPC needs to monitor, over a given period of time t , the activity of PU k and record the resulting probability θ_{ik}^t . This process can be seen as a sampling of the PU's activity distribution. Note that the time period t during which a single observation is recorded must be reasonably large so as to enable the CPC to record a reasonably accurate observation. Due to this, the number of observations L_{ik} for each PU channel k is, in practice, small. We note that, in a given time period, the observations \mathcal{L}_{ik} are the only information that a CPC node i has about the behavior of PU k .

Having recorded the observations \mathcal{L}_{ik} , each CPC $i \in \mathcal{N}$ must infer the distribution of every PU $k \in \mathcal{K}$. Given \mathcal{L}_{ik} , a CPC i can predict the distribution of the next observation $\theta_{ik}^{L_{ik}+1}$ using

the following expression:

$$H_{ik}(\theta_{ik}^{L_{ik}+1} \in \mathcal{A} | \theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}}) = \frac{\sum_{l=1}^{L_{ik}} \delta_{\theta_{ik}^l}(\mathcal{A})}{L_{ik}}, \quad (1)$$

where $\mathcal{A} \subseteq \Theta$ is a subset of the space Θ and $\delta_{\theta_{ik}^l}$ is the point mass located at θ_{ik}^l such that $\delta_{\theta_{ik}^l}(\mathcal{A}) = 1$ if $\theta_{ik}^l \in \mathcal{A}$ and 0 otherwise.

Non-cooperatively, each CPC can compute the distribution of $\theta_{ik}^{L_{ik}+1}$ using (1) which is discrete. Given the limited number of observations L_{ik} , using (1) can yield an inaccurate estimation. To improve this accuracy, a continuous estimate \tilde{H}_{ik} of the distribution H_{ik} in (1), can be generated by each CPC i using *kernel density estimation* [10]. Kernel density estimation methods aim at smoothing a discrete function using well-defined mathematical steps [10]. While a detailed treatment of kernel density estimation techniques is beyond the scope of this paper, for the proposed model, we assume that, when acting non-cooperatively, the CPCs utilize the generic kernel density estimation via the linear diffusion approach of [10], in order to obtain a continuous version $\tilde{H}(\theta_{ik}^{L_{ik}+1} \in \mathcal{A} | \theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}})$ of (1) which constitutes the non-cooperative *kernel estimate*.

The kernel estimate is the most reasonable estimate that any CPC $i \in \mathcal{N}$ can obtain non-cooperatively. However, as the number L_{ik} of available observations is small, the kernel estimate of the PUs distributions may not perform as well as needed. Hence, the CPCs need to seek alternative methods to improve their estimates. In particular, CPCs that are observing similar PUs distributions would have an incentive to form cooperative groups, i.e., coalitions, so as to share observations and improve their estimates, as discussed next.

III. COOPERATIVE BAYESIAN NONPARAMETRIC ESTIMATION OF PRIMARY USERS' ACTIVITY

In this section, we formulate the proposed CPC cooperation problem as a *coalitional game with a non-transferable utility* which consists of a pair (\mathcal{N}, V) in which \mathcal{N} represents the set of players (the CPCs in our game) and V is a mapping that assigns for any coalition $S \subseteq \mathcal{N}$ a set of payoff vectors that the members of S can achieve. Before determining V in our game, we first provide a cooperative procedure that the CPCs belonging to any potential coalition S can adopt. Whenever the CPC nodes decide to form a coalition $S \subseteq \mathcal{N}$, the CPCs in S would be able to share their non-cooperative estimates. Hence, within any potential coalition S , each CPC can obtain the PUs distribution estimates from its partners and, if deemed suitable, use these distributions as *prior distributions* so as to generate new, better estimates. For a coalition $S \subseteq \mathcal{N}$, each CPC $i \in S$ must perform three steps for every PU $k \in \mathcal{K}$: (i)- Step 1: check priors validity, (ii)-Step 2: generate new estimates, and (iii)- Step 3: assess the accuracy of the new distribution. In the next subsections, we detail each one of these steps.

A. Priors Validity Check

Given a coalition $S \subseteq \mathcal{N}$, any CPC $i \in S$ can use *goodness of fit* techniques [11] to assess whether the estimates received from the CPCs in $S \setminus \{i\}$ come from the same distribution as CPC i 's own set \mathcal{L}_{ik} . Goodness of fit tests provide a description of how well a certain model fits a set of observations [11]. For a coalition S , given a CPC $i \in S$ that receives, from another CPC $j \in S$, a certain kernel estimate \tilde{H}_{jk} for a PU k , CPC i must determine whether \tilde{H}_{jk} and its own estimate \tilde{H}_{ik} come from the same distribution. Thus, each CPC i must identify whether a given cooperating partner CPC j is observing a similar distribution regarding the activity of a certain PU k . To do so, CPC i first generates two sets of samples \mathcal{H}_{ik} and \mathcal{H}_{jk} from \tilde{H}_{ik} and \tilde{H}_{jk} , respectively. The samples in \mathcal{H}_{ik} can simply be the original observations \mathcal{L}_{ik} of CPC i or newly generated samples using the continuous kernel estimate \mathcal{H}_{ik} . Here, sampling refers to the process of obtaining samples from a distribution function which does not require observing the PU behavior and is commonly performed in wireless networks.

