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Spontaneous scalarization with massive fields

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We study the effect of a mass term in the spontaneous scalarization of neutron stars, for a wide range of scalar field parameters and neutron star equations of state. Even though massless scalars have been the focus of interest in spontaneous scalarization so far, recent observations of binary systems rule out most of their interesting parameter space. We point out that adding a mass term to the scalar field potential is a natural extension to the model that avoids these observational bounds if the Compton wavelength of the scalar is small compared to the binary separation. Our model is formally similar to the asymmetron scenario recently introduced in application to cosmology, though here we are interested in consequences for neutron stars and thus consider a mass term that does not modify the geometry on cosmological scales. We review the allowed values for the mass and scalarization parameters in the theory given current binary system observations and black hole spin measurements. We show that within the allowed ranges, spontaneous scalarization can have nonperturbative, strong effects that may lead to observable signatures in binary neutron star or black hole–neutron star mergers, or even in isolated neutron stars.

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I. INTRODUCTION

Spontaneous scalarization is a phenomenon that occurs in certain scalar-tensor theories of gravity where the scalar field vacuum can be unstable to condensation of the field in the presence of certain kinds of matter [1]. As we discuss in more detail below, at the linear level the instability is a long wavelength tachyon instability, where the minimum wavelength is inversely related to the density of matter. Nonlinear coupling of the scalar field to matter saturates the growth of the field at a value related to a parameter in the coupling potential. These facts together allow for the intriguing possibility that, with certain potentials, amongst all compact matter objects known in the Universe only neutron stars offer an environment where scalarization can occur. Moreover, the effects can be nonperturbative, allowing order-of-unity deviation in the structure of neutron stars. Hence this offers a (rare) example of an alternative theory to general relativity (GR) that is consistent with current weak-field observational bounds [2], yet could have sizable deviations in the dynamical strong field, in particular, as pertains to gravitational wave emission in merger events [3,4].

To date, most investigations of spontaneous scalarization have considered massless scalars. However, recent observations of a pulsar–white dwarf binary [5] have allowed rather stringent bounds to be placed on the massless theory, eliminating most of the range of relevance to future gravitational wave observations. Moreover, as we show below, a massless scalar would have condensed on cosmological scales and could be ruled out by cosmological observations alone.[1] A simple way to adjust the theory to evade these observational constraints, yet preserve the property of giving neutron stars large, nonperturbative corrections to the predictions of GR, is to give the scalar field a mass m_ϕ. Such a modification is also rather "natural" compared to the machinations theorists often resort to conform their favorite theory to observational constraints (questions about the naturalness of the original theory aside). The effect of the mass term is twofold. First, it suppresses the tachyon instability for wavelengths longer than the Compton wavelength $\lambda_\phi = 2\pi/m_\phi$ of the field (unless otherwise stated we use units where Newton's constant G, the speed of light c and Planck's constant $\hbar$ are set to 1). Thus a very light mass can prevent the instability on cosmological scales. Second, it screens the presence of the field outside a scalarized neutron star in that the field falls off as $e^{-r/\lambda_\phi}/r$ rather than the $1/r$ decay of a massless field. This will effectively shut off dipole radiation in a white dwarf–neutron star system if the orbital radius is significantly larger than λ_ϕ [8]; it is the lack of inferred dipole radiation that allows the observations presented in [5] to so tightly constrain the massless theory.

[1]On the flip side, this can actually be a "feature" if one wants to explain aspects of dark energy or dark matter with scalar-tensor gravity, rather than limit its effects to neutron stars. As far as we are aware two models similar to the spontaneous scalarization theory described here are the so-called "symmetron" [6] and more recently introduced "asymmetron" cosmologies [7]; we will discuss similarities with our work and this latter model further below.

Motivated by the above considerations, in this work we present an initial study of massive scalar field spontaneous scalarization in neutron stars. Our main goal is to investigate the static solutions representing isolated nonspinning neutron stars within this theory, for various parameters of the theory and neutron star equations of state (EOS). This is a first step toward exploring the mergers of binary neutron star and black hole–neutron star systems, which we are currently pursuing and will present the results for elsewhere. Much of what has been discussed above about scalarization is well known. Independent of our work, a mass term has recently been discussed [7] in what the authors dub the asymmetron scenario. While the primary motivation for the asymmetron is cosmological, we are interested in a modified theory which differs from GR only on small scales relevant to the late stages of compact object coalescence involving neutron stars. We will discuss the differences between our model and the asymmetron in the results section.

