

Classically Stable Nonsingular Cosmological Bounces

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One of the fundamental questions of theoretical cosmology is whether the Universe can undergo a nonsingular bounce, i.e., smoothly transit from a period of contraction to a period of expansion through violation of the null energy condition (NEC) at energies well below the Planck scale and at finite values of the scale factor such that the entire evolution remains classical. A common claim has been that a nonsingular bounce either leads to ghost or gradient instabilities or a cosmological singularity. In this Letter, we consider a well-motivated class of theories based on the cubic Galileon action and present a procedure for explicitly constructing examples of a nonsingular cosmological bounce without encountering any pathologies and maintaining a subluminal sound speed for comoving curvature modes throughout the NEC violating phase. We also discuss the relation between our procedure and earlier work.

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Introduction.—Demonstrating that classical bounces are possible is an important milestone in constructing theories of the origin and evolution of the Universe that avoid a big bang and its attendant singularity problem, or the invocation of large quantum gravity effects. The challenge has been to find examples that avoid instabilities or other pathologies so that it is possible to smoothly transit from a bounce to a homogeneous, isotropic and flat expanding universe that matches observations.

The necessary conditions for a bounce can be understood by following the evolution of the Hubble parameter $H \equiv \dot{a}/a$ assuming Einstein gravity and a spatially flat Friedmann-Robertson-Walker (FRW) universe with metric $ds^2 = -dt^2 + a^2(t)dx^i dx_i$ (where the dot denotes differentiation with respect to FRW time t): During a period of ordinary (not de Sitter or anti-de Sitter) contraction, H is becoming more negative as the scale factor $a(t)$ is shrinking, and the total energy density $\propto H^2$ is growing. During an ordinary expanding period, on the other hand, H is becoming less positive as $a(t)$ is expanding, and the total energy density $\propto H^2$ is shrinking. These two cosmological phases can only be connected *classically* if, towards the end of the ordinary contracting phase, H reverses its evolution and starts becoming less negative at a finite value of a , well before H^2 gets close to Planckian energies. During this “bounce stage,” the increasing value of H eventually hits zero and continues to grow until it reaches a large positive value (well below the Planck scale but above the nucleosynthesis scale), at which point the bounce stage ends and H begins to decrease. In a flat FRW universe, a growing Hubble parameter ($\dot{H} > 0$), as occurs during the bounce stage, corresponds to violating the null energy condition (NEC). Hence, we see the NEC violation is essential.

To achieve NEC violation, various forms of stress energy have been considered [1]. One of the best motivated

examples is a scalar field described by a cubic Galileon action. An ordinary, canonical scalar field with quadratic kinetic term does not violate the NEC at all. A pure ghost field with a wrong sign quadratic kinetic term violates the NEC but is inherently quantum unstable. If a right-sign quartic kinetic term, $\sim(\partial\phi)^4$, is added, as occurs in so-called $P(X)$ theories [where $X = -(1/2)(\partial\phi)^2$], the ghost instability can be avoided; in this case, though, when curvature fluctuations of the metric are considered, there always remain gradient instabilities [2]. The cubic Galileon action, which corresponds to adding a term proportional to $\sim\Box\phi(\partial\phi)^2$, is the simplest example of a scalar field that enables NEC violation while avoiding both types of instability [3]. In addition, Galilean invariance of the action (exact or approximate) suppresses radiative corrections [4]. Galileons have several other physics applications as well, including massive gravity and attempts to explain late-time acceleration [5].

Although linear perturbation theory suggests that, for some constructions, cubic Galileon theories can avoid pathologies during a period of NEC violation, it has been unclear until now whether this is possible when the NEC violating period includes a nonsingular bounce. In fact, the recent arguments suggest that either the speed of sound of comoving curvature modes becomes imaginary (i.e., ghost or gradient instability) for some wavelengths during the NEC violating phase [6,7], or the evolution must reach a singularity [8].