Then, in order to identify whether \mathcal{H}_{ik} and \mathcal{H}_{jk} come from the same distribution, CPC i uses the *two-sample Kolmogorov-Smirnov goodness of fit test*, defined as follows [11]:

Definition 1: Consider two sets of observations \mathcal{H}_{ik} and \mathcal{H}_{jk} having, respectively, $h_{ik} = |\mathcal{H}_{ik}|$ and $h_{jk} = |\mathcal{H}_{jk}|$ samples. The *Kolmogorov-Smirnov statistic* is defined as

$$D_{h_{ik}, h_{jk}} = \sup_x |F_{h_{ik}}(x) - F_{h_{jk}}(x)|, \quad (2)$$

where $F_{h_{ik}}$ and $F_{h_{jk}}$ represent the empirical cumulative distribution functions (CDFs) of the samples in \mathcal{H}_{ik} and \mathcal{H}_{jk} , respectively. Given $D_{h_{ik}, h_{jk}}$, the *two-sample Kolmogorov-Smirnov (KS) test* decides that the hypothesis: "The samples in \mathcal{H}_{ik} and \mathcal{H}_{jk} come from same distribution" is *true* with a significance level η , if $\sqrt{\frac{h_{ik}h_{jk}}{h_{ik}+h_{jk}}} D_{h_{ik}, h_{jk}} \leq M_\eta$, with M_η can be set according to well-defined tables [11].

The two-sample KS test determines whether two sets of samples come from the same distribution or not, without the need for any information on what that distribution is. This test will be used by each CPC i , a member of a coalition S , to determine whether the estimates received from the CPCs in $S \setminus \{i\}$ come from the same distribution as CPC i 's own estimate. As a result, a cooperative CPC i can decide whether a received estimate is valid to be used as a *prior* distribution so as to improve its estimate for some PU k .

Subsequently, given any coalition S and any CPC $i \in S$, we let $S_{ik}^{\text{KS}} \subseteq \{S \setminus \{i\}\}$ denote the set of CPCs in $S \setminus \{i\}$ whose estimates regarding the distribution of the activity of PU k have been approved as valid priors by CPC i , using the two-sample KS test. If, for a PU k , CPC i could not find any valid prior in S , then $S_{ik}^{\text{KS}} = \emptyset$. The next step for any CPC $i \in S$ is to choose the priors that can improve its estimate of the PUs distributions.

B. Cooperative Estimate Generation

Once a CPC i member of a coalition S determines the set S_{ik}^{KS} for every PU k using the KS test, this CPC would build a $|S| \times 1$ vector $\tilde{\mathbf{H}}_k$ whose elements are the validated priors as received from the CPCs in S_{ik}^{KS} . Given $\tilde{\mathbf{H}}_k$, the next step for CPC i is to combine these priors with its own estimate \tilde{H}_{ik} so as to find the posterior distribution $H_{ik}^S(\theta_{ik}^{L_{ik}+1} | \theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}})$. To do so, we propose to use Bayesian nonparametric models, namely, the concept of a *Dirichlet process (DP)* [12]. The use of such a Bayesian *nonparametric* model is motivated by the following properties [12]: (i)- DPs provide *flexible models* that enable one to control the impact of each set of information used in estimation and (ii)- Bayesian nonparametric models can automatically infer an adequate distribution model from a limited data set with little complexity. Mathematically, given a probability distribution H over a continuous space Θ and a positive real number α , the DP is defined as follows [12]:

Definition 2: A random distribution G on a continuous space Θ is said to be distributed according to a *Dirichlet process* $\text{DP}(\alpha, H)$ with base distribution H and concentration parameter α , i.e., $G \sim \text{DP}(\alpha, H)$, if

$$(G(\mathcal{A}_1), \dots, G(\mathcal{A}_r)) \sim \text{Dir}(\alpha H(\mathcal{A}_1), \dots, \alpha H(\mathcal{A}_r)), \quad (3)$$

for every finite measurable partition $\{\mathcal{A}_1, \dots, \mathcal{A}_r\}$ of Θ , where $\text{Dir}(\beta_1, \dots, \beta_M)$ is the Dirichlet distribution [12]. The base distribution H is the mean of the DP, i.e., $E[G(\mathcal{A})] = H(\mathcal{A})$ for any measurable set $\mathcal{A} \subset \Theta$ while α is a parameter that indicates the strength of a DP when it is used as a prior.

