

The Issue-Adjusted Ideal Point Model

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Abstract

We develop a model of issue-specific voting behavior. This model can be used to explore lawmakers’ personal voting patterns of voting by issue area, providing an exploratory window into how the language of the law is correlated with political support. We derive approximate posterior inference algorithms based on variational methods. Across 12 years of legislative data, we demonstrate both improvement in heldout prediction performance and the model’s utility in interpreting an inherently multi-dimensional space.

Key words: Item response theory, Probabilistic topic model, Variational inference, Legislative voting

1. INTRODUCTION

Legislative behavior centers around the votes made by lawmakers. These votes are captured in *roll call data*, a matrix with lawmakers in the rows and proposed legislation in the columns. We illustrate a sample of roll call votes for the United States Senate in Figure 1.

The seminal work of Poole and Rosenthal (1985) introduced the *ideal point model*, using roll call data to infer the latent political positions of the lawmakers. The ideal point model is a latent factor model of binary data and an application of item-response theory (Lord 1980) to roll call data. It gives each lawmaker a latent political position along a single dimension and then uses these points (called the ideal points) in a model of the votes. (Two lawmakers with the same position will have the same probability of voting in favor of each bill.) From roll call data, the ideal point model recovers the familiar division of Democrats and Republicans. See Figure 2 for an example.

Ideal point models can capture the broad political structure of a body of lawmakers, but they cannot tell the whole story. We illustrate this with votes on a bill in Figure 3. This figure shows lawmaker’s ideal points for their votes on an act *Recognizing the significant accomplishments of AmeriCorps*, H.R. 1338 in Congress 111. In this figure, “Yea” votes are colored orange, while “Nay” votes are violet; a classic ideal point model predicted that votes

Lawmaker	Item of legislation					
	Bill	S. 3930	H.R. 5631	H.R. 6061	H.R. 5682	S. 3711
Mitch McConnell (R)		Yea	Yea	Yea	Yea	Yea
Olympia Snowe (R)			Yea	Yea	Yea	Nay
John McCain (R)		Yea	Yea	Yea	Yea	Yea
Patrick Leahy (D)		Nay	Yea	Nay	Nay	Nay
Paul Sarbanes (D)		Nay	Yea	Nay	Yea	Nay
Debbie Stabenow (D)		Yea	Yea	Yea	Yea	Yea

Figure 1: A sample roll-call matrix illustrating lawmakers’ votes on items of legislation. These votes are from the Senate in the 109th Congress (2005-2006). The party of each Senator – (D)emocrat or (R)epublican – is provided in parentheses. This matrix is sometimes incomplete (see Snowe’s vote on S. 3930, for example).

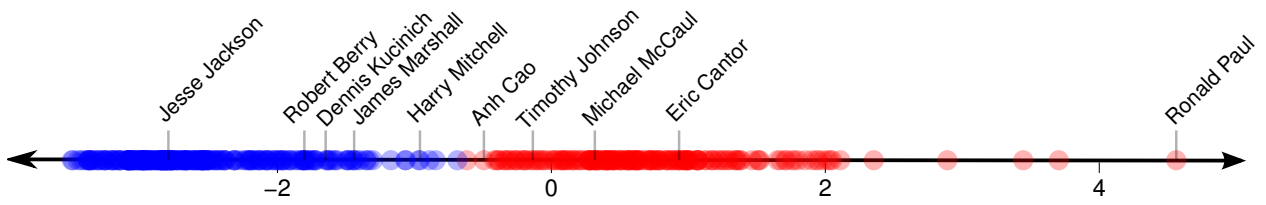


Figure 2: Traditional ideal points separate Republicans (red) from Democrats (blue).

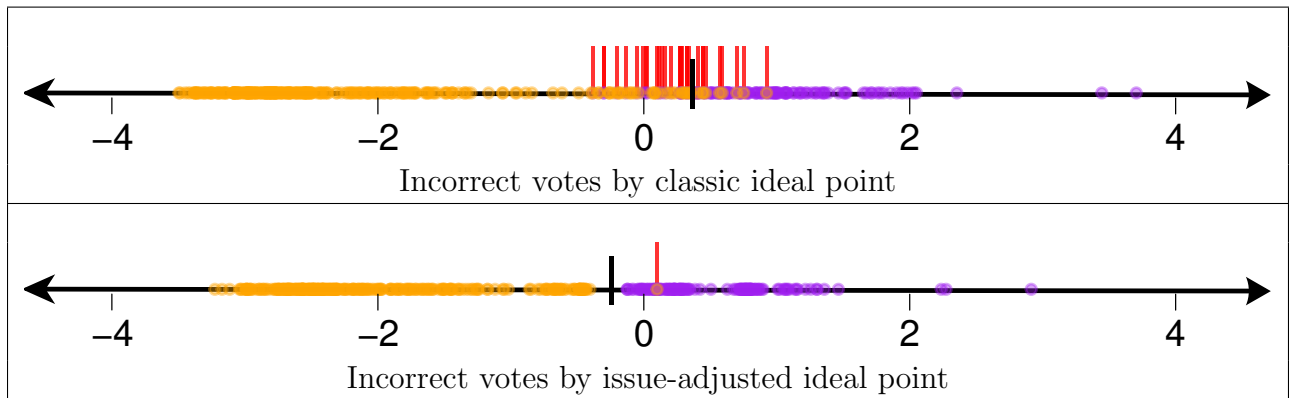


Figure 3: Classic ideal points (top) represent votes incorrectly when lawmakers hold issue-specific opinions, while issue-adjusted ideal points (bottom) can account for this. Classic ideal points assume that lawmakers hold fixed positions, while issue-adjusted ideal points allow their positions to change by issue. Each point above is the ideal point of a lawmaker voting on an act *Recognizing the significant accomplishments of AmeriCorps* [and raising community service] (H.R. 1338 in Congress 111); orange points represent lawmakers who voted “Yea”, and violet points represent lawmakers who voted “Nay” on this bill. The theory behind classic ideal points assumes that lawmakers’ votes on a bill can be described by their side of the cut point (black vertical line). Red lines mark lawmakers whose votes were incorrectly predicted with each model.

to the *right* of the vertical line were “Nay” while those to the left were “Yea”. Out of four hundred eight votes on this bill modeled by an ideal point model, thirty-one of these were modeled incorrectly.

Sometimes these votes are incorrectly predicted because of stochastic circumstances surrounding lawmakers and bills. More often, however, these votes can be explained because lawmakers are not one-dimensional: they each hold positions on different issues. For example, Ronald Paul, a Republican representative from Texas, and Dennis Kucinich, a Democratic representative from Ohio, hold consistent political opinions that an ideal point model systematically gets incorrect. Looking more closely at these errors, we would see that Paul differs from a typical Republican when it comes to foreign relations and social issues; Kucinich differs from a usual Democrat when it comes to foreign policy.

The problem is that classical ideal point models place each lawmaker in a single political position, but a lawmaker’s vote on a bill has to do with a number of factors—her political affiliation, the content of the proposed legislation, and her political position *on that content*. While classical ideal point models can capture the main regularities in lawmakers’ voting behavior, they cannot predict when and how a lawmaker will vote differently than we expect.

In this paper, we develop the *issue-adjusted ideal point model*, a model that captures issue-specific deviation in lawmaker behavior. We place the lawmakers on a political spectrum and identify how they deviate from their position as a function of specific issues. This results in inferences like those illustrated in Figure 3. An important component of our model is that we use the text of the proposed bills to encode which issues they are about. (We do this through a probabilistic topic model (Blei et al. 2003).) Unlike other attempts at developing multi-dimensional ideal point models (Jackman 2001), our approach explicitly ties the additional dimensions to the political discussion at hand.