The effects of scalar potential terms in scalar-tensor theories have been investigated in the post-Newtonian expansion [9–11], though to our knowledge no detailed work on neutron star structure or binary mergers has been performed for massive field scalarization in the fully nonlinear regime.[2] We aim to start this discussion by understanding the properties of isolated scalarized neutron stars. As such, after introducing the theory in Sec. II we give back-of-the-envelope calculations illustrating the properties discussed above. This suggests masses in the range 10^{-15} eV $\lesssim m_\phi \lesssim 10^{-9}$ eV are consistent with present observational constraints, yet can produce nonperturbative deviations in the structure of neutron stars. A midsection 10^{-13} eV $\lesssim m_\phi \lesssim 10^{-11}$ eV of this range can further be eliminated if putative measurements of the spin of solar mass black holes are accurate [13], and these are old black holes. The reason for this is that if a rapidly spinning black hole forms with Schwarzschild radius close to the Compton wavelength of a massive scalar field, superradiant amplification of the scalar field will occur, causing the black hole to lose most of its angular momentum on time scales sufficiently short to make observation of the initially highly spinning black hole unlikely [14,15].

In Sec. IV we solve the analog of the Tolman-Oppenheimer-Volkov (TOV) equations in this theory to find the static neutron star solutions. We search over the parameter space of the scalar-tensor theory to see where scalarization in neutron stars occur and ascertain its effect on the star's structure. We show that for λ_ϕ much larger than the radius of the neutron star, the mass term (unsurprisingly) has little effect compared to the massless theory. For λ_ϕ much smaller than the radius of the neutron star, the mass term prevents scalarization, and the usual neutron stars of GR result. Scalarization can lead to large observable effects in certain parts of the parameter space even for isolated stars. We discuss the implications of such effects and how future gravitational wave observations of binary mergers could constrain this theory. We investigate nonrotating stars in the $\beta < 0$ regime, which has been the primary interest in the literature, though some recent work has also explored $\beta > 0$ [16–18].

[2]After submission of our manuscript Stoytcho Yazadjiev and Dimitar Popchev informed us that they have independently been working on massive scalarization as part of Popchev's Diploma at Sofia University. Their work has recently appeared on the [12].

II. EQUATIONS OF MOTION AND THE ORIGIN OF SPONTANEOUS SCALARIZATION

The Lagrangian of the scalar-tensor theory that leads to spontaneous scalarization is given by [1],

$$\frac{1}{16\pi}\int d^4x\sqrt{g}[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2m_\phi^2\phi^2] + S_m[\psi_m, A^2(\phi)g_{\mu\nu}], \tag{1}$$

where $g_{\mu\nu}$ is the metric in the Einstein frame, ϕ is the scalar field and m_ϕ is the parameter coupling to the mass potential. S_m is any ordinary matter contribution to the Lagrangian with the matter degrees of freedom represented by ψ_m. Here, ψ_m couples to a conformally scaled version of the metric, $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$, rather than the minimal coupling in GR. The scaled metric defines the so-called Jordan frame and is the physical metric observers use to measure proper length scales. We use a tilde to denote any tensor defined in this frame. The equations of motion are

$$R_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) + 2\partial_\mu\phi\partial_\nu\phi + m_\phi^2\phi^2 g_{\mu\nu},$$
$$\Box_g\phi = -4\pi\alpha(\phi)T + m_\phi^2\phi, \tag{2}$$

where $\alpha = \partial(\ln A)/\partial\phi$ and $\Box_g$ is the wave operator with respect to the Einstein frame metric. We use $A(\phi) = e^{\beta\phi^2/2}$ throughout this study, with β a constant parameter, but other choices are also possible.[3] We only consider negative values as in [1].

Note that $\phi = 0$ is a solution in this theory and is equivalent to GR. Spontaneous scalarization occurs when this solution is unstable; i.e. an arbitrarily small perturbation of ϕ grows and the system ends up in a stable configuration with nonzero ϕ.

[3]Aside from the coefficient of the parabolic ϕ term, β, another important property of the potential A is its asymptotic value $A_\infty = A(\phi \to \infty)$. This parameter determines the deviation from GR for extremely strong scalar fields. Our choice of $A_\infty = 0$ gives the maximal possible deviation, whereas values closer to 1 set an *a priori* upper limit on the observable differences from GR. The asymmetron scenario considers such nonzero A_∞ values.

First, let us give a sketch of the physical mechanism behind spontaneous scalarization (see [1,4,7]) for alternative approaches). To relate the dynamics of the field to physical properties of matter, we rewrite the scalar field equation of motion as follows:

$$\Box_g \phi = -4\pi\beta e^{2\beta\phi^2}\phi\tilde{T} + m_\phi^2\phi, \tag{3}$$

where $\tilde{T}$ is the trace of the physical stress-energy tensor. Beginning with a small perturbation about the GR solution $\phi = 0$, we can expand to linear order in ϕ:

