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ABSTRACT
The transition between merging and bouncing outcomes for a drop impacting on a liquid film is critically controlled by the resistance from the microscopic interfacial gas layer trapped between the interacting and deformable drop and film surfaces. Using high-speed imaging and color interferometry, we have quantified and analyzed the gas layer dynamics during bouncing when the liquid film thickness is comparable to the drop radius. Results show that the gas layer morphology changes dramatically and non-monotonically with the film thickness and that in addition to the centrally located dimple previously observed for impact on thin films, a new, rim-dimple morphology is observed for larger film thicknesses. The effects of capillarity of the drop and film are also delineated by increasing the liquid viscosity and hence damping the respective surface waves.

I. INTRODUCTION
Drop impact on wet or dry surfaces may result in either merging [1–6] or bouncing [7–13]. The controlling factor is whether the thin gas layer trapped between the impacting surfaces can resist the impact inertia to bring them to close proximity for the van der Waals force to effect merging. However, due to the microscopic nature of this gas layer, it is difficult to observe experimentally the morphology and dynamics of the interfaces. Recently, optical techniques such as interferometry [4,10,11,14–16] and internal reflection microscopy [5,9] have been developed to indirectly interpret the gas layer thickness for drop impact on solid surfaces. Such studies have mostly been concerned with the merging outcomes [3,4,14,17,19,20], while corresponding studies on the morphology of the gas layer for the bouncing outcomes have been limited. To the authors’ best knowledge, only recently Kolinski et al. [9] and de Ruiter et al. [10,11] measured the gas layer profiles for a bouncing drop, albeit for impact on a solid surface, and showed that a bell-shaped dimple develops at the center and maintains its shape throughout the impact process until the drop departs from the solid surface. However, the superficially analogous problem of drop bouncing from a liquid film and the associated interfacial gas layer dynamics has not been investigated. Phenomenologically, while drop impact on a solid surface involves only one deformable surface and the internal dynamics of the drop, two deformable surfaces are involved for impact on a liquid film, with the additional film surface participating not only in the much richer dynamics of the gas layer but also through the internal dynamics of the film itself.

A critical system parameter controlling the impact response is the film thickness which transfers the resistance from the submerged solid surface. Previous studies on drop impact have been mostly concerned in the limiting situations of either a solid surface, [1–4,7–11,14–16,20,21] hence vanishing film thickness such that the drop motion is totally restricted by the
rigid surface, or a deep liquid pool\textsuperscript{[5,12,22–27]} representing infinite film thickness such that the drop motion is controlled by the deformable pool surface. Less study, however, has been directed at the intermediate situation\textsuperscript{[28–30]} for which the film thickness is comparable to the drop radius such that the resistance of the solid wall is closely felt, while the morphologies of the drop and film are also strongly coupled through the interfacial gas layer. Such situations are of particular technological relevance, including spray coating and 3D printing, in which drops impact on a liquid layer generated by previously deposited drops.

In this paper, we report, for the first time, a complete temporal characterization of the gas layer dynamics for drop bouncing on liquid surface with thickness comparable to the drop radius, using high-speed imaging and color interferometry. We have succeeded in identifying the role of the non-dimensional film thickness ($H^* = H/R$) and Weber number ($We$) in controlling the gas layer morphology, where $We = 2\rho RU^2/\sigma$, $\rho$, R, U, and $\sigma$ are the density, radius, velocity, and surface tension of the drop, and $H$ is the film thickness. In particular, we shall show that the presence of the deformable impacted surface drastically modifies the shape and evolution of the thin gas layer, observed previously during the impact on solid surfaces.

II. EXPERIMENTAL SETUP

Experimentally, a drop of about 1.6 mm in diameter was generated by pushing the test liquid through a vertically oriented stainless-steel needle using a syringe pump, with the drop separating from the needle tip when its weight overcame the capillary force. The drop landed on a liquid film contained in a $25 \times 25 \times 10$ mm$^3$ chamber with a 1.2 mm thick microscope glass slide at the bottom. The impact speed was changed by manipulating the distance between the needle and the impacted surface. The film thickness is controlled by adding or withdrawing liquid into the chamber using the syringe–needle system. Tetradecane (C14, $\rho$: 760 kg m$^{-3}$, $\sigma$: 27 mN m$^{-1}$, kinematic viscosity, $\nu$: 3.6 cSt) was used as the primary working fluid for both the drop and the liquid film.