By incorporating issues, we can model the AmeriCorp bill above much better than we could with classic ideal points (see Figure 3). By recognizing that this bill is about *social services*, and by modeling lawmakers’ positions on this issue, we are able to predict all but one of the lawmakers’ votes correctly. This is because we can learn to differentiate between lawmakers who are conservative and lawmakers who are conservative on *social services*. For example, the issue-adjusted model tells us that, while Doc Hastings (Republican of Washington) is considered more conservative than Timothy Johnson (Republican of Illinois) in the ideal point model, Hastings is much more liberal on social issues than Johnson—hence, he will more often generally side with Democrats on those votes.

In the following sections, we describe our model and develop efficient approximate posterior inference algorithms for computing with it. To handle the scale of the data we want to study, we replace the usual MCMC approach with a faster variational inference algorithm. We then study 12 years of legislative votes from the U.S. House of Representatives and Senate, a collection of 1,203,009 votes. We show that our model gives a better fit to the data than a classical ideal point model and demonstrate that it provides an interesting exploratory tool for analyzing legislative behavior.

Related work. Item response theory (IRT) has been used for decades in political science (Clinton et al. 2004; Martin and Quinn 2002; Poole and Rosenthal 1985); see Fox (2010) for an overview, Enelow and Hinich (1984) for a historical perspective, and Albert (1992) for

Bayesian treatments of the model. Some political scientists have used higher-dimensional ideal points, where each legislator is described by a vector of ideal points $\mathbf{x}_u \in \mathbb{R}^K$ and each bill polarization \mathbf{a}_d (i.e., how divisive it is) takes the same dimension K Heckman and Snyder (1996). The probability of a lawmaker voting “Yes” is $\sigma(\mathbf{x}_u^T \mathbf{a}_d + b_d)$ (we describe these assumptions further in the next section). The principle component of ideal points explains most of the variance and explains party affiliation. However, other dimensions are not attached to issues, and interpreting beyond the principal component is painstaking (Jackman 2001).

At the minimum, this painstaking analysis often requires careful study of the original roll-call votes or study of lawmakers’ ideal-point neighbors. The former obviates an IRT model, since we cannot make inferences from model parameters alone; while the latter begs the question, since it assumes we know in the first place how lawmakers vote on different issues. The model we discuss in this paper is intended to address this problem by providing interpretable multi-dimensional ideal points. Through posterior inference, we can estimate each lawmaker’s political position and how it changes on a variety of concrete issues.

The model we will outline takes advantage of recent advances in content analysis, which have received increasing attention because of their ability to incorporate large collections of text at a relatively small cost (see Grimmer and Stewart (2012) for an overview of these methods). For example, Quinn et al. (2006) used text-based methods to understand how legislators’ attention was being focused on different issues, to provide empirical evidence toward answering a variety of questions in the political science community.

We will draw heavily on content analytic methods in the machine learning community, which has developed useful tools for modeling both text and the behavior of individuals toward items. Recent work in this community has provided joint models of legislative text and votes. Gerrish and Blei (2011) aimed to predict votes on bills which had not yet received any votes. This model fitted predictors of each bill’s parameters using the bill’s text, but the underlying voting model was still one-dimensional—it could not model individual votes better than a one-dimensional ideal point model. In other work, Wang et al. (2010) developed a Bayesian nonparametric model of votes and text over time. Both of these models have different purposes from the model presented here; neither addresses individuals’ affinity toward different types of bills.

The issue-adjusted model is conceptually more similar to recent models for content recommendation. Specifically, Wang and Blei (2011) describe a method to recommend academic articles to users of a service based on what they have already read, and Agarwal and Chen (2010) proposed a similar model to match users to other items (i.e., Web content). Our model is related to these approaches, but it is specifically designed to analyze political data. These works, like ours, model users’ affinities to items. However, neither of them employ the notion of the orientation of an item (i.e., the political orientation of a bill) or that the users (i.e., lawmakers) have a position on a this spectrum. These are considerations which are required when analyzing political roll call data.

2. THE ISSUE-ADJUSTED IDEAL POINT MODEL

We first review ideal point models of legislative roll call data and discuss their limitations. We then present our model, the *issue-adjusted ideal point model*, that accounts for how legislators vote on specific issues.

2.1. Modeling Political Decisions with Ideal Point Models

Ideal point models are latent variable models that have become a mainstay in quantitative political science. These models are based on item response theory, a statistical theory that models how members of a population judge a set of items (see Fox (2010) for an overview). Applied to voting records, ideal point models place lawmakers on an interpretable political spectrum. They are widely used to help characterize and understand historical legislative and judicial decisions (Clinton et al. 2004; Poole and Rosenthal 1985; Martin and Quinn 2002).

One-dimensional ideal point models posit an *ideal point* $x_u \in \mathbb{R}$ for each lawmaker u . Each bill d is characterized by its *polarity* a_d and its *popularity* b_d . (The polarity is often called the “discrimination”, and the popularity is often called the “difficulty”; polarity and popularity are more accurate terms.) The probability that lawmaker u votes “Yes” on bill d is given by the logistic regression

$$p(v_{ud} = \text{yes} \mid x_u, a_d, b_d) = \sigma(x_u a_d + b_d), \quad (1)$$

where $\sigma(s) = \exp(s)/(1 + \exp(s))$ is the logistic function. (A probit function is sometimes used instead of the logistic. This choice is based on an assumption in the underlying model, but it has little empirical effect in legislative ideal point models.) When the popularity of a bill b_d is high, nearly everyone votes “Yes”; when the popularity is low, nearly everyone votes “No”. When the popularity is near zero, the probability that a lawmaker votes “Yes” is determined primarily by how her ideal point x_u interacts with bill polarity a_d .

In Bayesian ideal point modeling, the variables a_d , b_d , and x_u are usually assigned standard normal priors (Clinton et al. 2004). Given a matrix of votes $\mathbf{v} = \{v_{ud}\}$, we can estimate the posterior expectation of the ideal point of each lawmaker $\mathbb{E}[x_u \mid \mathbf{v}]$. Figure 2 illustrates ideal points estimated from votes in the U.S. House of Representatives from 2009-2010. The model has clearly separated lawmakers by their political party (color) and provides an intuitive measure of their political leanings.

2.2. Limitations of Ideal Point Models

The ideal point model fit to the House of Representatives from 2009-2010 correctly models 98% of all lawmakers’ votes on training data. (We correctly model an observed vote if its probability under the model is bigger than 1/2.) But it fits some lawmakers better than others. It only predicts 83.3% of Baron Hill’s (D-IN) votes and 80.0% of Ronald Paul’s (R-TX) votes. Why is this?

To understand why, we look at how the ideal point model works. The ideal point model assumes that lawmakers are ordered, and that each bill d splits them at a *cut point*. The cut point is a function of the bill’s popularity and polarity, $-b_d/a_d$. Lawmakers with ideal