$$\Box_g \phi \approx (-4\pi\beta\tilde{T} + m_\phi^2)\phi. \tag{4}$$

For matter that can be modeled as a perfect fluid, $\tilde{T} = -\tilde{\rho} + 3\tilde{P}$, where $\tilde{\rho}$ and $\tilde{P}$ are the (physical) rest-frame density and pressure of the fluid, respectively. For nonrelativistic matter, $\tilde{\rho} \gg \tilde{P}$, $\tilde{T} \approx -\tilde{\rho}$, and so for $\beta < 0$ the first term on the right in (4) is effectively a negative mass-squared term. At the linear level the theory thus suffers a tachyon instability where $\lambda_{\rm eff} < \lambda_\phi$, with

$$\lambda_{\rm eff} \equiv \sqrt{\frac{\pi}{|\beta|\tilde{\rho}}}. \tag{5}$$

Consequently all Fourier modes with wavelength $\lambda > \lambda_{\rm eff}/\sqrt{1 - (\lambda_{\rm eff}/\lambda_\phi)^2}$ that “fit” within the region where $\lambda_{\rm eff} < \lambda_\phi$ will initially experience exponential growth. This of course by itself would be disastrous for the theory, though from (3) one can see that the nonlinear term $e^{2\beta\phi^2}$ will eventually become important and suppress the growth, saturating ϕ at a value, order of magnitude, of $1/\sqrt{|\beta|}$.

For a star, approximating $\tilde{\rho} \sim M/R^3$, where M is the mass of the star and R its radius,

$$\lambda_{\rm eff,star} \sim \frac{R}{\sqrt{C|\beta|}}, \tag{6}$$

with $C \equiv 2M/R$ being its compactness. To be susceptible to scalarization, $\lambda_{\rm eff,star} < \lambda_\phi$, and the shortest wavelength unstable mode must fit inside the star, or roughly $R > \lambda_{\rm eff,star}$. Thus, for a given β, only stars that are sufficiently compact,

$$C \gtrsim 1/|\beta|, \tag{7}$$

can scalarize. For a typical $1.4M_\odot$ neutron star C is approximately between $1/5$ and $1/3$ (depending on the equation of state), a white dwarf has $C \sim 10^{-3}$, and a main sequence solar mass star has $C \sim 10^{-6}$. Note, however, that for very massive neutron stars, and again depending on the EOS, the core can become relativistic in that $\tilde{T} \gtrsim 0$, which will suppress scalarization.

On cosmological scales, during the radiation dominated epoch $\tilde{T} \approx 0$; however, in the matter dominated era $\tilde{T} \approx -\tilde{\rho}_m$, where $\tilde{\rho}_m$ is the average, redshift z-dependent energy density in matter. Thus, unless the scalar field has a sufficiently large mass term, the entire Universe would scalarize (see also the discussion in [19]). To estimate how large a mass term is required to prevent this, let us assume that the nonrelativistic component of matter became relevant at matter-radiation equilibrium $z_{eq} \approx 10^3$. Then, $\tilde{\rho} \equiv \tilde{\rho}_{m,eq} \approx \tilde{\rho}_{m0} z_{eq}^3$, where $\rho_{m0} \sim 3 \times 10^{-27}$ kg/m^3 is the present-day baryonic matter density. Relative to the matter density at z_{eq}, to prevent scalarization would thus require

$$\lambda_\phi \lesssim 10^5 \text{ pc}\sqrt{\frac{\tilde{\rho}_{m,eq}}{|\beta|\tilde{\rho}_m}}, \tag{8}$$

or

$$m_\phi \gtrsim 10^{-27} \text{ eV}\sqrt{\frac{|\beta|\tilde{\rho}_m}{\tilde{\rho}_{m,eq}}}. \tag{9}$$

Note that a similar order-of-magnitude estimate could have been obtained using the earlier analysis for stars, since when considered a uniform density sphere the compactness of the Universe reaches unity for a radius of order the Hubble length. If a tachyonic instability were excited in the Universe, a simple estimate for its growth rate can be obtained solving (4) on a flat background and assuming ρ_m is constant: $\phi \propto e^{at}$, with $a = \sqrt{4\pi|\beta|\tilde{\rho}_m}$. Scaled to z_{eq}, this gives an e-folding time of

$$t_s \sim 10^6 \text{ yr}\sqrt{\frac{\tilde{\rho}_{m,eq}}{|\beta|\tilde{\rho}_m}}. \tag{10}$$

Thus unless $|\beta|$ is extremely small, scalarization happens very rapidly on a cosmological time scale. However, note that regardless of the magnitude of β, once the instability saturates the effect is an order unity scaling between the Einstein and Jordan frame metrics ($\tilde{g}_{\mu\nu} = e^{4\beta\phi^2} g_{\mu\nu}$, $\phi \propto 1/\sqrt{|\beta|}$).

