Supplementary Material to “Fractional Chern Insulators beyond Laughlin states”

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In this Supplementary Material, we provide additional numerical results that might be relevant to a more specialized audience. In addition to the ruby lattice model, we have also studied the Kagome [26] and checkerboard [7, 8, 27, 28] lattice models for both fermions and bosons at filling factors $\nu = p/(np+1)$. Fig. 1 shows the energy spectrum for $\nu = 2/5$ (fermions) and $\nu = 2/3$ (bosons) on the Kagome lattice. We observe a low lying manifold separated from higher energy states by a manybody gap. The $np+1$ dimension of the ground state manifold is characteristic of a CF-like state. The ground state degeneracy lifting is more important than in the ruby lattice case, especially for bosons. Fig. 2 shows the energy spectrum for $\nu = 2/5$ (fermions) and $\nu = 2/3$ (bosons). For the fermionic case, the approximate degeneracy quickly deteriorates when increasing the system size despite tuning the band structure parameters. The bosonic case seems to have a better behavior but investigating the eigenstates with the PES reveals a less favorable picture. As shown in Fig. 3 for $N = 10$ particles, the PES does not display any gapped structure, thereby preventing the identification of the groundstate as a topological FQH state.

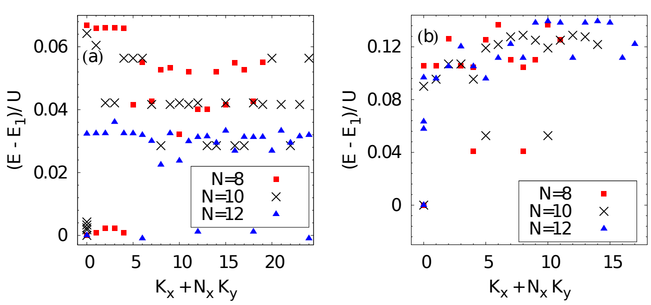

FIG. 1. Low energy spectra on the Kagome lattice for $N = 8,\ 10,\ 12$ fermions at filling factor $\nu = \frac{2}{5}$ and $N_x = 5, 5, 6$ (a) and $N = 8,\ 10,\ 12$ bosons at filling factor $\nu = \frac{2}{3}$ and $N_x = 4, 5, 6$ (b). The energies are shifted by the lowest energy E_1 for each system size. We only show the lowest energy per momentum sectors in addition to the approximate degenerate groundstate manifold.

We now give more details about the FCI on the ruby lattice. As mentioned in the article, we have checked that upon flux insertion the $\nu = p/(np+1)$ groundstate manifold does not mix with higher energy states. Also, the insertion of $np+1$ fluxes restores the original configuration. This can be observed for $N = 10$ particles in Fig. 4a for fermions at $\nu = 2/5$ and in Fig. 4b for bosons at $\nu = 2/3$. We have also looked at the excitation energy spectrum

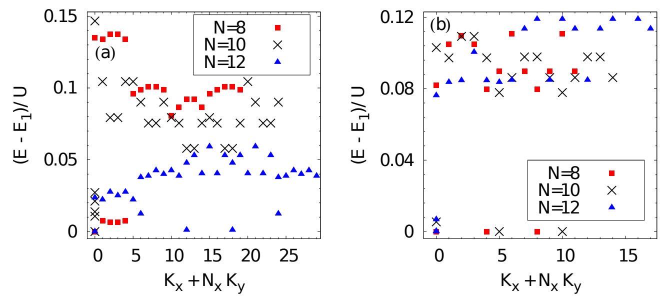

FIG. 2. (a) Low energy spectrum of $N = 8,\ 10,\ 12$ fermions on a checkerboard lattice at filling factor $\nu = \frac{2}{5}$. (b) Low energy spectrum of $N = 8,\ 10,\ 12$ bosons on a checkerboard lattice at filling factor $\nu = \frac{2}{3}$. We use the same conventions as in Fig. 1.

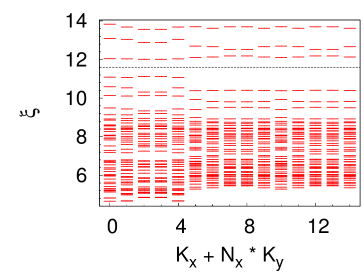

FIG. 3. PES for $N = 10$ bosons on a $N_x = 5, N_y = 3$ checkerboard lattice, with $N_A = 3$. The entanglement spectrum does not show any clear gapped structure except for the one (materialized by the black line) related to the Moore-Read counting.

both in the case of quasielectrons and quasiholes. Similarly to the case discussed in the article, one can generate two quasiholes at filling $\nu = 2/3$ with $N = 10$ bosons by looking at the system on a $4*4$ ruby lattice (see Fig. 5a). Once again we compare the low energy structure of this system to the FQH one (see Fig. 5b) and check that the number of states per momentum sector is predicted by the FQH-FCI mapping.