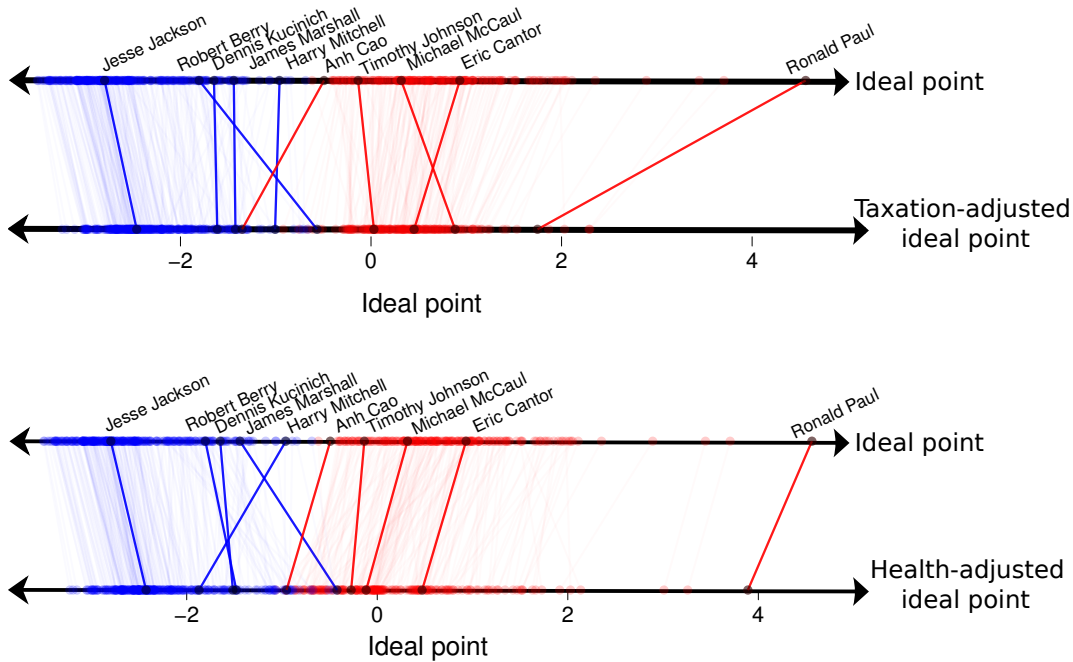


Figure 4: In a traditional ideal point model, lawmakers’ ideal points are static. In the issue-adjusted ideal point model, lawmakers’ ideal points change when they vote on certain issues, such as *taxation* (top panel) and *health* (bottom panel). A line segment connects select lawmakers’ ideal points (top row of each panel) to their issue-adjusted ideal points (bottom row of each panel). Unlabeled lawmakers are illustrated by the remaining, faint line segments. We have colored Democrats blue and Republicans red.

points x_u to one side of the cut point are more likely to support the bill; lawmakers with ideal points to the other side are more likely to reject it. The issue with lawmakers like Paul and Hill, however, is that this assumption is too strong—their voting behavior does not fit neatly into a single ordering. Rather, their location among the other lawmakers changes with different bills.

However, there are still patterns to how they vote. Paul and Hill vote consistently within individual areas of policy, such as foreign policy or education, though their voting on these issues diverges from their usual position on the political spectrum. In particular, Paul consistently votes against United States involvement in foreign military engagements, a position that contrasts with other Republicans. Hill, a “Blue Dog” Democrat, is a strong supporter of second-amendment rights, opposes same-sex adoption, and is wary of government-run health care—positions that put him at odds with many other Democrats. Particularly, the ideal point model would predict Paul and Hill as having muted positions along the classic left-right spectrum, when in fact they have different opinions about certain issues than their fellow legislators.

We refer to voting behavior like this as *issue voting*. An issue is any federal policy area, such as “financial regulation,” “foreign policy,” “civil liberties,” or “education,” on which lawmakers are expected to take positions. Lawmakers’ positions on these issues may diverge

from their traditional left/right stances, but traditional ideal point models cannot capture this. Our goal is to develop an ideal point model that allows lawmakers to deviate, depending on the issue under discussion, from their usual political position.

Figure 4 illustrates the kinds of hypotheses our model can make. Each panel represents an issue; *taxation* is on the top, and *health* is on the bottom. Within each panel, the top line illustrates the ideal points of various lawmakers—these represent the relative political positions of each lawmaker for most issues. The bottom line illustrates the position adjusted for the issue at hand. For example, the model posits that Charles Djou (Republican representative for Hawaii) is more similar to Republicans on *taxation* and more similar to Democrats on *health*, while Ronald Paul (Republican representative for Texas) is more Republican-leaning on *health* and less extreme on *taxation*. Posterior estimates like this give us a window into voting behavior that is not available to classic ideal point models.

2.3. Issue-adjusted Ideal Points

The issue-adjusted ideal point model is a latent variable model of roll call data. As with the classical ideal point model, bills and lawmakers are attached to popularity, polarity, and ideal points. In addition, the text of each bill encodes the issues it discusses and, for each vote, the ideal points of the lawmakers are adjusted according to those issues. (We obtain issue codes from text by using a probabilistic topic model. This is described below in Section 2.5.)

In more detail, each bill is associated with a popularity a_d and polarity b_d ; each lawmaker is associated with an ideal point x_u . Assume that there are K issues in the political landscape, such as *finance*, *taxation*, or *health care*. Each bill contains its text \mathbf{w}_d , a collection of observed words, from which we derive a K -vector of *issue proportions* $\boldsymbol{\theta}(\mathbf{w}_d)$. The issue proportions represent how much each bill is about each issue. A bill can be about multiple issues (e.g., a bill might be about the tax structure surrounding health care), but these values will sum to one. Finally, each lawmaker is associated with a real-valued K -vector of *issue adjustments* \mathbf{z}_u . Each component of this vector describes how his or her ideal point changes as a function of the issues being discussed. For example, a left-wing lawmaker may be more right wing on defense; a right-wing lawmaker may be more left wing on social issues.

For the vote on bill d , we linearly combine the issue proportions $\boldsymbol{\theta}(\mathbf{w}_d)$ with each lawmaker’s issue adjustment \mathbf{z}_u to give an adjusted ideal point $x_u + \mathbf{z}_u^\top \boldsymbol{\theta}(\mathbf{w}_d)$. The votes are then modeled with a logistic regression,

$$p(v_{ud} | a_d, b_d, \mathbf{z}_u, x_u, \mathbf{w}_d) = \sigma \left((x_u + \mathbf{z}_u^\top \boldsymbol{\theta}(\mathbf{w}_d)) a_d + b_d \right). \quad (2)$$

We put standard normal priors on the ideal points, polarity, and popularity variables. We use Laplace priors for the issue adjustments \mathbf{z}_u ,

$$p(z_{uk} | \lambda_1) \propto \exp(-\lambda_1 \|z_{uk}\|_1).$$

Using MAP inference, this finds sparse adjustments. With full Bayesian inference, it finds nearly-sparse adjustments. Sparsity is desirable for the issue adjustments because we do not expect each lawmaker to adjust her ideal point x_u for every issue; rather, the issue adjustments are meant to capture the handful of issues on which she does diverge.

Suppose there are U lawmakers, D bills, and K issues. The generative probabilistic process for the issue-adjusted ideal point model is the following.

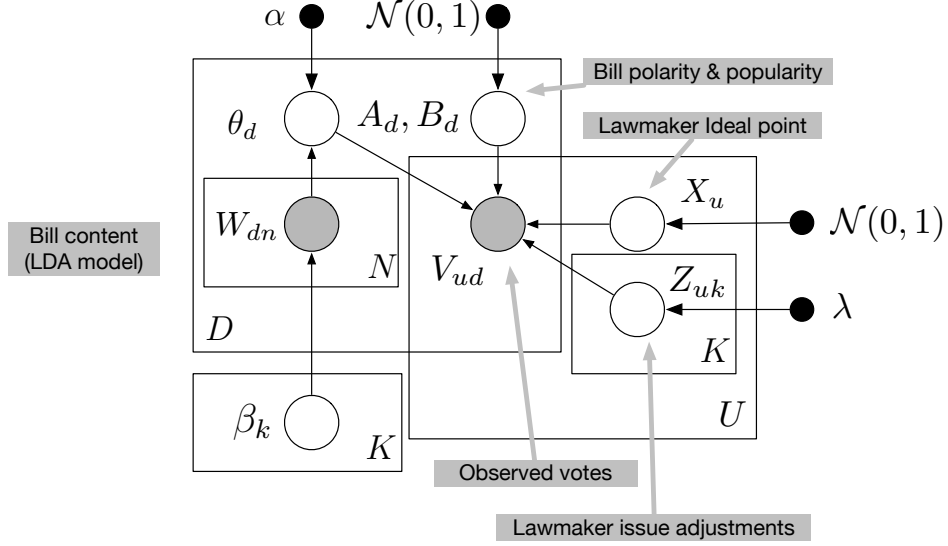


Figure 5: A graphical model for the issue-adjusted ideal point model, which models votes v_{ud} from lawmakers and legislative items. Lawmakers’ positions are determined by x_u and z_u , a k -vector which interacts with bill-specific issue mixtures θ_d (also k -vectors). Issue mixtures are fit from text using labeled latent Dirichlet allocation. As with ideal points models, a_d and b_d are bill-specific variables describing the bill’s polarization and popularity.