The DP is thus a stochastic process that can be seen as a distribution over distributions, as every draw from a DP represents a random distribution over Θ . The base distribution H of a $\text{DP}(\alpha, H)$ is interpreted as a prior distribution over Θ . As the real distributions of the PUs are unknown to the CPCs, the CPCs will assume these PUs distributions to be distributed according to a DP. Subsequently, each CPC i , member of a coalition S , needs to combine its own observations about a PU k with the DPs received from other cooperating CPCs in the set of validated priors S_{ik}^{KS} . Such a combination of a number of independent DPs can also be modeled as a DP with a strength parameter $\sum_{l \in S_{ik}^{\text{KS}}} \alpha_{lk}$ being the sum of the individual parameters and a prior being the weighted sum of the different priors [12]. For any cooperative CPC i the distribution G_{ik} of any PU k is modeled using a DP that combines the received estimates from the CPCs in S_{ik}^{KS} into a single prior, as follows:

$$G_{ik} \sim \text{DP} \left(\sum_{l \in S_{ik}^{\text{KS}}} \alpha_{lk}, \frac{\sum_{l \in S_{ik}^{\text{KS}}} \alpha_{lk} \tilde{H}_{lk}}{\sum_{l \in S_{ik}^{\text{KS}}} \alpha_{lk}} \right), \quad (4)$$

where \tilde{H}_{lk} is the non-cooperative kernel estimate of PU k that CPC i received from a CPC $l \in S_{ik}^{\text{KS}}$ and validated using

the two-sample KS goodness of fit test. In (4), the combined strength parameter $\sum_{l \in S_{ik}^{KS}} \alpha_{lk}$ is the total confidence level (e.g., trust level in the accuracy of this estimation) in using (4) as a nonparametric prior. The prior $\frac{\sum_{l \in S_{ik}^{KS}} \alpha_{lk} \tilde{H}_{lk}}{\sum_{l \in S_{ik}^{KS}} \alpha_{lk}}$ used in (4) represents a weighted sum of the priors received from the coalition partners. Each weight represents the relative confidence level of a certain prior \tilde{H}_{lk} with respect to the total strength parameter level $\sum_{l \in S_{ik}^{KS}} \alpha_{lk}$. Note that, the control and setting of the parameters α_{lk} in (4) will be discussed in detail later in this section.

For every CPC i , member of a coalition S , having its set of observations \mathcal{L}_{ik} and its vector of validated priors $\tilde{\mathbf{H}}_k$, using the DP model in (4), the predictive distribution on any new observation $\theta_{ik}^{L_{ik}+1}$ conditioned on \mathcal{L}_{ik} with G_{ik} marginalized out can be given by [12, Eq. (5)] (after algebraic manipulation):

$$H_{ik}^S(\theta_{ik}^{L_{ik}+1} \in \mathcal{A} | \theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}}) = \sum_{l \in S_{ik}^{KS}} w_l \tilde{H}_{lk}(\mathcal{A}) + w_0 \tilde{H}_{ik}(\mathcal{A}), \quad (5)$$

where $w_0 = \frac{L_{ik}}{\sum_{l \in S_{ik}^{KS}} \alpha_{lk} + L_{ik}}$ is a weight that quantifies the contribution of CPC i 's own kernel estimate in the predictive distribution H_{ik}^S and $w_l = \frac{\alpha_{lk}}{\sum_{l \in S_{ik}^{KS}} \alpha_{lk} + L_{ik}}$ represent weights that identify the strength or impact of the contribution of the priors \tilde{H}_{lk} in the final distribution H_{ik}^S . The resulting posterior distribution in (5) is composed mainly of two terms: a first term related to the received estimates and a second term related to the contribution of CPC i 's own observations.

From (5), we can see that the parameters α_{lk} , $\forall l \in S_{ik}^{KS}$ allow the CPCs to control the effect of each prior on the resulting distribution H_{ik}^S . In practice, each CPC has an incentive to give a higher weight to priors that were generated out of a larger number of observations. Hence, we allow each cooperative CPC $i \in \mathcal{N}$ to set the parameters α_{lk} , $\forall l \in S_{ik}^{KS}$ such that the weights in (5) are proportional to the number of observations, i.e.,

$$w_0 = \frac{L_{ik}}{\sum_{j \in S} L_{jk}} \text{ and } w_l = \frac{L_{lk}}{\sum_{j \in S} L_{jk}}, \quad \forall l \in S_{ik}^{KS}. \quad (6)$$

By using the definition of the weights in (5), each CPC i can compute the parameters α_{lk} , $\forall l \in S_{ik}^{KS}$ from (6). We note that, in the model studied so far, we assumed that the CPCs have no knowledge of the PU's locations. However, the proposed model can easily extend to the case in which each CPC $i \in \mathcal{N}$ has additional information about the PUs.