Similarly to the bosonic case, the PES for fermions at $\nu = 2/5$ exhibits a non-trivial structure. Fig 6 shows the PES for $N = 10$ fermions and $N_A = 3$ both for the FCI on the ruby lattice and its FQH counterpart. Once again, we observe similar structures between the two spectra.

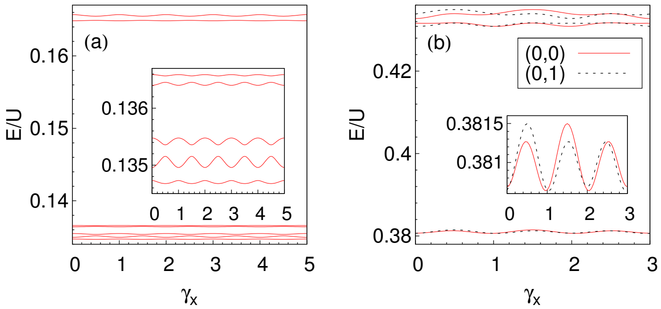

FIG. 4. (a) Evolution of the low-lying states of the ruby lattice model in momentum sector $(0,0)$ with $N = 10$ fermions on a $(N_x, N_y) = (5,5)$ lattice at $\nu = \frac{2}{5}$ upon flux insertion along the x direction. γ_x counts the number of inserted flux quanta. We only show the momentum sector $(K_x, K_y) = (0,0)$ where the almost 5 fold degenerate groundstate lies. (b) Evolution of the low-lying states of the ruby lattice model in momentum sectors $(0,0)$ and $(0,1)$ for $N = 10$ bosons on a $(N_x, N_y) = (5,3)$ lattice ($\nu = \frac{2}{3}$) upon flux insertion along the x direction. The third groundstate in the $(0,2)$ sector is not shown since it is related to the $(0,1)$ sector by the inversion symmetry. In both Figs. a and b, the inset is a zoom on the low energy part of the spectrum.

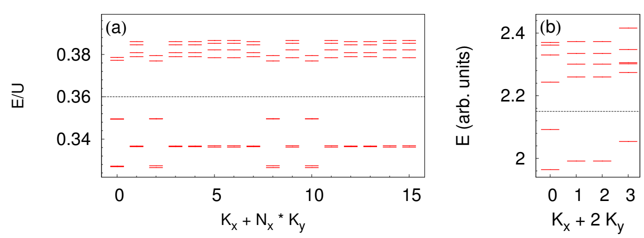

FIG. 5. Low energy spectra for the $N = 10$ bosons on the $N_x = N_y = 4$ ruby lattice (a) and the FQH equivalent for bosons on the torus with delta interaction and $N_\Phi = 16$ flux quanta (b). Both systems have one more flux (or one more site) than the $\nu = \frac{2}{3}$ groundstate and thus embed two quasiholes. For the FQH plot, the Brillouin zone is partially represented; we only display the sectors that are not related by the 8-fold center of mass translation symmetry. The number of states per momentum sector below the gap (materialized by the dashed line) in the FCI spectrum can be deduced from the FQH spectrum using the FQH-FCI mapping.

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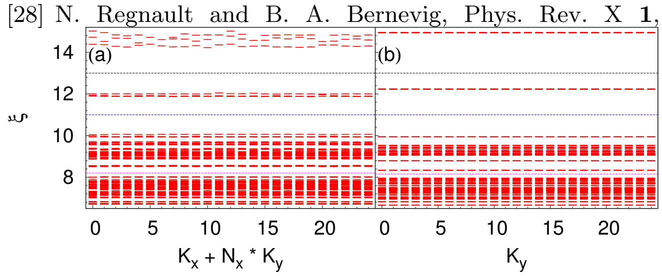

FIG. 6. PES of fermionic FCI and FQHE systems with $\nu = \frac{2}{5}$ and $N_A = 3$. (a) 10 fermions on a $5 * 5$ ruby lattice. (b) FQHE on torus with 10 fermions and $N_\Phi = 25$. We observe several gaps in both spectra (each depicted by a line). The counting below each of these gaps obeys the FQH-to-FCI mapping. The counting below the largest (and topmost) gap corresponds to the number of Moore-Read quasihole states. The number of states below the middle gap is related to the Gaffnian state.