1. For each user $u \in \{1, \dots, U\}$:
 - (a) Draw ideal points $x_u \sim \mathcal{N}(0, 1)$.
 - (b) Draw issue adjustments $z_{uk} \sim \text{Laplace}(\lambda_1)$ for each issue $k \in \{1, \dots, K\}$.
2. For each bill $d \in \{1, \dots, D\}$:
 - (a) Draw polarity $a_d \sim \mathcal{N}(0, 1)$.
 - (b) Draw popularity $b_d \sim \mathcal{N}(0, 1)$.
3. Draw vote v_{ud} from Equation 2 for each user/bill pair, $u \in \{1, \dots, U\}$ and $d \in \{1, \dots, D\}$.

Figure 5 illustrates the graphical model. Given roll call data and bill texts, we can use posterior expectations to estimate the latent variables. For each lawmaker, these are the expected ideal points and per-issue adjustments; these are the posterior estimates we illustrated in Figure 4. For each bill, these are the expected polarity and popularity.

We consider a simple example to better understand this model. Suppose a bill d is only about *finance*. This means that $\theta(\mathbf{w}_d)$ has a one in the *finance* dimension and zero everywhere else. With a classic ideal point model, a lawmaker u ’s ideal point x_u gives his position on every bill, regardless of the issue. With the issue-adjusted ideal point model, his *effective ideal point* for this bill is $x_u + z_{u,\text{Finance}}$, adjusting his position based on the bill’s content. The adjustment $z_{u,\text{Finance}}$ might move him to the right or the left, capturing an issue-dependent change in his ideal point.

In the next section we will describe a posterior inference algorithm that will allow us to estimate x_u and z_u from lawmakers’ votes. An eager reader can scan ahead to browse these effective ideal points for Ron Paul, Dennis Kucinich, and a handful of other lawmakers in Figure 13. This figure shows the posterior mean of issue-adjusted ideal points that have been inferred from votes about *finance* (top) and votes about *congressional sessions* (bottom).

In general, a bill might involve several issues; in that case the issue vector $\theta(\mathbf{w}_d)$ will include multiple positive components. We have not yet described this important function, $\theta(\mathbf{w}_d)$, which codes a bill with its issues. We describe that function in Section 2.5. First we discuss the relationship between the issue adjusted model and other models of political science data.

2.4. Relationship to Other Models of Roll-call Data

The issue-adjusted ideal point model recovers the classical ideal point model if all of the adjustments (for all of the lawmakers) are equal to zero. In that case, as for the classical model, each bill cuts the lawmakers at $-b_d/a_d$ to determine the probabilities of voting “yes.” With non-zero adjustments, however, the model asserts that the relative positions of lawmakers can change depending on the issue. Different bill texts, through the coding function $\theta(\mathbf{w}_d)$, will lead to different orderings of the lawmakers. Again, Figure 4 illustrates these re-orderings for idealized bills, i.e., those that are only about taxation or healthcare.

Issue adjusted models are an interpretable multidimensional ideal point model. In previous variants of multidimensional ideal point models, each lawmaker’s ideal point \mathbf{x}_u and each bill’s polarity \mathbf{a}_d are vectors; the probability of a “yes” vote is $\sigma(\mathbf{x}_u^\top \mathbf{a}_d + b_d)$ (Heckman and Snyder 1996; Jackman 2001). When fit to data from U.S. politics the principle dimension invariably explains most of the variance, separating left-wing and right-wing lawmakers, and subsequent dimensions capture other kinds of patterns in voting behavior. Researchers developed these models to capture the complexity of politics beyond the left/right divide. However, these models are difficult to use because (as for classical factor analysis) the dimensions are not readily interpretable—nothing ties them to concrete issues such as *Foreign Policy* or *Defense* (Jackman 2001). Our model circumvents the problem of interpreting higher dimensions of ideal points.

The problem is that classical models only analyze the votes. To coherently bring issues into the picture, we need to include what the bills are about. Thus, the issue-adjusted model is a multidimensional ideal point model where each additional dimension is explicitly tied to a political issue. The language of the bills determine which dimensions are “active” when modeling the votes. Unlike previous multidimensional ideal point models, we do not posit higher dimensions and then hope that they will correspond to known issues. Rather, we explicitly model lawmakers’ votes on different issues by capturing how the issues in a bill relate to deviations from issue-independent voting patterns.

2.5. Using Labeled LDA to Associate Bills with Issues

We now describe the issue-encoding function θ . This function takes the language of a bill as input and returns a K-vector that represents the proportions with which each issue is discussed. In particular, we use labeled latent Dirichlet allocation (Ramage et al. 2009). To

Top words in selected issues			
Terrorism	Commemorations	Transportation	Education
terrorist	nation	transportation	student
September	people	minor	school
attack	life	print	university
nation	world	tax	charter school
york	serve	land	history
terrorist attack	percent	guard	nation
Hezbollah	community	coast guard	child
national guard	family	substitute	college

Figure 6: The eight most frequent words from topics fit using labeled LDA (Ramage et al. 2009).

use this method, we estimate a set of “topics,” i.e., distributions over words, associated with an existing taxonomy of political issues. We then estimate the degree to which each bill exhibits these topics. This treats the text as a noisy signal of the issues that it encodes, and we can use both tagged bills (i.e., bills associated with a set of issues) and untagged bills to estimate the model.

Labeled LDA is a topic model, a model that assumes that our collection of bills can be described by a set of themes, and that each bill in this collection is a bag-of-words drawn from a mixture of those themes. The themes, called topics, are distributions over a fixed vocabulary. In unsupervised LDA—and many other topic models—these themes are fit to the data (Blei et al. 2003; Blei 2012). In labeled LDA, the themes are defined by using an existing tagging scheme. Each tag is associated with a topic, and its distribution is found by taking the empirical distribution of words for documents assigned to that tag, an approach heavily influenced by, but simpler than, that of Ramage et al. (2009). This gives interpretable names (the tags) to the topics. (We note that our method is readily applicable to the fully unsupervised case, i.e., for studying a political history with untagged bills. However, such analysis requires an additional step of interpreting the topics.)

We used tags provided by the Congressional Research Service (CRS 2012), a service that provides subject codes for all bills passing through Congress. These subject codes describe the bills using phrases which correspond to traditional issues, such as *civil rights* and *national security*. Each bill may cover multiple issues, so multiple codes may apply to each bill. (Many bills have more than twenty labels.) Figure 6 illustrates the top words from several of these labeled topics. We then performed two iterations of unsupervised LDA ((Blei et al. 2003) with variational inference to smooth the word counts in these topics. We used the 74 issues in all (the most-frequent issue labels); we summarize all 74 of them in Appendix B.1.

With topics in hand, we model each bill with a mixed-membership model: Each bill is drawn from a mixture of the topics, but each one exhibits them with different proportions. Denote the K topics by $\beta_{1:K}$ and let α be a vector of Dirichlet parameters. The generative process for each bill d is:

1. Choose topic proportions $\theta_d \sim \text{Dirichlet}(\alpha)$.

2. For each word $n \in \{1, \dots, N\}$:
 - (a) Choose a topic assignment $z_{d,n} \sim \theta_d$.
 - (b) Choose a word $w_{d,n} \sim \beta_{z_{d,n}}$.

The function $\theta(\mathbf{w}_d)$ is the posterior expectation of θ_d . It represents the degree to which the bill exhibits the K topics, where those topics are explicitly tied to political issues through the congressional codes, and it is estimated using variational inference at the document level (Blei et al. 2003). The topic modeling portion of the model is illustrated on the left hand side of the graphical model in Figure 5.