In this Letter, we demonstrate that a classically stable nonsingular bounce is possible. As an example, we study generalized cubic Galileon theories and employ an “inverse method” for explicitly constructing classically stable nonsingular bounces. After briefly reviewing the background evolution during the bounce stage, we derive the second-order Galileon action in comoving gauge and formulate

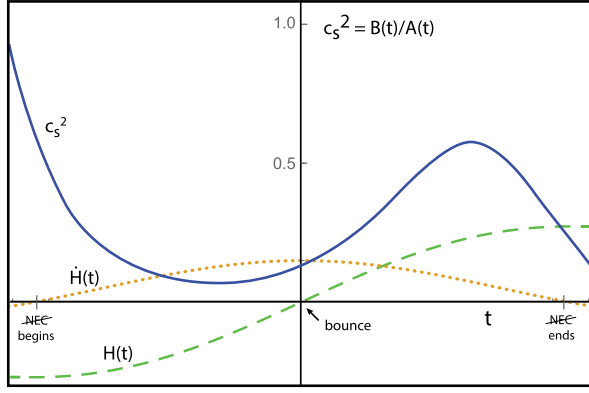


FIG. 1. A plot of the sound speed c_s^2 (solid blue curve) for comoving curvature perturbations as a function of time t . The time coordinate is given in Planck units and the value of c_s^2 is given in units where the speed of light is unity. Superimposed for illustration purposes are the shapes of the background solutions for $H(t)$ (dashed green curve; also shown in Figs. 2 and 3) and $\dot{H}(t)$ (dotted red curve). More specifically, the results correspond to $H(t) = H_0 t e^{-F(t-t^*)^2}$ and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ with the parameter values $H_0 = 3 \times 10^{-5}$, $t^* = 0.5$, $F = 9 \times 10^{-5}$, $\gamma_0 = -0.0044$, $\Theta = 0.0046$. Notably, throughout, the sound speed is real [$A(t), B(t) > 0$] and subluminal, with $0 < c_s^2 < 1$. The characteristic energy scale $\sim H^2$ is well below the Planck scale, and the NEC violating phase lasts ~ 150 Planck times; it starts when \dot{H} becomes positive at $t_{\text{beg}} \approx -74 M_{\text{Pl}}^{-1}$ and ends when \dot{H} becomes negative at $t_{\text{end}} \approx 75 M_{\text{Pl}}^{-1}$; the bounce [$H(t) = 0$] occurs at $t = 0$. Note that the bounce stage occurs well within the classical regime.

the linear stability criteria for gauge-invariant curvature perturbations. We then describe how to produce examples that have no ghost or gradient instability and maintain a subluminal sound speed throughout the NEC violating phase. Those more interested in the existence of stable bouncing solutions than in the method for obtaining them may wish to jump ahead to Figs. 1 and 2 to see an explicit example. Finally, we discuss the relation between our results and earlier work.

Background evolution during the bounce stage.—We assume that the bounce stage is driven by a single scalar field ϕ that is described by the generalized cubic Galileon action $S = \int d^4x \sqrt{-g} \mathcal{L}$ with the defining Lagrangian density

$$\mathcal{L} = \frac{1}{2} M_{\text{Pl}}^2 R - \frac{1}{2} k(\phi) (\partial\phi)^2 + \frac{1}{4} M_{\text{Pl}}^{-4} q(\phi) (\partial\phi)^4 + \frac{1}{2} M_{\text{Pl}}^{-3} b(\phi) (\partial\phi)^2 \square\phi - V(\phi). \quad (1)$$

Here, M_{Pl} is the reduced Planck mass, R is the Ricci scalar, g is the metric determinant, $k(\phi)$ is the dimensionless quadratic coupling and $q(\phi)$ is the dimensionless quartic coupling, $b(\phi)$ is the dimensionless coupling of the scalar