C. Utility Function

For any coalition $S \subseteq \mathcal{N}$ we define, for every CPC $i \in S$ and PU $k \in \mathcal{K}$, the following utility function yielded from a given estimate of the distribution of PU k :

$$u_{ik}(S) = -\rho \left(H_{ik}^S(\theta_{ik}^{L_{ik}+1} | \theta_{ik}^{[1, \dots, L_{ik}]}), H_{ik}^S(\theta_{ik}^{(1+\Delta_{ik})L_{ik}+1} | \theta_{ik}^{[1, \dots, (1+\Delta_{ik})L_{ik}]}) \right) \quad (7)$$

where $\theta_{ik}^{[1, \dots, L_{ik}]} \triangleq \{\theta_{ik}^1, \dots, \theta_{ik}^{L_{ik}}\}$, $\theta_{ik}^{[1, \dots, (1+\Delta_{ik})L_{ik}]} \triangleq \{\theta_{ik}^1, \dots, \theta_{ik}^{(1+\Delta_{ik})L_{ik}}\}$ H_{ik}^S is given by (5), $0 < \Delta_{ik} \leq 1$ is a real number, and $\rho(P, Q)$ is the Kullback-Leibler distance between two probability distributions P and Q , given by [13]:

$$\rho(P, Q) = \int_{-\infty}^{\infty} P(x) \log \frac{P(x)}{Q(x)} dx, \quad (8)$$

where the log is taken as the natural logarithm. The KL distance in (8) is a well-known nonsymmetric measure of the difference between two probability distributions P and Q [13].

The utility in (7) measures, using (8), the distance between an estimate of the distribution of PU k when CPC i computes this distribution using L_{ik} observations and an estimate of the distribution of PU k when CPC i uses an extra $\Delta_{ik}L_{ik}$ set of observations to find the estimate. The rationale behind (7) is that, as the accuracy of the estimate H_{ik}^S improves, the KL distance in (7) would decrease, since the extra $\Delta_{ik}L_{ik}$ observations have a smaller impact on the overall distribution. As a result, the objective of each CPC $i \in \mathcal{N}$ is to cooperate and join a coalition S so as to maximize (7) by reducing the KL distance $\rho(H_{ik}^S(\theta_{ik}^{L_{ik}+1} | \cdot), H_{ik}^S(\theta_{ik}^{(1+\Delta_{ik})L_{ik}+1} | \cdot))$, on every PU channel k . It is interesting to note that (7) allows the CPCs to evaluate the validity of their distribution estimates without requiring *any knowledge* on the actual or real distribution of the PU.

While cooperation allows the CPCs to improve the estimates as per (5) and (7), these gains are limited by inherent costs that accompany any cooperative process. For every CPC i member of a coalition S , we define the following payoff function that captures both the costs and benefits from cooperation:

$$\phi_i(S) = \sum_{k \in \mathcal{K}} u_{ik}(S) - c(S), \quad (9)$$

where $u_{ik}(S)$ is given by (7) and $\phi(\emptyset) = 0$. The first term in (9) represents the sum of KL distances over all PU channels $k \in \mathcal{K}$, as given in (7), while the second term represents the cost for cooperation $c(S)$. Hereinafter, without loss of generality, we consider a cost function that varies linearly with the coalition size, i.e.,

$$c(S) = \begin{cases} \kappa \cdot (|S| - 1), & \text{if } |S| > 1, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

with $0 < \kappa \leq 1$ representing a pricing factor. (10) represents the costs for the synchronization, coordination, and communication overhead that occur during cooperation and grow linearly with the number of involved CPCs.

Given (9), the proposed CPC cooperation problem is formally modeled as a *coalitional game with non-transferable utility* (\mathcal{N}, V) in which \mathcal{N} is the set of CPCs and $V(S)$ is a singleton set that assigns for every coalition S a *single* payoff vector ϕ whose elements $\phi_i(S)$ are given by (9). The proposed CPC coalitional game is classified as a coalition formation game [14] in which

the objective is to develop an algorithm that enables the CPCs to cooperate and form coalitions. In this paper, we restrict our attention to games in which the outcome is a set of *disjoint* CPC coalitions due to the known complexity of dealing with overlapping, non-disjoint coalitions [14].

IV. A DISTRIBUTED COALITION FORMATION ALGORITHM

The proposed CPC coalitional game can be modeled using *hedonic coalition formation games* [15] which are games in which the payoffs depend *only* on the identity of the members in each coalition (as seen in (7)) and the coalition formation process is done using *preference relations* defined as follows [15]:

Definition 3: For any CPC $i \in \mathcal{N}$, a *preference relation* or *order* \succeq_i is a complete, reflexive, and transitive binary relation over the set of all coalitions that CPC i can possibly belong to, i.e., the set $\{S_k \subseteq \mathcal{N} : i \in S_k\}$.