We have completed our specification of the model. Given roll call data and bill texts, we first compute the issue vectors for each bill. We then use these in the issue-adjusted ideal point model of Figure 5 to infer each legislator’s posterior ideal point and per-issue adjustment. We now turn to the central computational problem for this model, posterior inference.

3. POSTERIOR ESTIMATION

Given roll call data and an encoding of the bills to issues, we form inferences and predictions through the posterior distribution of the latent ideal points, issue adjustments, and bill variables, $p(x, \mathbf{z}, a, b | \mathbf{v}, \boldsymbol{\theta})$. In the next section, we inspect this posterior to explore lawmakers’ positions about specific issues.

As for most interesting Bayesian models, this posterior is not tractable to compute; we must approximate it. Approximate posterior inference for Bayesian ideal point models is usually performed with MCMC methods, such as Gibbs sampling (Johnson and Albert 1999; Jackman 2001; Martin and Quinn 2002; Clinton et al. 2004). Here we will develop an alternative algorithm based on variational inference. Variational inference tends to be faster than MCMC, can handle larger data sets, and is attractive when fast Gibbs updates are not available. In the next section, we will use variational inference to analyze twelve years of roll call data.

3.1. Mean-field Variational Inference

In variational inference we select a simplified family of candidate distributions over the latent variables and then find the member of that family which is closest in KL divergence to the posterior of interest (Jordan et al. 1999; Wainwright and Jordan 2008). This turns the problem of posterior inference into an optimization problem. For posterior inference in the issue-adjusted model, we use the fully-factorized family of distributions over the latent variables, i.e., the mean-field family,

$$q(x, \mathbf{y}, \mathbf{z}, a, b | \boldsymbol{\eta}) = \left(\prod_U \mathcal{N}(x_u | \tilde{x}_u, \sigma_x^2) \mathcal{N}(\mathbf{z}_u | \tilde{\mathbf{z}}_u, \sigma_z^2) \right) \left(\prod_D \mathcal{N}(a_d | \tilde{a}_d, \sigma_a^2) \mathcal{N}(b_d | \tilde{b}_d, \sigma_b^2) \right). \quad (3)$$

This family is indexed by the *variational parameters* $\boldsymbol{\eta} = \{(\tilde{x}_u, \sigma_x), (\tilde{\mathbf{z}}_u, \sigma_{\mathbf{z}_u}), (\tilde{a}, \sigma_a), (\tilde{b}, \sigma_b)\}$, which specify the means and variances of the random variables in the variational posterior.

While the model specifies priors over the latent variables, in the variational family each instance of each latent variable, such as each lawmaker’s issue adjustment for *Taxation*, is endowed with its own variational distribution. This lets us capture data-specific marginals—for example, that one lawmaker is more conservative about *Taxation* while another is more liberal.

We fit the variational parameters to minimize the KL divergence between the variational posterior and the true posterior. Once fit, we can use the variational means to form predictions and posterior descriptive statistics of the lawmakers’ issue adjustments. In ideal point models, the means of a variational distribution can be excellent proxies for those of the true posterior (Gerrish and Blei 2011).

3.2. The Variational Objective

Variational inference proceeds by taking the fully-factorized distribution (Equation 3) and successively updating the parameters $\boldsymbol{\eta}$ to minimize the KL divergence between the variational distribution (Equation 3) and the true posterior:

$$\hat{\boldsymbol{\eta}} = \arg \min_{\boldsymbol{\eta}} \text{KL} (q_{\boldsymbol{\eta}}(x, \mathbf{z}, a, b) || p(x, \mathbf{a}, a, b|v)) \tag{4}$$

This optimization is usually reformulated as the problem of maximizing a lower bound (found via Jensen’s inequality) on the marginal probability of the observations:

$$\begin{aligned} p(v) &= \int_{\boldsymbol{\eta}} p(x, \mathbf{z}, a, b, v) dx dz da db \\ &\geq \int_{\boldsymbol{\eta}} q_{\boldsymbol{\eta}}(x, \mathbf{z}, a, b) \log \frac{p(x, \mathbf{z}, a, b, v)}{q_{\boldsymbol{\eta}}(x, \mathbf{z}, a, b)} dx dz da db \\ &= \mathbb{E}_q [p(x, \mathbf{z}, a, b, v)] - \mathbb{E}_q [q_{\boldsymbol{\eta}}(x, \mathbf{z}, a, b)] = \mathcal{L}_{\boldsymbol{\eta}}. \end{aligned} \tag{5}$$

We follow the example of Braun and McAuliffe (2010) by referring to the lower bound $\mathcal{L}_{\boldsymbol{\eta}}$ as the *evidence lower bound* (ELBO).

For many models, the ELBO can be expanded as a closed-form function of the variational parameters and then optimized with gradient ascent or coordinate ascent. However, the issue-adjusted ideal point model does not allow for a closed-form objective. Previous research on such non-conjugate models overcomes this by approximating the ELBO (Braun and McAuliffe 2010; Gerrish and Blei 2011). Such methods are effective, but they require many model-specific algebraic tricks and tedious derivations. Here we take an alternative approach, where we approximate the gradient of the ELBO with Monte-Carlo integration and perform stochastic gradient ascent with this approximation. This gave us an easier way to fit the variational objective for our complex model.

3.3. Optimizing the Variational Objective with a Stochastic Gradient

We begin by computing the gradient of the ELBO in Equation 5. We rewrite it in terms of integrals, then exchange the order of integration and differentiation, and apply the chain

rule:

$$\begin{aligned}
\nabla \mathcal{L}_\eta &= \nabla \left[\int q_\eta(x, \mathbf{z}, a, b) (\log p(x, \mathbf{z}, a, b, v) - \log q_\eta(x, \mathbf{z}, a, b)) dx \right] \\
&= \int \nabla \left[q_\eta(x, \mathbf{z}, a, b) (\log p(x, \mathbf{z}, a, b, v) - \log q_\eta(x, \mathbf{z}, a, b)) \right] dx \\
&= \int \nabla q_\eta(x, \mathbf{z}, a, b) (\log p(x, \mathbf{z}, a, b, v) - \log q_\eta(x, \mathbf{z}, a, b)) - q_\eta(x, \mathbf{z}, a, b) \nabla \log q_\eta dx.
\end{aligned} \tag{6}$$

Above we have assumed that the support of q_η is not a function of η , and that $\log q_\eta(x, \mathbf{z}, a, b)$ and $\nabla \log q_\eta(x, \mathbf{z}, a, b)$ are continuous with respect to η .

We can rewrite Equation 6 as an expectation by using the identity $q_\eta(x) \nabla \log q_\eta(x) = \nabla q_\eta(x)$:

$$\nabla \mathcal{L}_\eta = \mathbb{E}_q [\nabla \log q_\eta(x, \mathbf{z}, a, b) (\log p(x, \mathbf{z}, a, b, v) - \log q_\eta(x, \mathbf{z}, a, b) - 1)]. \tag{7}$$

Next we use Monte Carlo integration to form an unbiased estimate of the gradient at $\eta = \eta_0$. We obtain M *iid* samples $(x_1, \dots, x_M, \dots, b_1, \dots, b_M)$ from the variational distribution q_{η_0} for the approximation

$$\begin{aligned}
\nabla \mathcal{L}_\eta \Big|_{\eta_0} &\approx \\
&\frac{1}{M} \sum_{m=1}^M \nabla \log q_\eta(x_m, \mathbf{z}_m, a_m, b_m) \Big|_{\eta_0} (\log p(x_m, \mathbf{z}_m, a_m, b_m, y) - \log q_{\eta_0}(x_m, \mathbf{z}_m, a_m, b_m) - C).
\end{aligned} \tag{8}$$

We denote this approximation $\tilde{\nabla} \mathcal{L}_\eta \Big|_{\eta_0}$. Note we replaced the 1 in Equation 7 with a constant C , which does not affect the expected value of the gradient (this follows because $\mathbb{E}_q [\nabla \log q_\eta(x, \mathbf{z}, a, b)] = 0$). We discuss in the supplementary materials how to set C to minimize variance. Related estimates of similar gradients have been studied in recent work (Carbonetto et al. 2009; Graves 2011; Paisley et al. 2012) and in the context of expectation maximization (Wei and Tanner 1990).