A monochromatic high-speed camera (Phantom V7.3) along with a 50 mm lens (Nikon) and a $2\times$ tele-converter was used to capture the side-view shadowgraph images of the impact, with the aid of a halogen light. In addition, a high-speed color camera (Phantom V710) with a long-distance microscope system (Navitar 6000) connected to a 5x objective (Olympus) and a coaxially ported broadband white light (Olympus ILP2) was used to capture the bottom-view interference pattern created by the gas layer between the drop and film surfaces. The depth of field of the microscopic lens system is about 100 $\mu$m and, since the experiment is highly repeatable, for each condition, we scan the focal plane across the film depth to capture the full dynamics. The gas layer thicknesses were extracted by comparing it with a known reference profile of a lens. Details of this high-speed color interferometry are given in Ref. 14. Note that the slope of the interface does not affect the measurement (as shown in the Appendix) but reduces the local fringe intensity. If the intensity is reduced beyond the camera noise, the fringes are not observable as shown later. All the images were recorded at 15 037 fps, with spatial resolutions of 17 $\mu$m/pixel for the side-view images and 0.75 $\mu$m/pixel for the bottom-view images. The instantaneous drop diameter, impact speed, and liquid film thickness were measured from the side-view images.

III. RESULTS AND DISCUSSIONS

A. Gas layer dynamics

Based on the film thickness, the bouncing response can be divided into three regimes: thin film ($H^* < 1$), thick film ($1 < H^* < 1.5$), and deep pool ($H^* > h_{\text{max}}^*$)\textsuperscript{29}. The thin film regime bears close resemblance to impact on a solid surface since the effect of the rigid surface is prominent despite the presence of a thin layer of liquid. $H^* < h_{\text{max}}^*$ constitutes the deep pool limit, where $h_{\text{max}}^* = 1 + We/12$ (for C14) is the normalized maximum penetration of the drop into the liquid pool. Thus, for the deep pool, the drop is far from the bottom surface during impact, and as such, the gas layer dynamics is only affected by the drop and the liquid surface. In between these two extremes lies the thick film regime, in which the process is influenced by both the resistive solid surface and the adaptive liquid film. While there is no clear discernable boundary between the thin and thick film regimes, it is reasonable to identify that the transition occurs when the film thickness is close to the drop radius, i.e., $H^* \approx 1$.

A typical bouncing event is shown in Fig. 1, where the top row consists of the side-view images (global behavior), while the middle and bottom rows show the interference patterns and the corresponding gas layer thickness profiles, respectively. Figure 2(a) shows the schematic of the gas layer globally.
and locally, with $h_c$ being the center thickness of the axisymmetric gas layer and $h(r)$ being the radial variation. The gas layer dynamics will now be discussed in terms of the evolution of $h_c$ and $h(r)$.

We first consider the evolution of $h_c$ with time, normalized by the inertial time scale, $t^* = t/(2R/U)$, for impacts with similar We but different film thicknesses in Fig. 2(b). Here $t^* = 0$ represents the instant when the bottom of the drop reaches the location of the unperturbed liquid surface. In our experiments, where the impact results in bouncing of the drop, we observe two distinct stages in the evolution of $h_c$: the approaching stage, when the gas at the center continuously diminishes to reach a minimum point, and the rebounding stage, when the gas increases. During the approaching stage, for all $H^*$, $h_c$ decreases almost linearly with time. While the slopes are similar for thick film and thin film regimes, it is slightly smaller for the deep pool. During the rebounding stage, however, the response is dramatically different in that while the rate of increase of $h_c$ is rather steady for the thin film; for the deep pool, it first increases rapidly and then slows down substantially. For the intermediate case of the thick film, $h_c$ responds non-monotonically by first sharply jumping to a value of 1.7 $\mu$m, then immediately dropping to 0.4 $\mu$m, and then increasing again within a very short $t^*$ (increment of about 0.5).