Returning to stellar sources, far from the star, the spherically symmetric static solution for the case with a mass term is

$$\phi(r \to \infty) \sim a\frac{e^{-2\pi r/\lambda_\phi}}{r}. \tag{11}$$

For the massless case, this changes radically to

$$\phi(r \to \infty) \sim \phi_\infty + \frac{a}{r}, \tag{12}$$

where ϕ_∞ and a are constants. Thus it is apparent that the mass term effectively screens the potential on scales larger that λ_ϕ, and also removes the ambiguity in the vacuum state of the field (i.e. $\phi_\infty = 0$).

A. Theoretical and observational bounds on the parameters of the theory

Our main theoretical motivation for considering spontaneous scalarization is to explore the consequences of an alternative theory of gravity to GR that (i) is consistent with GR in all regimes where it has been tested by experiment or observation, (ii) predicts large deviations from GR in the dynamics and consequently gravitational wave emission during strong-field merger events, and (iii) has a classical, mathematically well-posed initial value problem. There are several reasons why these restrictions are important. First is to understand the issue of theoretical bias in the gravitational wave detection effort [20]. This can arise due to the heavy reliance on theoretical templates for gravitational wave observation: if GR does not exactly describe the dynamical strong-field regime relevant to the late stages of merger, unless templates are used that explicitly measure this, the result could likely be detections erroneously attributed to pure GR with "wrong" parameters for the binary. Various methods, such as the parametrized post-Einsteinian (ppE) approach [20], have been proposed to try to measure the consistency of a signal with GR, though without explicit examples beyond perturbation theory for how the waveforms can differ, it is unclear how effective these approaches may be. Case in point is the "dynamical scalarization" effect noted in binary neutron star mergers within the massless theory in [3], where at close separations prior to merger a scalarized neutron star is able to induce scalarization in its initially unscalarized companion; this affects the waveform in a manner not well captured by the original ppE parametrization. Second, it is unclear whether using a theory that violates (i) is useful, even if only to use as a strawman to measure the effects of deviations from GR; i.e. consistency in the weak field and with the leading order radiative dynamics of GR may, in general, severely constrain possible deviations in the strong field. Lastly, if a theory violates (iii), aside from the obvious doubts that would place on its viability, it will not be solvable using standard numerical methods. It is rather remarkable that (to our knowledge) scalar-tensor theories with spontaneous scalarization are the only class of alternatives that have been demonstrated to satisfy (i), (ii) and (iii).

These considerations thus guide the following choice of parameters within the theory that we consider viable. First, the observational constraints inferred from the pulsar–white dwarf binary PSR J0348 + 0432 [5] have come close to ruling out essentially the entire range of parameters in the massless theory that lead to neutron stars being scalarized. The massless theory also necessarily affects the Universe on cosmological scales. We thus require a mass term (see [7] for an alternative cosmological view). To avoid bounds from PSR J0348 + 0432 requires that the mass be sufficiently large such that $\lambda_\phi \ll r_p$, with $r_p \sim 10^{10}$ m the periapse of the orbit (the actual orbital parameters of the binary are very accurately measured, though here we just give the order-of-magnitude results). This translates to a lower limit for the mass

$$m_\phi \gg 10^{-16}\ \text{eV}, \tag{13}$$

which also easily suppresses cosmological effects.[4] From (7), to allow a star as compact as a neutron star to scalarize, but not a white dwarf, bounds β to the range

$$3 \lesssim -\beta \lesssim 10^3. \tag{14}$$

To not have the mass term prevent scalarization in a neutron star requires $\lambda_{\text{eff,star}} < \lambda_\phi$, which depends on the structure of the star [see (6)]. For a strict upper limit consider a neutron star where $C|\beta|$ is maximal ($C \sim 1/3$ and $|\beta| = 10^3$); with $R \approx 10$ km this gives

$$m_\phi \lesssim 10^{-9}\ \text{eV}. \tag{15}$$

For $|\beta|$ approaching its lower limit, this upper limit on m_ϕ increases by about 2 orders of magnitude.

A further restriction on the scalar mass can be leveled if claimed measurements of high spins for several candidate stellar mass black holes are correct [13]. The reason is that highly spinning black holes are superradiantly unstable in the presence of a massive scalar field with Compton wavelength on order the size of the black hole [14]. The effect of the instability is to spin down the black hole; thus observation of a highly spinning, old black hole rules out the existence of a related range of scalar field masses. Taking he present observations of black hole spins rules out the mass window from roughly 10^{-11} eV to 10^{-13} eV.

III. RESULTS

With the above guidance on restrictions to the scalar-tensor theory parameters, we investigate the spontaneous scalarization scenario for various equations of state and various values of β and m_ϕ.