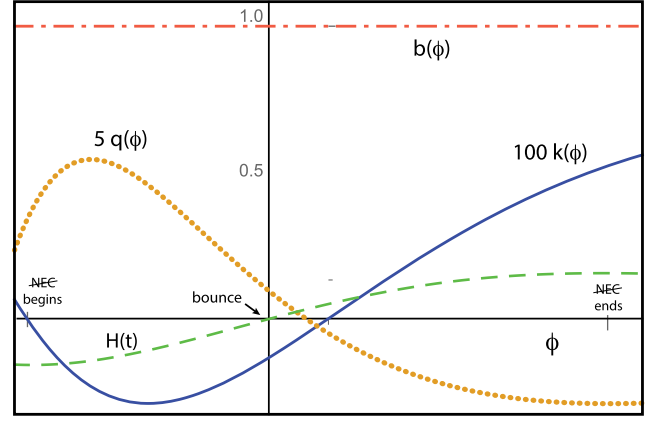


FIG. 2. A plot of the dimensionless couplings k (solid blue curve), q (dotted orange curve), and b (dot-dashed red line) as a function of ϕ for the example given in Fig. 1. The x axis has Planck units and the y axis has dimensionless units. As indicated by the labeling, we have rescaled the quadratic and quartic couplings for the purpose of illustration.

field ϕ to the cubic Galileon term, and $V(\phi)$ is the scalar potential. Other energy components, such as radiation, matter, dark energy, or other scalars that drive different stages of cosmic evolution are subdominant and, hence, negligible during the bounce stage (though they play an important role after the bounce stage, as we will describe in the Discussion section).

Varying the action with respect to the metric $g_{\mu\nu}$, we find the corresponding Friedmann equations for a spatially flat geometry,

$$3H^2 = \rho = \frac{1}{2} k \dot{\phi}^2 + \frac{1}{4} (3q - 2b') \dot{\phi}^4 + 3Hb \dot{\phi}^3 + V, \quad (2)$$

$$-2\dot{H} = \rho + p = k \dot{\phi}^2 + (q - b') \dot{\phi}^4 + 3Hb \dot{\phi}^3 - b \ddot{\phi} \dot{\phi}^2, \quad (3)$$

where prime denotes differentiation with respect to the scalar field ϕ . Throughout, we work in reduced Planck units ($M_{\text{Pl}} \equiv 1$). The first Friedmann equation describes the different contributions to the total energy density ρ while the second Friedmann equation describes the sum of energy density and pressure p of the scalar field ϕ . The ratio $-\dot{H}/H^2$ is equal to the equation-of-state parameter $\epsilon \equiv (3/2)(\rho + p)/\rho$.

Variation of the action with respect to ϕ yields the FRW scalar field equation

$$\begin{aligned} & \left(k + (3q - 2b') \dot{\phi}^2 + 6Hb \dot{\phi} + \frac{3}{2} b^2 \dot{\phi}^4 \right) \ddot{\phi} \\ &= -\frac{1}{2} k' \dot{\phi}^2 - \frac{1}{4} (3q' - 2b'') \dot{\phi}^4 - \frac{3}{4} b (q \dot{\phi}^4 + 4V) \dot{\phi}^2 \\ & - \left(k + q \dot{\phi}^2 + \frac{3}{2} b^2 \dot{\phi}^4 \right) 3H \dot{\phi} - V'. \end{aligned} \quad (4)$$

The Universe enters the bounce stage when the Galileon field's kinetic energy becomes the dominant energy component and the sum of pressure and energy density turns negative (NEC violation). Since $\rho \geq 0$ throughout, the bounce stage is characterized by negative pressure $-p > \rho$ and a super-stiff equation of state $\epsilon < 0$ both commonly associated with potential ghost or gradient instabilities. To understand why NEC violating theories are potentially unstable and under which conditions instabilities can be avoided, we next derive the stability criteria for Galileon theories. [The criteria were obtained previously in Ref. [9], but we rederive them here to emphasize some pedagogical points, especially the importance of the lapse equation of motion and the origin of $\gamma(t)$ in the analysis that follows.]