Next, we discuss the instantaneous radial variation of the gas layer thickness and its evolution for all three conditions (Fig. 3); the left and right columns, respectively, indicate the approaching and rebounding stages. For the thin film, as the drop approaches the liquid surface, it squeezes the interfacial gas layer and hence creates a high-pressure zone, which in turn deforms both the drop and film surfaces and forms a bell-shaped gap with the center elevated relative to the rim [Fig. 3(a)]. This is analogous to the well-known dimple shape formed at the center when a drop impacts a solid surface, except now the interfacial gap for the thin film is formed from the deformation of both the drop and film surfaces. The trapped gas layer is subsequently squeezed out as the drop moves down further. Consequently, the center thick dimple profile morphs into a flatter and even reversed profile with elevated rim thickness compared to the center. The minimum thickness in the profile reaches around 0.2 $\mu$m before the rebounding stage begins. For the deep pool [Fig. 3(e)], although the shape of the center dimple is maintained, the gas layer thickness profiles are flatter and shorter. Due to the large pool thickness and the diminished constraint from the solid surface, the deformable liquid surface is now more adaptive to the drop impact, thereby reducing the radial variation in the gas layer thickness and hence the pressure gradient. The unrestrained deformation in the liquid surface also results in a steeper interface [side-view image in the inset of Fig. 3(e)], deflecting the incident light from the bottom away from the camera, which progressively reduces the extent of the observed/measured profile as the drop penetrates the liquid pool. For the thick film, the gas layer thickness profiles [Fig. 3(o)] show a mixed influence of the deep pool and thin film regimes, in that the profile is flatter due to the adaptive liquid surface and its radial extent increases with time due to spreading against the bottom surface.

During the rebounding stage for the thin film, when the drop as a whole rises upward [Fig. 3(b)], the center of the gas layer gradually thickens, forming another, albeit much narrower, bell-shaped profile. At the same time, the rim also thickens forming an annular dimple and is connected to the center dimple through a thin neck (0.15 $\mu$m). Both dimples become thicker as the drop moves upward during the rest of the rebounding stage, with the neck moving inward and becoming thicker. Note that the radial extent of the visible interference pattern, and thus measured gas layer thickness, reaches beyond $r^* = 0.5$. Mechanistically, deformation of the liquid surface is now largely constrained by the solid surface such that spreading of the drop over the surface closely resembles that for drop impact on the dry surface. For the deep pool [Fig. 3(d)], the center thickens first but with a flatter profile. The gas layer becomes thicker as the drop leaves the liquid surface without changing its shape. The observable extension is now larger, up to $r^* = 0.5$, as compared to the approaching stage.

For the thick film, however, evolution of the profiles is markedly different [Fig. 3(d)]. At the beginning of the rebounding stage, the center of the gas layer thickens, with the familiar bell-shape dimple. It then changes immediately, within 0.28 ms ($\Delta t^* = 0.7$), as the profile becomes flatter and thinner at the center with a thicker rim, forming a reverse bell-shape, with the rim being thicker than the center. Finally, with the rim remaining the same, the center starts to become thicker and approaches the thickness of the rim until the drop leaves the liquid surface.

Phenomenologically such plethora of possible shapes in the rebounding stage can be attributed to the dynamics of capillary oscillation of the drop and the liquid film surface. During the approaching stage, both the drop and the liquid film are deformed and stretched by the impact inertia, while during the rebounding stage, they will be retreating toward
FIG. 3. Gas layer thickness profiles at approaching [(a), (c), and (e)] and rebounding [(b), (d), and (f)] stages. [(a) and (b)] Thin film, We = 9.91, \(H^*=0.12\). [(c) and (d)] Thick film, We = 8.04, \(H^*=1.19\). [(e) and (f)] Deep pool, We = 9.26, \(H^*=2.72\).

their initial shape causing capillary controlled shape oscillation in both drop and liquid surfaces. For example, the unaltered rim thickness during the latter part of the rebounding stage for the impact on the thick film indicates that the relaxation of the drop and film surface are phase matched, and as such, they move together near the rim. Since the complexity in the gas layer dynamics increases as the capillary relaxation of the drop and film surfaces is affected by the solid substrate, the rebounding stage of the impact on thick-film shows the most complex behavior.