A. TOV framework

We seek the static solutions for perfect fluid neutron stars in the spontaneous scalarization theory with a mass term; the massless-case study was originally carried out in [1]. We use the following Einstein-frame ansatz for the metric:

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(r)}dt^2 + \frac{dr^2}{1-2\mu(r)/r} + r^2 d\Omega^2. \tag{16}$$

In terms of physical quantities, the perfect fluid stress energy tensor is

[4]The asymmetron scenario proposes a lower bound of 10^{-11} eV [7]. This arises from a completely different consideration motivated by cosmology.

$$\tilde{T}^{\mu\nu} = (\tilde{\rho} + \tilde{p})\tilde{u}^{\mu}\tilde{u}^{\nu} + \tilde{p}\tilde{g}^{\mu\nu}, \tag{17}$$

where the energy density $\tilde{\rho}$, pressure $\tilde{p}$, and components of the fluid 4-velocity $\tilde{u}^{\alpha}$ only depend on the radial coordinate r [and due to the symmetries $\tilde{u}^{\alpha} = e^{-\nu/2}(\partial/\partial t)^{\alpha}$]. The equations of motion (2) reduce to the following set of ordinary differential equations (ODEs):

$$\begin{aligned}
\mu' &= 4\pi r^2 A^4(\phi)\tilde{\rho} + \frac{1}{2} r(r-2\mu)\psi^2 + \frac{1}{2} r^2 m_\phi^2 \phi^2 \\
\nu' &= r\psi^2 + \frac{1}{r(r-2\mu)}[r^3[8\pi A^4(\phi)\tilde{p} - m_\phi^2\phi^2] + 2\mu] \\
\phi' &= \psi \\
\psi'(r-2\mu) &= 4\pi r A^4(\phi)[\alpha(\phi)(\tilde{\rho} - 3\tilde{p}) + r\psi(\tilde{\rho} - \tilde{p})] \\
&\quad + m_\phi^2(r^2\phi^2\psi + r\phi) - 2\psi(1 - \mu/r) \\
\tilde{p}' &= -(\tilde{\rho} + \tilde{p})(\nu'/2 + \alpha(\phi)\psi),
\end{aligned} \tag{18}$$

where $'$ denotes a derivative with respect to r. This system of equations is closed by supplying an equation of state of the form $\tilde{\rho} = \tilde{\rho}(\tilde{p})$. Then, to solve the equations requires specifying initial conditions at $r = 0$, and integrating outwards. At the surface of the star the pressure goes to zero, and beyond this point we set $\tilde{\rho}$ and $\tilde{p}$ to zero, integrating only the scalar field and metric equations further outward. Regularity at the origin requires $\mu(0) = \nu(0) = \psi(0) = 0$. In general, one can freely specify $\tilde{p}(0) = \tilde{p}_0$ and $\phi(0) = \phi_0$, for which the asymptotic solution for the scalar field of an isolated star takes the form $r\phi(r) = Ae^{-2\pi r/\lambda_\phi} + Be^{2\pi r/\lambda_\phi}$. Only solutions with $B = 0$ are physically relevant, and hence for a given $\tilde{p}_0$, if a scalarized solution exists it will correspond to a particular (nonzero) value of ϕ_0. We numerically find these solutions using the shooting method. This begins with a guess for values of ϕ_0 that bracket $B = 0$, then using a bisection search to reduce the range to $|B| < \epsilon$, for some predetermined small tolerance ϵ. Of course, for any finite B the solution will eventually blow up, though if sufficiently small it will match the $B = 0$ solution to a desired accuracy for $r < r_1$, with r_1 a chosen outer boundary location. We use a 4th-order Runge-Kutta method to integrate the ODEs. A further issue for the $B = 0$ solutions is that a subclass of them turns out to be dynamically unstable to perturbations. As we will describe later, to investigate this we evolved a set of neutron stars using the numerical code described in [21,22] (imposing axisymmetry).

IV. RESULTS

We investigate the existence and the strength of spontaneous scalarization for different values of β and m_ϕ, as well as different EOS. We use the piecewise-polytropic parametrization for the EOS introduced in [23], designed to approximate the zero temperature limit of many of the current nuclear-physics-inspired EOS, and bracket much of the theoretically plausible range. These equations of state named 2B, B, HB, H, 2H correspond to successively increasing stiffness for the neutron star matter, 2B being the softest, and 2H the stiffest. We measure the strength of the scalar field by its value at the center of the neutron star, where it is maximal, and also discuss its dependence on the radius for various scalar field parameters.

Radial profiles of the density and scalar field of a spherically symmetric neutron star with representative values of β and m_ϕ, and for the HB equation of state, are shown in Figs. 1 and 2. For the cases with stronger scalar fields, the structure of the star changes noticeably, especially near the center where the deviation from general relativity is greatest. Also, the compactness of the star can be significantly different for higher values of $|\beta| \gtrsim 10$.