Using this method for finding an approximate gradient, we optimize the ELBO with stochastic optimization (Robbins and Monro 1951; Spall 2003; Bottou and Cun 2004). Stochastic optimization follows noisy estimates of the gradient with a decreasing step-size. While stochastic optimization alone is sufficient to achieve convergence, it may take a long time to converge. To improve convergence rates, we used two additional ideas: quasi-Monte Carlo samples (which minimize variance) and second-order updates (which eliminate the need to select an optimization parameter). We provide details of these improvements in the appendix.

Let us return briefly to the problem that motivated this section. Our goal is to estimate the mean of the hidden random variables—such as lawmakers’ issue adjustments \mathbf{z} —from their votes on bills. We achieved this by variational Bayes, which amounts to maximizing the ELBO (Equation 5) with respect to the variational parameters. This maximization is achieved with stochastic optimization on Equation 8. In the next section we will empirically study these inferred variables (i.e., the expectations induced by the variational distribution) to better understand distinctive voting behavior.

4. ISSUE ADJUSTMENTS IN THE UNITED STATES CONGRESS

We used the issue-adjusted ideal point model to study the complete roll call record from the United States Senate and House of Representatives during the years 1999-2010. We report on this study in this and the next section. We first evaluate the model fitness to this data, confirming that issue-adjustments give a better model of roll call data and that the encoding of bills to issues is responsible for the improvement. We then use our inferences to give a qualitative look at U.S. lawmakers’ issue preferences, demonstrating how to use our richer model of lawmaker behavior to explore a political history.

4.1. The United States Congress from 1999-2010

We studied U.S. Senate and House of Representative roll-call votes from 1999 to 2010. This period spanned Congresses 106 to 111, the majority of which Republican President George W. Bush held office. Bush’s inauguration and the attacks of September 11th, 2001 marked the first quarter of this period, followed by the wars in Iraq and Afghanistan. Democrats gained a significant share of seats from 2007 to 2010, taking the majority from Republicans in both the House and the Senate. Democratic President Barack Obama was inaugurated in January 2009.

The roll-call votes are recorded when at least one lawmaker wants an explicit record of the votes on the bill. For a lawmaker, such records are useful to demonstrate his or her positions on issues. Roll calls serve as an incontrovertible record for any lawmaker who wants one. We downloaded both roll-call tables and bills from www.govtrack.us, a nonpartisan website which provides records of U.S. Congressional voting. Not all bill texts were available, and we ignored votes on bills that did not receive a roll call, but we had over one hundred for each Congress. Table 7 summarizes the statistics of our data.

We fit our models to two-year periods in the House and (separately) to two-year periods in the Senate. Some bills received votes in both the House and Senate; in those cases, the issue-adjusted model’s treatment of the bill in the House was completely independent of its treatment by the model in the Senate.

Vocabulary. To fit the labeled topic model to each bill, we represented each bill as a vector of phrase counts. This “bag of phrases” is similar to the “bag of words” assumption commonly used in natural language processing. To select this vocabulary, we considered all phrases of length one word to five words. We then omitted content-free phrases such as “and”, “when”, and “to the”. The full vocabulary consisted of 5,000 n -grams (further details of vocabulary selection are in Appendix B.2). We used these phrases to algorithmically define topics and assign issue weights to bills as described in Section 2.5.

Identification. When using ideal-point models for interpretation, we must address the issue of identification. The signs of ideal points x_u and bill polarities a_d are arbitrary, for example, because $x_u a_d = (-x_u)(-a_d)$. This leads to a multimodal posterior (Jackman 2001). We address this by flipping ideal points and bill polarities if necessary to follow the convention that Republicans are generally on the right (positive on the line) and Democrats are generally on the left (negative on the line).

Figure 7: Roll-call data sets used in the experiments. These counts include votes in both the House and Senate. Congress 107 had fewer votes than the remaining congresses in part because this period included large shifts in party power, in addition to the attacks on September 11th, 2001. The number of lawmakers within each House and Senate varies by congress because there was some turnover within each Congress. In addition, some lawmakers never voted on legislation in our experiments (recall, we used legislation for which both text was available and for which the roll-call was recorded).

Statistics for the U.S. Senate				
Congress	Years	Lawmakers	Bills	Votes
106	1999-2000	81	101	7,612
107	2001-2002	78	76	5,547
108	2003-2004	101	83	7,830
109	2005-2006	102	74	7,071
110	2007-2008	103	97	9,019
111	2009-2010	110	62	5,936

Statistics for the U.S. House of Representatives				
Congress	Years	Lawmakers	Bills	Votes
106	1999-2000	437	345	142,623
107	2001-2002	61	360	18,449
108	2003-2004	440	490	200,154
109	2005-2006	441	458	187,067
110	2007-2008	449	705	287,645
111	2009-2010	446	810	330,956

4.2. Ideal Point Models vs. Issue-adjusted Ideal Point Models

The issue-adjusted ideal point model in Equation 2 is a generalization of the traditional ideal point model (see Section 2.4). Before using this more complicated model to explore our data, we empirically justify this increased complexity. We first outline empirical differences between issue-adjusted ideal points and traditional ideal points. We then report on a quantitative validation of the issue-adjusted model.

Examples: adjusting for issues. To give a sense of how the issue-adjusted ideal point model works, Table 8 gives a side-by-side comparison of traditional ideal points x_u and issue-adjusted ideal points ($x_u + \mathbf{z}_u^T \boldsymbol{\theta}$) for the ten most-improved bills of Congress 111 (2009-2010). For each bill, the top row shows the ideal points of lawmakers who voted “Yea” on the bill and the bottom row shows lawmakers who voted “Nay”. The top and bottom rows are a partition of votes rather than separate treatments of the same votes. In a good model of roll call data, these two sets of points will be separated, and the model can place the bill parameters at the correct cut point. Over the whole data set, the cut point of the votes improved in 14,347 heldout votes. (It got worse in 8,304 votes and stayed the same in 5.7M.)

Comparing issue-adjusted ideal points to traditional ideal points. The traditional ideal point model (Equation 1) uses one variable per lawmaker, the ideal point x_u , to explain all of her voting behavior. In contrast, the issue-adjusted model (Equation 2) uses x_u along with K issue adjustments. Here we ask, how does x_u under these two models differ? We fit ideal points to the 111th House (2009 to 2010) and issue-adjusted ideal points to the same period with regularization $\lambda = 1$.

The top panel of Figure 4.2 compares the classical ideal points to the global ideal points from the issue-adjusted model. In this parallel plot, the top axis of this represents a lawmaker’s ideal point x_u under the classical model, while the bottom axis represents his global ideal point under the issue-adjusted model. (We will use plots like this again in this paper. It is called a parallel plot, and it compares separate treatments of lawmakers. Lines between the same lawmakers under different treatment are shaded based on their deviation from a linear model to highlight unique lawmakers.) The ideal points in Figure 4.2 are similar; their correlation coefficient is 0.998. The most noteworthy difference is that lawmakers appear more partisan under the traditional ideal point model—enough that Democrats are completely separated from Republicans by x_u —while issue-adjusted ideal points provide a softer split.

This is not surprising, because the issue-adjusted model is able to use lawmakers’ adjustments to explain their votes. In fact, the political parties are *better* separated with issue adjustments than they are by ideal points alone. We checked this by writing each lawmaker u as the vector $w_u := (x_u, z_{u,1}, \dots, z_{u,k})$ and performing linear discriminant analysis to find that vector β which “best” separates lawmakers by party along $w_u^T \beta$.