To evaluate the effects of \(We\), evolution of the center thickness, \(h_c\), [Fig. 4(a) for the thick film, Fig. 4(d) for the deep pool] and the radial variation, \(h(r)\), [Figs. 3(c) and 3(d) vs 4(b) and 4(c) for the thick film, Figs. 3(e) and 3(f) vs 4(e) and 4(f) for the deep pool] of the gas layer for both thick film and deep pool are compared for different values of \(We\). For the evolution of \(h_c\), the slope of the linear approaching stage and the general behavior of the nonmonotonic (thick film) or monotonic (deep pool) rebounding stages are the same for both \(We\). However, for higher \(We\), there is a delay between the end of the approaching stage and the increase in the center thickness. Furthermore, the higher \(We\) case has a longer duration of the rebounding stage, appearing as it stretches the \(h_c\) evolution of the rebounding stage for both thick film and deep pool. The delay and the stretched rebounding stage can be attributed to the higher penetration depth of the drop into the liquid film (for the deep pool), and the larger spreading of the drop, on the solid substrate (thick film) during the approaching stage at higher \(We\) to allow enhanced drop and film deformation affecting their interaction during the rebounding stage.

The gas layer thickness profiles during the approaching stage are also similar with typical features of the extending radial extension for the thick film and the predominant flat profile and reduced radial extension for the deep pool. The behavior at the rebounding stage show different features at higher \(We\). For the thick film [Fig. 4(c)], although the center becomes thick first and forms a neck at the rim, unlike the low
We case, the center thickness does not reduce. Instead, the neck starts to thicken and simultaneously moves towards the center. Consequently, the rim becomes thicker than the center forming a reversed bell shape and the center thickness reduces slightly. Eventually, the center starts to thicken again and the reversed bell shape flattens out to retrieve the regular bell shape as the drop leaves the liquid surface. For the deep pool at higher $We$ [Fig. 4(f)], unlike low $We$, the rim becomes thicker first and develops an "annular dimple," which grows and starts propagating towards the center because of the radially inward entrainment of gas from outside. Finally, the dimple arrives at the center as the profile takes a bell-shape and is maintained until the drop leaves the liquid surface. This specific characteristic of rim thickening followed by its propagation towards center, which appears only at higher $We$ for both thick film and deep pool, likely arises from a phase delay between relaxation of the deformed film and the oscillating drop. Although the drop relaxation is solely controlled by its own capillarity, inception of the relaxation process for the liquid surface can only occur after it reaches maximum

![FIG. 4. Gas layer dynamics with higher $We$ for the thick film [(a)–(c); $We = 13.24$, $H^* = 1.48$] and deep pool [(d)–(f); $We = 12.96$, $H^* = 2.67$]. (a) Comparison of center thickness evolution for lower (8.04) and higher (13.24) $We$ for the thick film. (d) Comparison of center thickness evolution for lower (9.26) and higher (12.96) $We$ for the deep pool. Gas layer thickness profiles at [(b) and (e)] approaching and [(c) and (f)] rebounding stages.](image-url)
deformation. Both the duration and degree of the deformation depend on We, affecting the phase delay between the drop and the liquid surface. In Sec. III B, we will delineate the role of these relaxation processes on the gas layer profile qualitatively.