The strength of spontaneous scalarization for several different points in $\beta - m_\phi$ parameter space and for various EOS is illustrated in Fig. 3. For a given point in parameter space (i.e. one of the subplots in Fig. 3) and a given EOS, there is a finite ADM mass range for the star that allows spontaneous scalarization. Even though the EOS can affect the neutron star mass range where spontaneous scalarization can occur, it does not have a strong effect on its qualitative behavior; i.e. the existence of the effect and the maximum strength of the scalar field has a comparatively

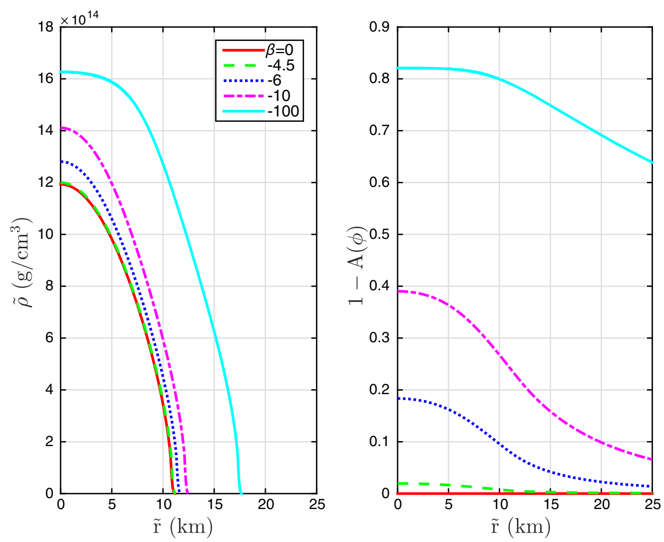

FIG. 1. Effect of varying β on the radial profiles of the matter density (left) and conformal factor $1 - A(\phi)$ (right) for neutron stars. The physical radius $\tilde{r} = A(\phi(r))r$ is the radial coordinate associated with the Jordan frame metric. All cases have a fixed ADM mass of $1.70M_\odot$, $m_\phi = 1.6 \times 10^{-13}$ eV and HB EOS. The baryon mass of the neutron star is 2.06, 2.07, 2.10, 2.32, $5.37M_\odot$ for $\beta = 0, -4.5, -6, -10, -100$, respectively. Even for moderate values of β, the structure of the star is altered significantly. For large values such as $\beta = -100$, observations of isolated stars might already be able to test spontaneous scalarization.

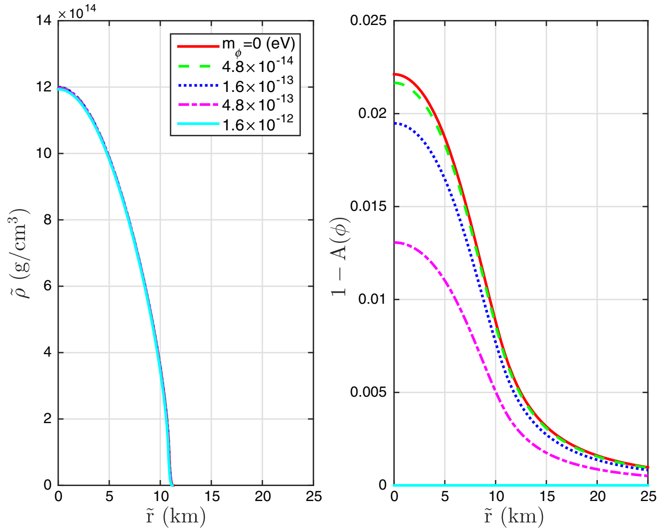

FIG. 2. Effect of varying m_ϕ on the radial profiles of the matter density (left) and conformal factor $1 - A(\phi)$ (right) for neutron stars. The physical radius $\tilde{r} = A(\phi(r))r$ is the radial coordinate associated with the Jordan frame metric. All cases have a fixed ADM mass of $1.70M_\odot$, $\beta = -4.5$ and HB EOS. Increasing m_ϕ always inhibits the scalarization of the star and eventually kills it. Below a certain value ($m_\phi \sim 10^{-13}$ eV for this case) the field profile changes only marginally, asymptoting to the case of a massless scalar. This characteristic dependence on m_ϕ holds qualitatively for higher β as well, but the cutoff value of m_ϕ that allows scalarization is typically higher for more negative β. The baryon mass of the neutron star is within a percent of $2.06M_\odot$ for all cases.

weak dependence on the EOS compared to varying β and m_ϕ.

One interesting feature of the dependence of the scalar field strength on the ADM mass is that in certain cases two different scalar field profiles are possible for a given ADM mass. Anticipating that only one of these solutions is dynamically stable, we evolved these stars using the code described in [21,22], and found that the star with the lower $\phi(r = 0)$ value is unstable in all cases (the thin parts of the curves in Fig. 3). These solutions quickly evolve to either a stable configuration with similar ADM mass, or collapse to a black hole. This is analogous to behavior seen in boson star solutions within pure GR, e.g. [24]. All unstable configurations we investigated have more than one extrema as a function of radius, whereas the stable stars have monotonically decreasing scalar field profiles.