We illustrate lawmakers’ projections $w_u^T \beta$ along the discriminant vector β in the bottom figure of Figure 4.2 (we normalized variance of these projections to match that of the ideal points). The correlation coefficient between this prediction and political party is 0.979, much higher than the correlation between ideal points x_u and political party (0.921).

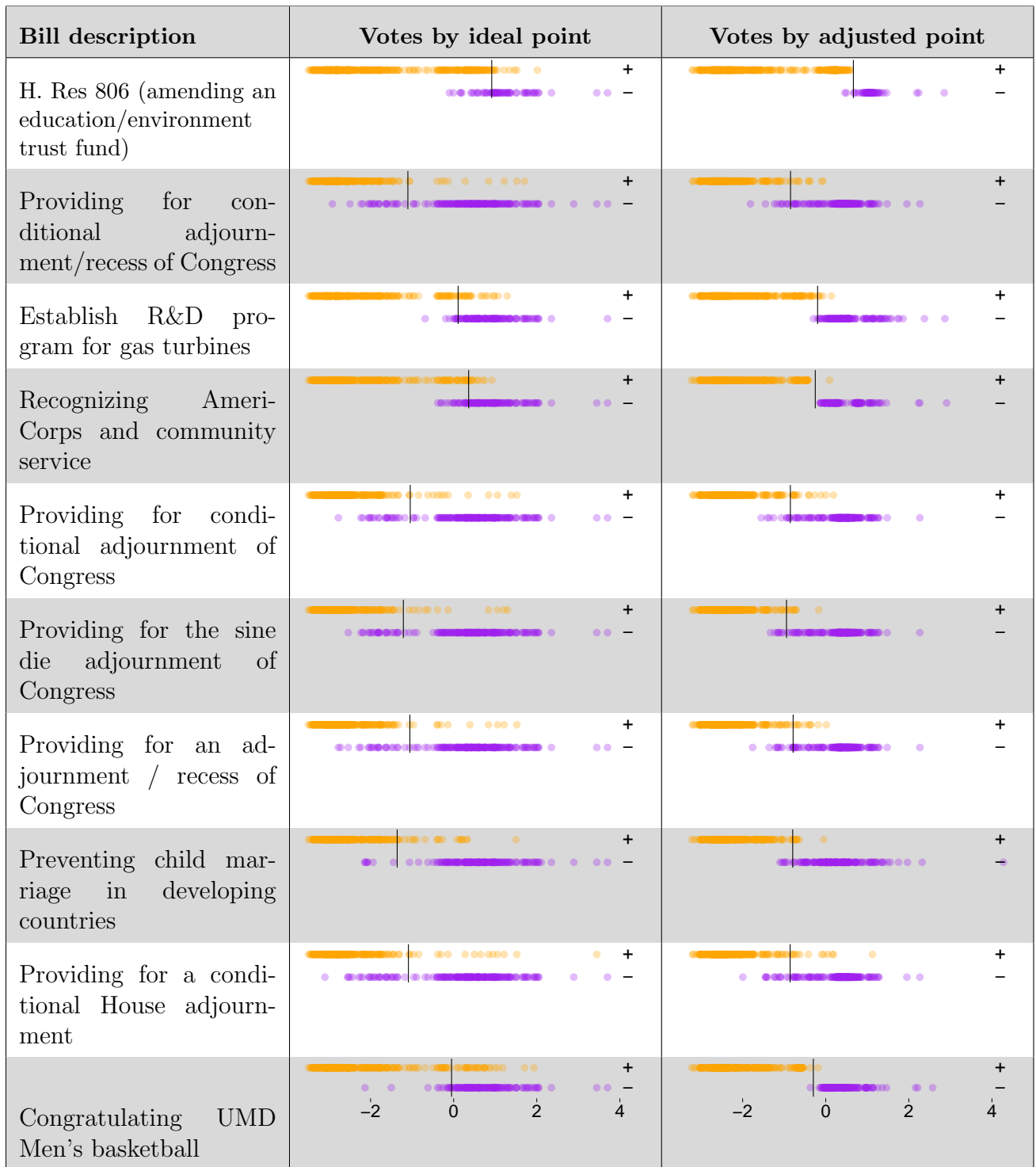


Figure 8: Issue-adjusted ideal points can explain votes better than standard ideal points. The x-axis of each small plot shows ideal point or issue-adjusted ideal point for a lawmaker. Each bill's indifference point $-b_d/a_d$ is shown as a vertical line. Positive votes (orange) and negative votes (purple) are better-divided by issue-adjusted ideal points.

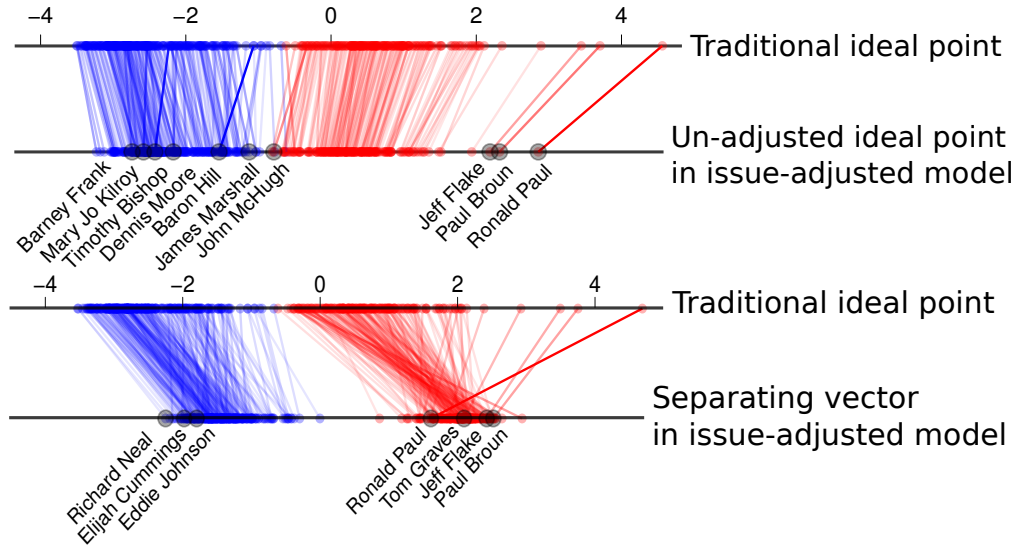


Figure 9: Classic issue-adjusted ideal points x_u (top row, both figures) separate lawmakers by party better than un-adjusted ideal points x_u from the issue-adjusted model (bottom row, top figure). The issue-adjusted model can still separate Republicans from Democrats better than the ideal point model along a separating vector (bottom row, bottom figure). In each figure, Republicans are colored red, and Democrats are blue. These ideal points were estimated in the 111th House of Representatives. The line connecting ideal points from each model has opacity proportional to the squared residuals in a linear model fit to predict issue-adjusted ideal points from ideal points. The separating vector was defined using linear discriminant analysis.

To be sure, some of this can be explained by random variation in the additional 74 dimensions. To check the extent of this improvement due only to dimension, we draw random issue adjustments from normal random variables with the same variance as the empirically observed issue adjustments. In 100 tests like this, the correlation coefficient was higher than for classical ideal points, but not by much: 0.933 ± 0.004 . Thus, the posterior issue adjustments provide a signal for separating the political parties *better* than ideal points alone. In fact, we will see in Section 5.2 that procedural votes driven by political ideology is one of the factors driving this improvement.

Changes in bills’ parameters. Bills’ polarity a_d and popularity b_d are similar under both the traditional ideal point model and the issue-adjusted model. We illustrate bills’ parameters in these two models in Figure 10 and note some exceptions.