B. Role of drop and liquid film deformation on gas layer profile

Unlike impact on a flat solid surface where the gas layer thickness profile inherently depicts the local shape of the drop, for drop impact on a liquid film, it is the relative distance between the deformed surfaces of the drop and the film. To understand which contributes more toward the observed gas layer dynamics, the drop or the film surface, liquids of higher viscosity but similar surface tension were additionally examined. The liquid properties are n-heptadecane, C17: \( \rho = 777 \text{ kg m}^{-3}, \sigma = 27.5 \text{ mN m}^{-1}, \) and \( \nu = 4.8 \text{ cSt}; \) silicone oil, S5: \( \rho = 910 \text{ kg m}^{-3}, \sigma = 20 \text{ mN m}^{-1}, \) and \( \nu = 5 \text{ cSt}; \) and silicone oil, S100: \( \rho = 960 \text{ kg m}^{-3}, \sigma = 20 \text{ mN m}^{-1}, \) and \( \nu = 100 \text{ cSt}. \) As discussed earlier, the complex dynamics of the gas layer evolution reported herein arises from the interplay of capillary waves on the drop and liquid surfaces, which would be significantly damped for high viscosity liquids. Here, we compare the gas layer profiles for impact on the deep pool where the drop and film deformations are not restricted by the bottom substrate. First, we note that the gas layer profiles in the approaching and rebounding stages are qualitatively similar between C14 (3.6 cSt) [Figs. 4(e) and 4(f)] and C17 [Figs. 5(a) and 5(b)]. For the significantly higher viscosity S100 [Figs. 5(c) and 5(d)], the capillary waves on both drop and liquid surfaces decay quickly, and as such, the rich dynamics in the gas layer profile disappears. The profiles maintain the bell-shape with minimal changes in thickness throughout the approaching and rebounding stages.

To further delineate the effects of drop and liquid surfaces on the gas layer dynamics, we purposefully weaken the capillary waves either in the drop or in the liquid film by mismatching their viscosities. For the first configuration of low viscosity drop (S5) impacting on the high viscosity liquid pool (S100), no interference fringe was observed during the rebounding stage. It signifies rapid departure of the drop from the liquid surface without further interaction. Since high viscosity dissipates the kinetic energy and weakens the capillary wave, the liquid surface recovers much slower than the drop, causing a large gap between them. It shows that for observable fringes to form during the rebounding stage, it is imperative for the drop and liquid surface to recover at similar rates. For the second configuration of the high viscosity drop (S100) impacting on the low viscosity pool (S5) [Figs. 5(e) and 5(f)], the capillary wave in the drop is significantly weaker. Consequently, in the rebounding stage, we observe formation of the neck region in the rim propagating to the center, similar to the dynamics observed for C14 and C17. The similarity among features observed for C14, C17, and S100 (drop)/S5 (pool) and the minimized variation in the gas layer dynamics for S100 (drop)/S100 (pool) suggest that the capillarity or surface wave in the liquid film indeed plays a critical role in creating gas layer morphologies different from the traditional bell-shaped (dimple) profiles. It is noted that Beilharz\(^{31}\) recently studied a highly viscous drop submerged in a liquid pool after impact, with interferometric measurement of the gas layer thickness at the side of the drop.
The important role of liquid surface capillarity on the gas layer dynamics can be further highlighted by comparing the impact on a solid surface [Figs. 11(b1) and 11(b2) from Ref. 11] and a thin film [Figs. 3(a) and 3(b)]. In Ref. 11, de Ruiter et al. reported dynamical changes in the gas layer profile and interfacial pressure for a water drop impacting on a hydrophobic glass wafer at \( \text{We} = 1.34 \). While the gas layer thickness profiles are bell-shaped for both of these experiments during the approaching stage, the profile becomes comparatively thinner and flatter for impact on the thin film (current experiment) due to the adaptive liquid surface that moderates pressure buildup. Furthermore, during the rebounding stage, the radial profile for the thin film develops a dimple at the rim, whose thickness can be larger than that at the center, which is absent for impact on the solid surface. As the deformed drop relaxes with time, the dimple at the rim propagates toward the center, indicating a wave behavior and hence the presence of capillary oscillation of the liquid surface, which is not observed for impact on the solid surface. Furthermore, the presence of the deformable thin film in our experiments significantly reduces the pressure buildup in the interfacial gas layer and the maximum dimple height compared to the impact on the solid surface, as reported in Ref. 11. Recently, this adaptive nature of the impacted surface has been discovered to reduce the pressure buildup for impact on soft solid surfaces as well, leading to suppression of pressure buildup and thus splashing.32