For any CPC $i \in \mathcal{N}$, given two coalitions $S_1 \subseteq \mathcal{N}$ and $S_2 \subseteq \mathcal{N}$ such that $i \in S_1$ and $i \in S_2$, the preference relation $S_1 \succeq_i S_2$ implies that CPC i prefers to join coalition S_1 rather than coalition S_2 , or is indifferent between S_1 and S_2 . When using the asymmetric counterpart \succ_i of \succeq_i , $S_1 \succ_i S_2$ implies that CPC i *strictly* prefers joining S_1 rather than S_2 . For the proposed model, the preferences of each CPC must capture this CPC's two, often conflicting, objectives: (i)- Maximize its own individual benefit as quantified by (9) and (ii)- Ensure that the overall network benefit, i.e., the social welfare, is maintained at a reasonable level. To capture these objectives, we propose the following preference relation for any CPC $i \in \mathcal{N}$:

$$S_1 \succeq_i S_2 \Leftrightarrow q_i(S_1) \geq q_i(S_2) \\ \text{and } v(S_1) + v(S_2 \setminus \{i\}) > v(S_1 \setminus \{i\}) + v(S_2), \quad (11)$$

where $S_1, S_2 \subseteq \mathcal{N}$, are any two coalitions containing CPC i , i.e., $i \in S_1$ and $i \in S_2$, $v(S) = \sum_{j \in S} \phi_j(S)$ is the total utility generated by any coalition S , and $q_i : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is a preference function defined as follows:

$$q_i(S) = \begin{cases} \phi_i(S), & \text{if } (\phi_j(S) \geq \phi_j(S \setminus \{i\}), \forall j \in S \setminus \{i\}) \\ -\infty, & \text{otherwise,} \end{cases} \quad (12)$$

where $\phi_i(S)$ is the payoff of a CPC i as given by (9). The preference function in (12) implies that the preference value that a CPC assigns to a certain coalition S is equal to the payoff that i achieves in S , if the payoffs of the CPCs in $S \setminus \{i\}$ do not decrease when i cooperates with them. Alternatively, the preference value is set to $-\infty$ to convey the fact that if, by being part of a coalition S , a CPC i decreases any of the payoffs of the other members in $S \setminus \{i\}$, then, CPC i will be *rejected* by the members of $S \setminus \{i\}$. The usefulness of (12) will become clearer as we define the following rule:

TABLE I
THE PROPOSED CPCs COALITION FORMATION ALGORITHM

Starting Network

The network is governed by a partition $\Pi_{\text{init}} = \{S_1, \dots, S_M\}$.

The algorithm consists of three phases

Phase 1 - PUs monitoring phase:

- a) Each individual CPC discovers its neighboring PUs.
- b) Each CPC records, over a period of time, a number of observations L_{ik} on the activity of each PU k .

Phase 2 - Distributed Coalition Formation:

repeat

- a) Each CPC i performs pairwise negotiations with its neighboring CPCs to identify potential join operations.
- b) Each CPC $i \in \mathcal{N}$ can identify a join operation by estimating the utility in (9) using the three steps described in Section III.
- c) If a join is possible using (11), CPC i leaves its current coalition S_m and joins a new coalition S_k .

until guaranteed convergence to a Nash-stable partition Π_{final} .

Phase 3 - Cooperative Bayesian Nonparametric Estimation:

Inside every formed coalition $S_m \in \Pi_{\text{final}}$, the CPCs perform the Bayesian nonparametric approach of Section III.

Definition 4: Join Coalition Rule - Given a network partition $\Pi = \{S_1, \dots, S_M\}$ of the CPCs' set \mathcal{N} , a CPC i chooses to move from its current coalition S_m , for some $m \in \{1, \dots, M\}$, and *join* a different coalition $S_k \in \Pi \cup \{\emptyset\}$, $S_k \neq S_m$, hence forming $\Pi' = \{\Pi \setminus \{S_m, S_k\}\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$, if and only if $S_k \cup \{i\} \succ_i S_m, \Pi$. Hence, $\{S_m, S_k\} \rightarrow \{S_m \setminus \{i\}, S_k \cup \{i\}\}$ and $\Pi \rightarrow \Pi'$. The strict preference \succ_i is formally given by (11) with strict inequality.