Stability criteria from linear perturbation theory.—On a homogeneous FRW background with $\rho \geq 0$, any leading-order instability comes from the kinetic or gradient terms of the linear theory. Hence, to properly identify the stability behavior, we will study first-order perturbations around the smooth background given by Eqs. (2)–(4).

To perform the stability analysis, it proves useful to employ the ADM formalism and decompose the metric as

$$ds^2 = -(N^2 - N^i N_i) dt^2 + 2N_i dx^i dt + g_{ij} dx^i dx^j, \quad (5)$$

where N is the lapse, N_i is the shift, and g_{ij} is the spatial metric. For the homogeneous FRW background, $\bar{N} = 1$, $\bar{N}_i = 0$, and $\bar{g}_{ij} = a^2(t)\delta_{ij}$ (bar denotes background quantities). We introduce linear perturbations to N , and N_i using the standard parametrization

$$\delta N = N - \bar{N} = \alpha, \quad \delta N_i = N_i - \bar{N}_i = \partial_i \beta. \quad (6)$$

For scalar perturbations of the spatial metric, we have the freedom to choose a particular slicing of space-time, i.e., fix the gauge. We choose to work in the comoving gauge, where space-time is sliced such that spatial inhomogeneities are all promoted to the metric and the scalar field does not carry any perturbations,

$$\delta \phi = 0, \quad g_{ij} = a^2(t)[1 + 2\zeta(t, \mathbf{x})]\delta_{ij}. \quad (7)$$

This gauge has the advantage that the comoving curvature perturbation ζ is gauge invariant and, hence, ensures that our conclusions about stability do not entail gauge artifacts. In addition, the lack of scalar-field perturbations significantly simplifies the calculation in the presence of higher-order kinetic terms.

In the comoving gauge, the second-order action takes the form

$$S_\zeta^{(2)} = \frac{1}{2} \int d^4 x a^3 \mathcal{L}_\zeta^{(2)}, \quad (8)$$

with the Lagrangian density

$$\begin{aligned} \mathcal{L}_\zeta^{(2)} = & \alpha^2(-6H^2 + k\dot{\phi}^2 + (3q - 2b')\dot{\phi}^4 + 12Hb\dot{\phi}^3) \\ & + 4\alpha \left(3 \left(H - \frac{1}{2} b\dot{\phi}^3 \right) \dot{\zeta} - \frac{\Delta\zeta}{a^2} \right) - 6\dot{\zeta}^2 + 2 \left(\frac{\nabla\zeta}{a} \right)^2 \\ & + 4 \frac{\Delta\beta}{a^2} \left(\dot{\zeta} - \left(H - \frac{1}{2} b\dot{\phi}^3 \right) \alpha \right) - 6(6H - b\dot{\phi}^3)\zeta\dot{\zeta} \\ & - 9 \left(3H^2 - \frac{1}{2} k\dot{\phi}^2 - \frac{3q - 2b'}{12} \dot{\phi}^4 - Hb\dot{\phi}^3 + V \right) \zeta^2, \end{aligned} \quad (9)$$

where we introduced the spatial gradient $\nabla = \partial_i$ and $\Delta = \nabla^2 = \partial_i \partial^i$. Using the background equations (2) and (3) and integrating by parts, the last two terms cancel. Varying the action with respect to the shift, we find the equation of motion for the lapse,

$$\left(H - \frac{1}{2} b\dot{\phi}^3 \right) \alpha = \dot{\zeta}. \quad (10)$$

The coefficient of α plays a subtle but important role in the analysis that follows. Note that $H - (1/2)b\dot{\phi}^3 = 0$ renders $\dot{\zeta} = 0$. In particular, no singular behavior appears at this point. For our stability analysis, we will consider solutions in which $H - (1/2)b\dot{\phi}^3 \neq 0$ throughout, and, as we comment below, stability forbids $H - (1/2)b\dot{\phi}^3$ passing continuously through zero.