IV. CONCLUSION

In summary, we have experimentally investigated the evolution of the profile of the gas layer thickness when a drop bounces upon impacting the liquid surface of various film thicknesses and Weber numbers, leading to the observation, quantification, and explanation of several phenomenologically new and significant features. We have demonstrated that, unlike impact on a solid surface, here the liquid surface absorbs the impact inertia by deforming itself and subsequently reshapes itself to moderate the pressure gradient. Furthermore, the deformed liquid surface at the later stage relaxes along with the deformed drop producing capillary oscillations, which internally interact with each other and the bottom solid substrate. As a result, the bell-shaped profile, which is universal and omnipresent for impact on solid surfaces, is found to transform into new shapes such as the “annular dimple” at the rim and the “reversed bell shape,” signifying interaction between the drop and the film surface. We conclude this exposition by noting that the results of the gas layer dynamics reported here are first of its kind, and as such, they are mostly explained phenomenologically. It is anticipated that dissemination of such complex dynamics would stimulate further theoretical and computational studies to explore such behavior.

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APPENDIX: EFFECT OF SLOPE OF THE INTERFACE ON INTERFEROMETRY

In recent years, use of interferometry for measuring small gaps between surfaces has been adopted in many fields. These studies have paved the way for its use in droplet studies. The theoretical formulation for this technique using a single wavelength light source has been provided in Refs. 33–35. Most of the theoretical analyses described in these references use an example of two parallel interfaces, which represents an ideal configuration, while in many applications, one or more surfaces are curved. Weigand et al.33,34 provided a correction function for inclined (or curved) interfaces. de Ruiter et al.35 showed that for drop impact on solid surfaces, the inclination or slope of the drop surface is less that 3°, which is negligible. Nevertheless, for completeness, we present, herein, an analysis with ray-diagrams to estimate errors due to curved surfaces. Below we considered a few configurations which would represent all possible geometries for drop impact studies.

First, we consider the simplest case of two parallel interfaces, which are also horizontal, as shown in Fig. 6(a) (case I).
From the theory of interferometry, we know that the interference pattern is dependent on the optical path difference (OPD) between the two reflected rays [1st and 2nd reflected rays, in Fig. 6(a)], which we term as dL. Since the interference pattern is used to measure the vertical gap between the surfaces (h), we seek a relationship, h = f(dL). From the geometry, we can see dL = AB + BC and thus, dL = 2AN/cos β. The vertical gap between the surfaces (h) for this configuration is AN. Thus, dL = 2h/cos β, where β is the angle of refraction, which depends on both the refractive index of liquids 1 and 2 and the incident angle, θ0. So we can write h1 = (dL/2)cos β (sub-script 1 in h1 represents case 1). This analysis is also valid for a configuration where two interfaces are curved, but the tangents to the interfaces are parallel and horizontal.

Next, we consider the case (case 2) of two parallel curved surfaces, such as the tangents of these surfaces are parallel but not horizontal [Fig. 6(b)]. Here we introduce an inclination angle for interface 1 (angle between the tangent of interface 1 and the horizontal axis), β. This configuration illustrates the effect of the slope of interface 1 on the measurement. To be consistent with case 1, we consider the analysis for the same refractive index case and the incident angle, θ0. From geometry, we find dL = 2AN/cos β, which is the same as case 1. However, in this configuration, AN is the normal distance from interface 1, while AM is the vertical distance between two interfaces (h). Simple geometrical analysis shows AN = AM cos β and thus h2 = (dL/2)cos β.

Now, we can compare the analyses for case 1 and case 2 to calculate the error due to the slope of the first interface, C2-1 = (θ2 - θ1)/h1 = (1/ cos β – 1)/h. In our experiments, the angle of the local tangents (effect of the slope) of the liquid film with the horizontal axis, which can be estimated from the side-view images (Fig. 1). Within the range of the radial distance where the interference pattern can be measured in our experiments (examples in Fig. 1), we find that the inclination angle β < 5° and thus the error C2-1 < 0.4%, which is negligible.

We can further extend the analysis for more generic yet complicated geometry of two non-parallel interfaces.