The join coalition rule enables every CPC to autonomously decide whether or not to leave its current coalition S_m and join another coalition $S_k \in \Pi$, as long as $S_k \cup \{i\} \succ_i S_m$ as per (11). Based on (11), a CPC would move to a new coalition if this move can strictly improve its individual payoff and increase the overall utility generated by the two involved coalitions *without* decreasing the payoff of any member of the newly joined coalition as per (11) and (12). Using this rule, we develop a coalition formation algorithm, shown in Table I, consisting of three phases: PUs monitoring phase, distributed coalition formation phase, and cooperative Bayesian nonparametric estimation phase. In the first phase, each CPC monitors the activity of all the PUs in its area, records a limited number of observations, and generates the non-cooperative kernel estimates. In the second phase, the CPCs engage in pairwise negotiations so as to form coalitions using the join rule. Performing a join operation implies that a CPC leaves its current coalition and coordinates the joining of the new, preferred coalition. The members of the new coalition must give their consent on the joining of every new CPC as captured by (11) and (12). Once coalition formation converges, each formed coalition estimates the PUs' activity distributions using the cooperative Bayesian nonparametric techniques based on the three steps described in Section III. The distributed coalition formation phase is guaranteed to converge to a final partition:

Theorem 1: Starting from any initial partition Π_{init} , the distributed coalition formation phase of the proposed algorithm will

always converge to a final network partition Π_{final} .

Proof: Given any initial network partition Π_{init} , the proposed coalition formation process can be mapped into a sequence of join operations performed by the CPC and which transform the network's partition as follows (as an example):

$$\Pi_0 = \Pi_{\text{init}} \rightarrow \Pi_1 \rightarrow \Pi_2 \rightarrow \dots, \quad (13)$$

where $\Pi_l = \{S_1, \dots, S_M\}$ is a partition composed of M coalitions that emerges after the occurrence of l join operations. As per (11), every join operation performed by a CPC that moves from a coalition $S_1 \in \Pi_{l-1}$ to a coalition $S_2 \in \Pi_l$, yields: $\sum_{j \in S_1} \phi_j(S_1 \setminus \{i\}) + \sum_{j \in S_2} \phi_j(S_2 \cup \{i\}) > \sum_{j \in S_1} \phi_j(S_1) + \sum_{j \in S_2} \phi_j(S_2)$. As the proposed game is hedonic, this implies that any join operation is accompanied by an increase in the overall social welfare of the network, i.e., $\Pi_{l-1} \rightarrow \Pi_l$ yields

$$\sum_{S \in \Pi_l} v(S) > \sum_{S' \in \Pi_{l-1}} v(S'). \quad (14)$$

(14) implies that each join operation $\Pi_{l-1} \rightarrow \Pi_l$ constitutes a transitive and irreflexive order. Given that the number of partitions of the set \mathcal{N} is finite [14], then, the sequence in (13) is guaranteed to converge to a final partition Π_{final} . ■

We can study the stability of any network partition Π_{final} resulting from our algorithm, using the concept of a Nash-stable partition defined as follows [15]:

Definition 5: A partition $\Pi = \{S_1, \dots, S_M\}$ of a set \mathcal{N} is said to be *Nash-stable* if $\forall i \in \mathcal{N}$ s. t. $i \in S_m, S_m \in \Pi$, $(S_m, \Pi) \succeq_i (S_k \cup \{i\}, \Pi')$ for all $S_k \in \Pi \cup \{\emptyset\}$ with $\Pi' = (\Pi \setminus \{S_m, S_k\}) \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$.

In other words, a partition Π is Nash-stable, if no CPC prefers to leave its current coalition and join another coalition in Π . For the proposed CPC coalitional game, we have the following result:

Proposition 1: Any partition Π_f resulting from the proposed algorithm in Table I is Nash-stable.

Proof: Assume that this partition Π_{final} is not Nash-stable, then, there exists a CPC $i \in S_1$, $S_1 \in \Pi_{\text{final}}$ and a coalition $S_2 \in \Pi_{\text{final}}$, such that $S_2 \cup \{i\} \succ_i S_1 \setminus \{i\}$, i.e., a join operation is possible. Such a case contradicts with the result of Theorem 1 which ensures that no join operations are possible in Π_{final} . Thus, Π_{final} must be Nash-stable. ■

V. SIMULATION RESULTS AND ANALYSIS

For our simulations, we consider a square area of $2 \text{ km} \times 2 \text{ km}$ in which the CPC nodes and the PUs are randomly deployed. The model proposed in this paper does not make any assumptions on the distributions of the PUs' activity, and, thus, it can be applied to any such distributions. In the simulations, we will use *beta distributions* [12] that are generated in such a way that each CPC perceives a different distribution depending on its location with respect to the PU. The parameters of the simulations are consequently set as follows. The number of PUs is set to $K = 4$

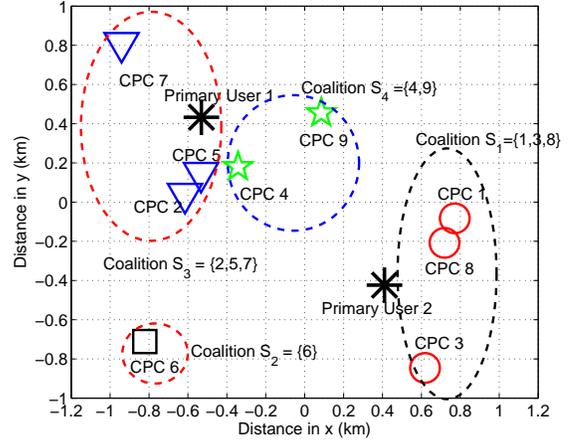


Fig. 1. A snapshot showing a network partition $\Pi = \{S_1, S_2, S_3, S_4\}$ resulting from the proposed coalition formation algorithm.