For $H - (1/2)b\dot{\phi}^3 \neq 0$, variation of the second-order action with respect to the lapse α and substituting the expression for the shift $\Delta\beta/a^2$ together with the expression for α from Eq. (10) back into the original action in Eq. (8) and doing a series of integrations by parts yields the second-order action for comoving curvature perturbations ζ ,

$$S_\zeta^{(2)} = \int d^4 x a^3(t) \left(A(t) \dot{\zeta}^2 - \frac{B(t)}{a^2(t)} (\nabla\zeta)^2 \right), \quad (11)$$

with the dimensionless coefficients

$$A(t) = \frac{k\dot{\phi}^2 + (3q - 2b')\dot{\phi}^4 + 6Hb\dot{\phi}^3 + \frac{3}{2}b^2\dot{\phi}^6}{2(H - \frac{1}{2}b\dot{\phi}^3)^2}, \quad (12)$$

$$B(t) = \frac{k\dot{\phi}^2 + q\dot{\phi}^4 + 2b\dot{\phi}\dot{\phi}^2 + 4Hb\dot{\phi}^3 - \frac{1}{2}b^2\dot{\phi}^6}{2(H - \frac{1}{2}b\dot{\phi}^3)^2}. \quad (13)$$

The conditions for stable NEC-violation correspond to positivity of $A(t)$ (no-ghost condition), and positivity of $B(t)$ (no gradient instability). Obviously, in the absence of ghost, the square of the sound speed $c_S^2 = B(t)/A(t) > 0$ if $B(t) > 0$. To achieve subluminal evolution for comoving curvature modes, we have to additionally demand that $c_S^2 < 1$.

Inverse method.—Using the background equations (2)–(3), it is straightforward to check that Eq. (13) can be recast to the simple form

$$\frac{d}{dt}\gamma(t)^{-1} + H(t)\gamma(t)^{-1} = B(t) + 1, \quad (14)$$

where $\gamma(t) = H(t) - (1/2)b(\phi)\dot{\phi}^3(t)$ and γ is defined to carry the same dimension as H ($[\gamma] = [H] = M_{\text{Pl}}$). Equation (14) is a linear first-order differential equation for $\gamma^{-1}(t)$ with the unique solution

$$\gamma(t) = \frac{N(t)}{\gamma_0^{-1} + \int_{t_0}^t (B(t) + 1)N(t)dt}, \quad \gamma(t_0) = \gamma_0, \quad (15)$$

where the auxiliary function $N(t)$ is defined as

$$N(t) = \exp\left(\int_{t_0}^t H(t)dt\right) = \frac{a(t)}{a(t_0)}. \quad (16)$$

In addition, we can reexpress $A(t)$ in Eq. (12) as a function of H and γ ,

$$A(t) = 3 + \frac{6H^2 - 4V + 2\dot{H} + \dot{\gamma} + 3H\gamma}{\gamma^2}. \quad (17)$$

These relations have many uses. For example, for any choice of $H(t)$ and $B(t)$ we can immediately determine the corresponding $\gamma(t)$ and $A(t)$, or, equivalently, for any choice of background solutions $H(t)$ and $\phi(t)$, we can immediately determine the corresponding $A(t)$ and $B(t)$ and, hence, infer the stability behavior for comoving curvature modes. This feature allows us to rapidly search through forms for $H(t)$ and $\phi(t)$ that describe bounces and identify choices for which both $A, B > 0$ and the sound speed for comoving curvature modes is subluminal, as shown in Fig. 1.