In Fig. 7(a), we show such a configuration (case 3), where interface 1 is horizontal. Here, we define the inclination angle for interface 2, α, as the angle between the tangents of interfaces 1 and 2. From geometry, we can show AB = AN(cosβ1 + sinβ1 tan(α + θ1)) and BC = AB cos θ1/cos(2α + θ1). Since dL = AB + BC and AN is the vertical height (h) between the interfaces, we can write h3 = dL/[(cosβ1 + sinβ1 tan(α + θ1)) (1 + cos θ1/cos(2α + θ1))].

The error C3-1 = |(θ3 - θ1)/h3|, which depends on both α and θ1, captures the effect of the slope of the second surface on the measurements. In interferometry measurements, the incident light comes from a coaxial microscope, which acts as both a light source and an objective, and as such, the incident angle, θ0, is close to 0, and thus θ1 is also small. As shown by previous work (Ref. 35), the inclination angle for interface 2 in drop impact solid surface studies is less than 3° (α < 3°), for which the error is very small (e.g., C3-1 < 2% for θ1 = 5°). This was also manifested through successful measurement of gas layer thicknesses for impact on solid surfaces in a series of work including Refs. 9–11, 14–19, and 35. It is noted that for α = 0, we get h3 = (dL/2)cos θ1 which is the solution for case 1.

For the final step, we consider the most generic case, where the interfaces are non-parallel, and both have different inclination angles (case 4). This case, shown in Fig. 7(b), represents the configuration of drop impact on a liquid surface, where both surfaces are curved. To be consistent with previous cases, we define the inclination angle of interface 1, β, as the angle between its tangent and the horizontal axis while the inclination of interface 2, α, as the angle between its tangent and the tangent of interface 1. Just like case 3, we find that AN = dL/[(cosβ1 + sinβ1 tan(α + θ1)) (1 + cos θ1/cos(2α + θ1))], which is the surface normal distance from interface 1. The vertical distance between the interfaces, h4, is AM. From geometry, it can be shown that AM = AN(cosβ + sinβ tan(α + β)). So the final relationship we get is h4 = dL/[(cosβ1 + sinβ1 tan(α + θ1)) (1 + cos θ1/cos(2α + θ1)) (cosβ + sinβ tan(α + β))].

The effect of the curved film surface on the interferometry measurement for drop impact on liquid surfaces thus

![Fig. 7. Ray diagram for (a) two non-parallel interfaces, where interface 1 is curved and interface 2 is horizontal, and (b) two curved interfaces, whose tangents are not horizontal.](image-url)
can be evaluated by the error $C_{4-3} = \frac{|h_4 - h_3|}{h_3} = |(\cos \beta + \sin \beta \tan(\alpha + \beta)) - 1|$. As it was discussed before, $\beta < 5^\circ$ and $\alpha < 3^\circ$ and thus $C_{4-3} < 0.8\%$.

In summary, the slope or the inclination of the interfaces does affect the measurement of gas layer thickness using the interferometry. However, since these inclinations are very small in drop impact on solid or liquid surfaces, the estimated error is negligible.

To further show that the slope of the bottom surface indeed does not introduce significant error in the measurements, we performed some validating experiments with different lenses. Particularly, we measured the thickness of the gas layer between a plano-convex lens and a flat glass slide using the color interferometry technique for both configurations [Figs. 8(a) and 8(b)] below.

As one can see for the normal configuration, the incident light passes through a flat horizontal surface (zero slope) before reaching the reflecting surface of the lens. This configuration also represents that used for calibration. On the other hand, in the inverted configuration, the incident light passes through the curved lens surface (with radially varying non-zero slope) before reaching the flat surface. Although the gas layer thickness $h(r)$ as a function of radial location ($r$) is the same for both of them, the inverted configuration has a curved first interface with non-zero local slope, just like our experimental situation except for the direction of curvature (concave vs convex). We measured the gas layer profile using the color interferometry technique for both configurations and found that the differences are negligible for three lenses of different curvatures (Fig. 9). This proves our point that the presence of non-zero slope of the liquid surface does not introduce significant error; it simply reduces the intensity of the light.

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