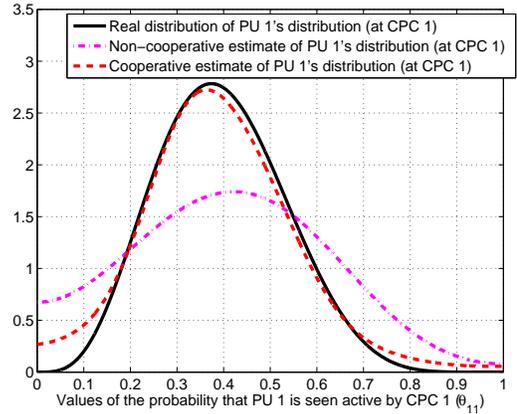


Fig. 2. Comparison of the estimates with the actual real distribution of PU 1 as seen by CPC node 1 in the network of Fig. 1.

and the pricing factor $\kappa = 10^{-3}$, unless stated otherwise. We let $\Delta_{ik} = 0.5$, $\forall i \in \mathcal{N}, k \in \mathcal{K}$. The number of observations L_{ik} for a CPC i is assumed to be uniformly distributed over the integers in the interval $[5, 20]$. The KS significance level is set to a typical value of $\eta = 0.05$ [11]. The transmit power of any PU $k \in \mathcal{K}$ is assumed to be $P_k = 100 \text{ mW}$ while the path loss exponent and the Gaussian noise are, respectively, set to $\mu = 3$ and $\sigma^2 = -90 \text{ dBm}$. All statistical results are averaged over the random locations of the CPCs and the PUs.

In Fig. 1, we present a snapshot of a Nash-stable partition $\Pi = \{S_1, S_2, S_3, S_4\}$ resulting from the proposed coalition formation game for a randomly generated network having $N = 9$ CPC nodes and $K = 2$ PUs. Fig. 1 shows how the nodes that are experiencing somewhat similar PUs' activity can decide to form a coalition. For example, consider coalition S_1 that consists of CPC nodes 1, 3, and 8. In this coalition, the distribution of PU 1 is seen by CPC nodes 1, 3, 8 as beta distributions with parameters

($\beta_{11} = 4.71, \gamma_{11} = 7.29$), ($\beta_{31} = 3.72, \gamma_{31} = 8.28$), and ($\beta_{81} = 4.6, \gamma_{81} = 7.4$), respectively while the distribution of PU 2 is seen by CPC nodes 1, 3, 8 as beta distributions with parameters ($\beta_{12} = 8.21, \gamma_{12} = 3.79$), ($\beta_{32} = 8.23, \gamma_{32} = 3.77$), and ($\beta_{82} = 8.27, \gamma_{82} = 3.73$). Clearly, CPC nodes 1, 3, and 8 have a benefit to use cooperative Bayesian nonparametric estimation to improve their estimate of the distribution of PU 2 which is seen by all three CPCs with an almost similar distribution (i.e., it passes the KS test for all three CPCs). However, for PU 1, although CPCs 1 and 8 see a comparable distribution, CPC 3 has a different view on PU 1's activity. In fact, the KS test fails when CPC node 3 uses it to compare its samples of PU 1's distribution to samples from CPCs 1 or 8. Nonetheless, all three CPCs find it beneficial to join forces and form a single coalition S_1 as it significantly improve their KL distance as per (9), on both PUs for CPCs 1 and 8, and only on PU 2 for CPC 3. Inside S_1 , CPC 3 discards the priors received from 1 and 8 regarding PU 1's distribution and only utilizes the received priors related to PU 2 in order to compute its DP estimate as in (5) for PU 2.

For the network of Fig. 1, we show, in Fig. 2, a plot of the real distribution of PU 1 as seen by CPC node 1, compared with the estimates generated from the proposed cooperative Bayesian nonparametric approach and with the non-cooperative kernel estimate. Fig. 2 clearly shows that, by performing cooperative Bayesian nonparametric estimation, CPC 1 was able to significantly improve its non-cooperative kernel estimate of PU 1's distribution by operating within coalition S_1 . We note that, in Fig. 1, the number of non-cooperative observations that CPCs 1, 3, and 8 record regarding the distribution of PU 1 are $L_{11} = 10, L_{31} = 8$ and $L_{81} = 20$ observations. Therefore, Fig. 2 demonstrates that by using the proposed cooperative Bayesian nonparametric approach while sharing observations (mainly with CPC 8 in S_1), CPC 1 was able to obtain an almost perfect estimate of PU 1's distribution *without any prior knowledge of this distribution* and by using only $L_{11} = 10$ own observations. Note that, analogous results can be seen for all CPCs in Fig. 1 as well as for all other simulated networks.