It is then straightforward to identify the corresponding couplings in the cubic Galileon Lagrangian, Eq. (1), using the background equations (2)–(3):

$$k(t) = -2(3H^2 - 2V + 2\dot{H} + \dot{\gamma} + 3H\gamma)/\dot{\phi}^2(t), \quad (18)$$

$$q(t) = \frac{4}{3}(2\dot{H} + \dot{\gamma} + 9H\gamma - 3V)/\dot{\phi}^4(t) + \frac{2}{3}b'. \quad (19)$$

In the examples discussed in the remainder of this Letter, we assume for simplicity that $V(\phi)$ is negligible. As we will discuss in the following section, if γ is finite and nonzero throughout, we have the freedom to set the Galileon coupling $b \equiv 1$. Finally, inverting $\phi(t) = \phi_0 + \int_{t_0}^t \sqrt[3]{2(H - \gamma)}dt$ and substituting $t(\phi)$ into Eqs. (18)–(19), we find the expressions for the couplings as a function of ϕ . The three coupling functions corresponding to our example in Fig. 1 are depicted in Fig. 2. Note that both $k(\phi)$ and $q(\phi)$ have simple forms. These can be well approximated

by simple functions of ϕ : we have checked that one can start with these approximate coupling functions and the initial conditions $\phi_0 = \phi(t_0)$, $\dot{\phi}_0 = \dot{\phi}(t_0)$ to find the background solutions describing a classically stable bounce stage with subluminal sound speed for comoving curvature modes. In sum, we have demonstrated that it is possible for the Universe to have a classically stable, NEC violating bounce stage without a singularity or any bad behavior.

Special case: $\gamma \rightarrow -\infty$.—If there is no gradient instability [$B(t) > 0$], the strict positivity of the function $N(t)$ defined in Eq. (16) implies that the integral in the denominator of the expression for $\gamma(t)$ in Eq. (15) is positive definite and increasing monotonically for all $t > t_0$. Hence, for any $\gamma_0 < 0$, there will be some $\bar{t} > t_0$ such that the denominator reaches zero and $\gamma \rightarrow -\infty$. A Taylor series of the denominator about \bar{t} results in a leading linear contribution: $[(B(\bar{t}) + 1)]N(\bar{t})(t - \bar{t})$. Hence, we see that γ must approach $-\infty$ as $t \rightarrow \bar{t}$ from $t < \bar{t}$ and $+\infty$ as $t \rightarrow \bar{t}$ from $t > \bar{t}$. In the previous section, we have shown that γ can be chosen such that it is negative and finite throughout the bounce stage, which means that, formally, \bar{t} would be reached after the NEC is restored, by which time other contributions to the stress energy (such as matter and radiation) that could be ignored during the bounce stage may dominate and the formal calculation of \bar{t} has no actual physical relevance.

However, in Fig. 3, we intentionally chose as an academic exercise an example in which \bar{t} occurs during the NEC-violating bounce stage and show that, even if the derived quantity γ diverges during the bounce stage, all

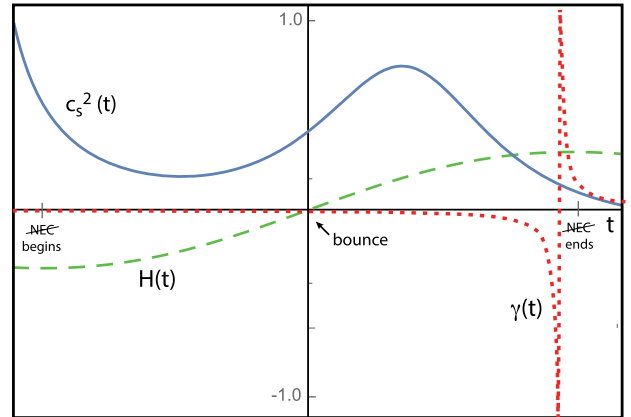


FIG. 3. A plot of the sound speed c_s^2 (solid blue curve) for comoving curvature modes as a function of time t during the bounce stage corresponding to the background given by $H(t) = H_0 t e^{-F(t-t_*)^2}$ (dashed green curve; rescaled to show shape) and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ (dotted red curve) for the parameter values $H_0 = 3 \times 10^{-5}$, $t_* = 0.5$, $F = 7 \times 10^{-5}$, $\gamma_0 = -0.0044$, and $\Theta = 4.6 \times 10^{-6}$. After the bounce and shortly before the end of NEC violation, $\gamma(t)$ goes from $-\infty$ to $+\infty$ while all fundamental physical quantities including $H(t)$ and c_s^2 remain finite and positive.