A. Discussion

Figures 1–3 clearly demonstrate that increasing m_ϕ weakens and eventually prevents spontaneous scalarization for all β, which is to be expected from (4). Theoretical studies of the massless theory have already shown that the maximal value of the spontaneously scalarized field (or total scalar charge) drops by a few orders of magnitude around $\beta = -4.5$, and then disappears completely around $\beta = -4$ for a wide variety of EOS [3,4]. This is consistent with the estimate [(7)], and still qualitatively true for massive scalars (see Fig. 3). Although this is not surprising for low m_ϕ, where the TOV-like equations are small perturbations of the massless scalar case near the star, it holds true even near the upper scalar mass limits imposed by the radius of the neutron star. In short, a scalar mass term does not significantly alter the least negative value of β below which spontaneous scalarization occurs, as compared to the massless theory. For even more negative β values, neutron stars can again support significant scalar fields for a wide range of m_ϕ values, with an upper bound that depends on the radius of the star.

On the other hand, as discussed in the Introduction, even though the range of β that allows strong spontaneous scalarization is similar for the massive and massless cases, a significant difference appears once observations are used to restrict the parameter space. Namely, more negative β values ($\lesssim -4.5$) that lead to strong scalarization for a massless scalar but which are ruled out by the white dwarf–pulsar binary [5] are still viable for a massive scalar field, due to its suppression of dipole radiation for large binary separations. However, significantly more negative values of β start to induce radical changes to the structure of a star that might be constrained by observations of single neutron stars (see also [18]). For example, at fixed ADM mass, scalarization leads to a more than 50% increase in the stellar radius around $\beta \sim -100$ (see Fig. 1). Such large radii could likely be ruled out by existing observations of thermonuclear bursts from neutron stars (see e.g. [25]). Though to properly connect these observations to inferred mass/radii requires models of the bursts and subsequent light propagation within the geometry of the star, and how this is affected by scalarization. This would be an interesting line of inquiry for a future study.

We also note that the effect of scalarization on the star's structure depends strongly on the EOS, but always *increases* the maximum allowed ADM mass. This is especially evident for the softest 2B EOS, which cannot support a neutron star with the highest observed mass under GR [5,26], but can do so for $\beta \lesssim -5$.

Another avenue to test spontaneous scalarization is via gravitational wave observations of compact object mergers involving neutron stars. We expect the dynamical scalarization effect [3], the phenomena where the strong scalar field of one neutron star can induce scalarization in a companion which in isolation would not carry a significant field, to become less pronounced as the mass of the field increases. This is due to the exponential decay of the field outside the star. As scalarization also allows significantly higher ADM mass values for $\beta \lesssim -10$ (see Fig. 3), if that is the case a larger fraction of binary neutron star mergers will leave a massive neutron star remnant versus a black hole. These starkly contrasting outcomes will produce very