In Fig. 3, we assess the performance of the proposed cooperative approach by plotting the average achieved KL distance between the real, yet unknown (by the CPCs), distributions of the PUs and the estimates computed by the CPCs for a network with $K = 4$ PUs as the number of CPCs, N , varies. This KL distance allows us to assess how accurate the computed estimate is with respect to the actual real PUs' distributions. The results in Fig. 3 show the average KL distance per CPC and per PU. Fig. 3 shows that, as the number of CPCs N increases, the average KL distance between the estimates and the real distributions decreases for the proposed approach and remains comparable for the non-cooperative case. This result demonstrates that, for the proposed approach, as N increases, the CPCs become more apt to find partners with whom to cooperate and, thus, their performance

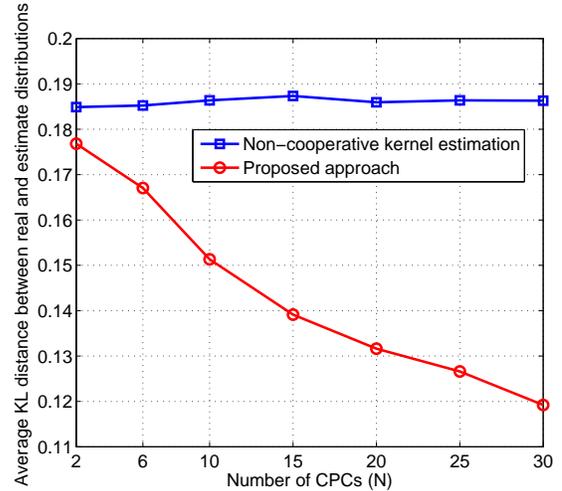


Fig. 3. Average KL distance between the real distributions and the estimates generated by the CPCs as the number of CPCs N varies.

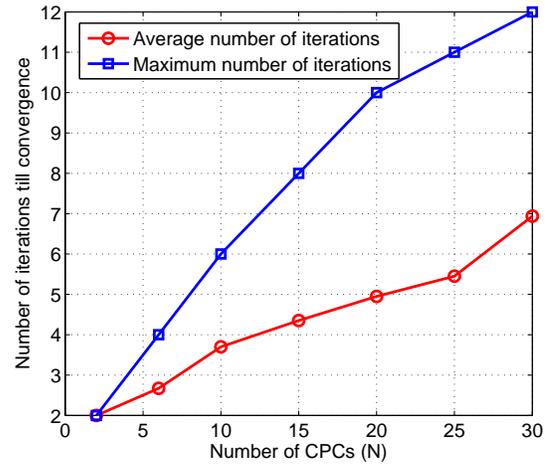


Fig. 4. Average and maximum number of iterations required till convergence to a Nash-stable partition for our algorithm.

improves as their estimates become more accurate, i.e., closer to the actual PUs' distributions. Fig. 3 shows that, at all network sizes, the proposed cooperative approach reduces significantly the KL distance between the real and estimated distributions relative to the non-cooperative case. This performance advantage is increasing with the network size N and reaching up to 36.5% improvement over the non-cooperative kernel estimation scheme at $N = 30$ CPCs. Fig. 3 also shows that our approach allows the average KL distance (average per PU and per CPC) to approach the ideal case of 0, as more cooperative partners exist in the network, i.e., as the network size N increases.

The convergence of the algorithm is assessed in Fig. 4 which shows the average and maximum number of iterations required until convergence to a Nash-stable partition. Each iteration consists of a number of join operations performed by the CPCs.

In Fig. 4, we can see that as the network size N increases, a larger number of iterations is needed for the CPCs to reach a Nash-stable partition. In this respect, the average and maximum number of iterations range from around 2 at $N = 2$ CPCs to 6.94 and 12, respectively, at $N = 30$ CPCs. Fig. 4 clearly show that the proposed algorithm has a low complexity as it enables the CPCs to cooperate, in a distributed manner, while requiring a very reasonable number of iterations and join operations.

VI. CONCLUSIONS

In this paper, we have introduced a novel cooperative approach between the CPC nodes of a cognitive radio network that is suitable for modeling the activity of primary users which is often unknown in practice. Using the proposed cooperative scheme, the CPC nodes can cooperate and form coalitions in order to perform joint Bayesian nonparametric estimation of the distributions of the primary users' activity. We have addressed this problem by formulating a coalitional game between the CPCs and proposing an algorithm for coalition formation. The proposed algorithm allows the CPC nodes to self-organize into disjoint, independent coalitions. We have shown the convergence of the proposed algorithm to a Nash-stable partition and we have assessed the properties of the resulting partitions using simulations.

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