fundamental, physical quantities can remain well-defined and finite throughout the bounce stage. For this to happen while keeping $H(\bar{t})$, $\dot{\phi}(\bar{t})$, and $\phi(\bar{t})$ finite, it is necessary that the coupling $k(\phi)$, $q(\phi)$, and $b(\phi)$ diverge at $t \rightarrow \bar{t}$. Remarkably, the sound speed c_s^2 remains continuous and positive and below 1 despite the diverging coupling.

It is also possible to construct examples with $\gamma_0 > 0$ and no gradient instability [$B(t) > 0$] throughout the entire bounce stage. In this case, the denominator in Eq. (15) remains positive definite and monotonically increasing; so the only option is that $\gamma(t)$ monotonically approaches zero from above as t increases, analogous to the examples in Figs. 1 and 2, where γ is negative and approaches zero as t decreases.

Discussion.—In this Letter we presented an inverse method that makes it possible to achieve a long-standing goal: to construct field theories with a classically stable, NEC-violating bounce stage that avoids ghost and gradient instabilities, maintains a real, subluminal sound speed for comoving curvature modes, and does not encounter a singularity.

Our results build on earlier studies on NEC violation and bounces. In Refs. [10,11], it was argued that cubic Galileons can smoothly pass through $H = 0$ but encountered gradient instabilities before exiting the NEC-violating stage. In Ref. [12], the authors considered Galilean genesis scenarios with NEC violation, and showed that Galilean scalar field perturbations are stable during the early stages when H is small and gravity is negligible, but this study did not include an exit from the NEC-violating stage when H becomes large and the scalar field can create curvature perturbations with ghost and gradient instabilities. Indeed, gradient instabilities were encountered during the NEC-violating stage in Refs. [6,7,13,14] when the authors attempted to construct nonsingular bouncing models, leaving the impression that linear instabilities are unavoidable. However, this Letter definitively shows these instabilities can be safely avoided.

Most recently, a no-go theorem claiming that singularities are unavoidable in NEC-violating Galileon theories was presented in Ref. [8] and further generalized in Ref. [15]. Their argument is equivalent to the statement that $\gamma(t)$ in Eq. (15) must have a zero-point crossing or diverge for some value of \bar{t} for the Galileon Lagrangian in Eq. (1). It is important to note, though, that the conclusion only depends on the cubic Galileon form of the action. The zero-point crossing or divergence occurs whether or not there is NEC violation or a bounce, so long as the action is described by Eq. (1). In particular, when there is a bounce, the theorem says nothing about whether \bar{t} occurs during, before or after the bounce. We have demonstrated, in fact, that \bar{t} does not have to occur during the NEC-violating bounce stage. As a practical matter, that may suffice for constructing fully stable nonsingular bouncing cosmologies since, before and after the bounce stage, it is expected

that other forms of energy are non-negligible and even dominant, so that the Lagrangian in Eq. (1) is no longer applicable. Although one can easily envisage how to construct examples in which the Galileon decays into or is overtaken by a NEC satisfying component before or after the bounce, as considered in Refs. [8,15,16], a reliable calculation of stability requires an expanded gauge-invariant treatment compared to these earlier analyses. We will present the analysis and examples in forthcoming work [17]. Also, as we will show in Ref. [18], simply extending the Galileon action to include the next-order interaction suffices to obtain nonsingular cosmological bouncing solutions that are fully stable for all finite times.

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