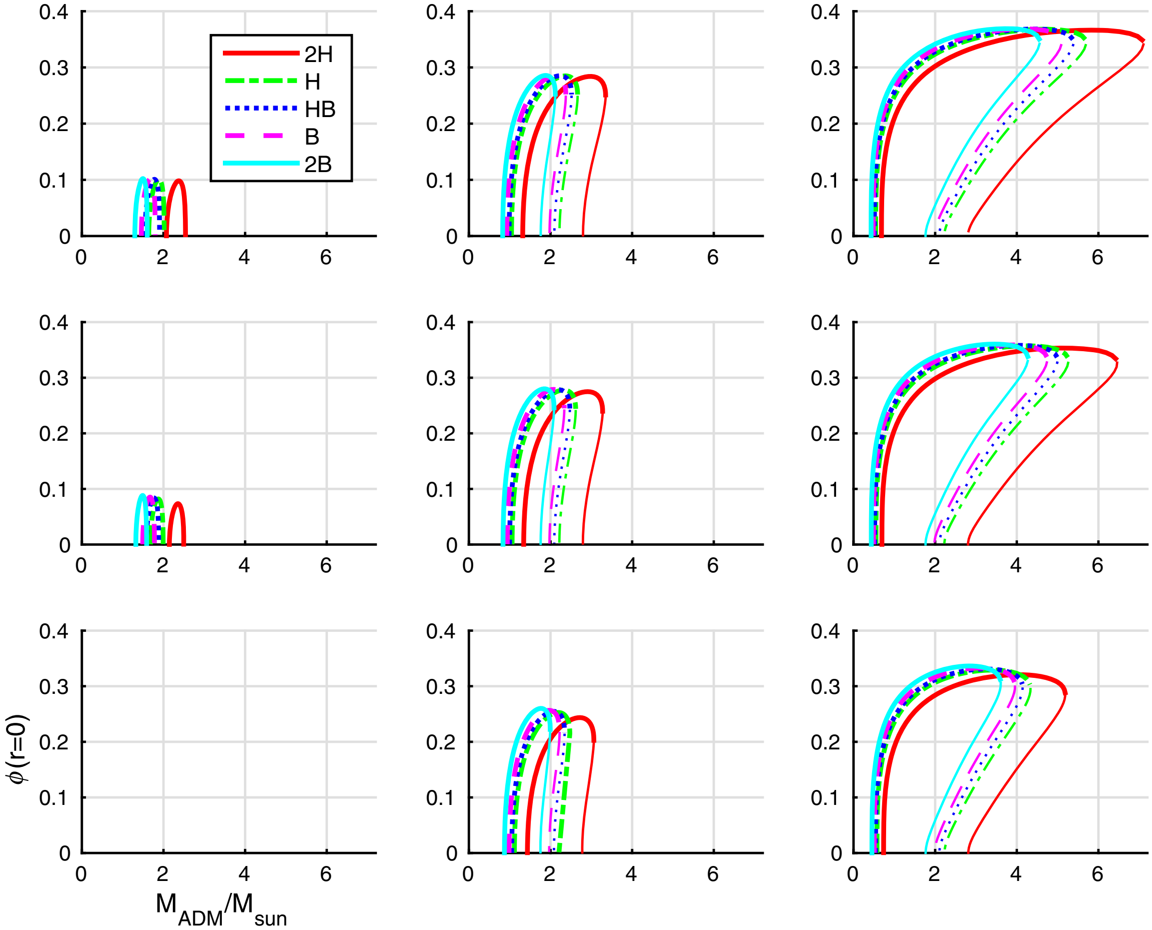

FIG. 3. Maximum value of the scalar field for different representative values of β and scalar field mass m_ϕ. Each subplot shows the maximum scalar field value with respect to the ADM mass of the neutron star for various EOSs described in [23]; 2H is the stiffest and 2B is the softest. Upper row: $m_\phi = 1.6 \times 10^{-13}$ eV; middle row: $m_\phi = 4.8 \times 10^{-13}$; lower row: $m_\phi = 1.6 \times 10^{-12}$ eV; left column: $\beta = -4.5$; middle column: $\beta = -6$; right column: $\beta = -10$. When two scalar field values are possible for a given ADM mass, the solution corresponding to the lower scalar field is unstable (thinner lines on the right end of the curves). Spontaneous scalarization becomes weaker with increasing m_ϕ and decreasing β, and eventually disappears (lower left). Note that the allowed maximum ADM mass of scalarized neutron stars can be quite large compared to the GR maximum mass even for moderately negative values of β.

different gravitational wave signals after coalescence. We plan to pursue some of these directions of study in a follow-up work.

Lastly, we give brief comments contrasting our results to the asymmetron model [7], focusing on consequences for scalarized stars. The asymmetron imposes $m_\phi \gtrsim 10^{-11}$ eV and $\beta \ll -1$. However, a direct comparison in terms of these two parameters is not straightforward, as the asymmetron model imposes a different limiting value on the conformal scaling function A, namely that $A_\infty = A(\phi \to \infty)$ is a positive, order-of-unity value not close to 0. $A_\infty = 1$ corresponds to GR, while $A_\infty = 0$ is the strongest possible deviation from GR, as in our model. Thus some of the radical changes mentioned above to neutron star structure for very large, negative β values can be ameliorated in the asymmetron model by varying the asymptotic behavior of A. However, in general (without more careful "engineering" of the conformal scaling function A), parameters in our model designed to give large but viable deviations to compact object physics are not relevant to physics on cosmological scales, and vice versa.

V. CONCLUSIONS

A significant feature of the original spontaneous scalarization scenario was that it was immune to existing weak-field and binary pulsar constraints, allowing for the intriguing possibility that large deviations to the structure of neutron stars were possible compared to the predictions within pure general relativity. The more recently discovered white dwarf–pulsar binary system has now almost ruled out this massless version of spontaneous scalarization. In this work we pointed out that the addition of a mass term to the scalar field potential can restore this feature of spontaneous scalarization without being in conflict with these observations.

Our preliminary calculations show that roughly a 5 order-of-magnitude range for the scalar field mass m_ϕ is viable. We computed the static solutions for isolated neutron stars for representative values of m_ϕ within this range, showing that spontaneous scalarization exists and can be strong (depending on the coupling parameter β). Our primary goal for exploring this theory is to have a well-posed vehicle to explore deviations to general relativity in the dynamical strong field, of relevance to the late stages of binary compact object coalescence. As such this study is a first step toward studies of merger simulations of scalarized binary neutron star and black hole–neutron star systems in the massive theory, which we plan to pursue in future work. A simpler alternative to studying mergers with full nonlinear simulations would be to adapt the semi-analytic approach developed in [27] to the massive theory, though this will likely first require calibration with full simulations.

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