First, procedural bills stand out from other bills in becoming more popular overall. In Figure 10, procedural bills have been separated from traditional ideal points. We attribute the difference in procedural bills’ parameters to *procedural cartel theory*, which we describe further in Section 5.2.

The remaining bills have also become less popular but more polarized under the issue-adjusted model. This is because the issue-adjusted model represents the interaction between lawmakers and bills with K additional bill-specific variables, all of which are mediated by

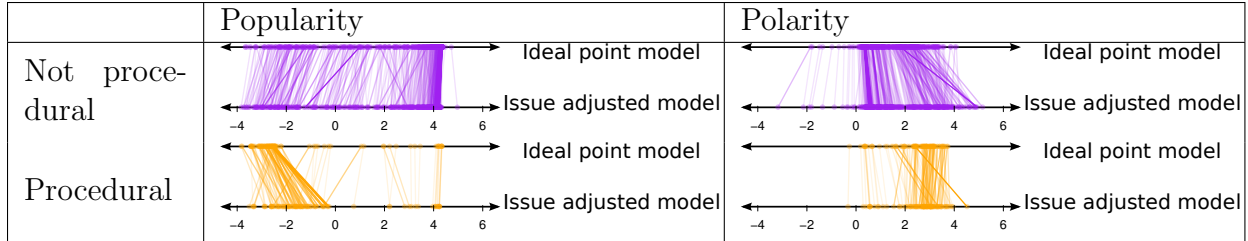


Figure 10: Procedural bills are more popular under the issue-adjusted voting model. Top: popularity b_d of procedural bills under the issue-adjusted voting model is greater than with traditional ideal points. Bottom: consistent with Cox and Poole (2002) and *procedural cartel theory*, the polarity of procedural bills is generally more extreme than that of non-procedural bills. However, issue adjustments lead to increased polarity (i.e., certainty) among non-procedural votes as well. The procedural issues include *congressional reporting requirements*, *government operations and politics*, *House of Representatives*, *House rules and procedure*, *legislative rules and procedure*, and *Congress*.

the bill’s polarity. This means that the the model is able to depend more on bills’ polarities than bills’ popularities to explain votes. For example, Donald Young regularly voted *against* honorary names for regional post offices. These bills—usually very popular—would have high popularity under the ideal point model. The issue-adjusted model also assigns high popularity to these bills, but it takes advantage of lawmaker’s positions on the *postal facilities* issue to explain votes, decreasing reliance on the bill’s popularity (*postal facilities* was more common than 50% of other issues, including *human rights*, *finance*, and *terrorism*).

4.3. Evaluation of the Predictive Distribution

We have described the qualitative differences between the issue-adjusted model and the traditional ideal point model. We now turn to a quantitative evaluation: Does the issue-adjusted model give a better fit to legislative data?

We answer this question via cross validation and the predictive distribution of votes. For each session, we divide the votes, i.e., individual lawmaker/bill pairs, into folds. For each fold, we hold out the votes assigned to it, fit our models to the remaining votes, and then evaluate the log probability of the held out votes under the predictive distribution. A better model will assign higher probability to the held-out data. We compared several methods:

1. The issue-adjusted ideal point model with topics found by labeled LDA: This is the model and algorithm described above. We used a regularization parameter $\lambda = 1$. (See Appendix A.3 for a study of the effect of regularization.)
2. The issue-adjusted ideal point model with explicit labels on the bills: Rather than infer topics with labeled LDA, we used the CRS labels explicitly. If a bill contains J labels, we gave it weight $1/J$ at each of the corresponding components of the topic vector θ .
3. The traditional ideal point model of Clinton et al. (2004): This model makes no reference to issues. To manage the scale of the data, and keep the comparison fair, we

Figure 11: Average log-likelihood of heldout votes across all sessions for the House and Senate. Log-likelihood was averaged across folds using six-fold cross validation for Congresses 106 to 111 (1999-2010) with regularization $\lambda = 1$. The variational distribution had higher heldout log-likelihood for all congresses in both chambers than either

Heldout log likelihood of Senate votes						
Congress	106	107	108	109	110	111
Traditional ideal point model (IPM)	-0.209	-0.209	-0.182	-0.189	-0.206	-0.182
Issue-adjusted IPM (with labeled LDA)	-0.208	-0.209	-0.181	-0.188	-0.205	-0.180
Issue-adjusted IPM (with direct labels)	-0.208	-0.209	-0.182	-0.189	-0.206	-0.181
Standard LDA	-0.208	-0.210	-0.184	-0.189	-0.207	-0.181
Issue-adjusted IPM (with permuted issues)	-0.210	-0.210	-0.183	-0.203	-0.211	-0.186

Heldout log likelihood of House votes						
	106	107	108	109	110	111
Traditional ideal point model (IPM)	-0.168	-0.154	-0.096	-0.120	-0.090	-0.077
Issue-adjusted IPM (with labeled LDA)	-0.167	-0.151	-0.095	-0.118	-0.089	-0.076
Issue-adjusted IPM (with direct labels)	-0.167	-0.151	-0.094	-0.117	-0.088	-0.075
Standard LDA	-0.171	-0.154	-0.097	-0.121	-0.091	-0.078
Issue-adjusted IPM (with permuted issues)	-0.167	-0.155	-0.096	-0.122	-0.090	-0.077

used variational inference. (In Gerrish and Blei (2011), we showed that variational approximations find as good approximate posteriors as MCMC in ideal point models.)

4. A permuted issue-adjusted model: Here, we selected a random permutation $\pi \in S_D$, to shuffle the D topic vectors $\theta_d \rightarrow \theta_{\pi(d)}$ and fit the issue-adjusted model with the permuted vectors. This permutation test removes the information contained in matching bills to issues, though it maintains the same empirical distribution over topic mixtures. It can indicate that improvement we see over traditional ideal points is due to the bills' topics, not due to spurious factors (such as the change in dimension). In this method we used five random permutations.

We summarize the results in Table 11. In all chambers in both Congresses, the issue-adjusted model represents heldout votes with higher log-likelihood than an ideal point model. Further, every permutation represented votes with lower log-likelihood than the issue-adjusted model. In most cases they were also lower than an ideal point model. These tables validate the additional complexity of the issue-adjusted ideal point model.

5. EXPLORING ISSUES AND LAWMAKERS

In the previous section, we demonstrated that the issue-adjusted IPM gives a better fit to roll call data than the traditional ideal point model. While we used prediction to validate the model, we emphasize that it is primarily an exploratory tool. As for the traditional ideal point model, it is useful for summarizing and characterizing roll call data. In this section, we demonstrate how to use the approximate posterior to explore a collection of bills, lawmakers, and votes through the lens of the issue-adjusted model.

We will focus on the 111th Congress (2009-2010). First, we show on which issues the issue-adjusted model best fits. We then discuss several specific lawmakers, showing voting patterns that identify lawmakers who transcend their party lines. We finally describe procedural cartel theory (Cox and Poole 2002), which explains why certain lawmakers have such different preferences on procedural issues like *congressional sessions* than substantive issues like *finance*.

5.1. Issues Improved by Issue Adjustment

Which issues give the issue-adjusted model an edge over the traditional model? We measured this with a metric we will refer to as *issue improvement*. Issue improvement is the weighted improvement in log likelihood for the issue-adjusted model relative to the traditional model. We formalize this by defining the log likelihood of each lawmaker’s vote

$$J_{ud} = 1_{\{v_{ud}=\text{yes}\}}p - \log(1 + \exp(p)), \quad (9)$$

where $p = (x_u + \mathbf{z}_u^T \boldsymbol{\theta}_d)a_d + b_d$ is the log-odds of a vote under the issue-adjusted voting model. We also measure the corresponding log-likelihood I_{ud} under the ideal point model, using $p = x_u a_d + b_d$. The improvement of issue k is then the sum of the improvement in log-likelihood, weighted by how much each vote represents issue k :

$$\text{Imp}_k = \frac{\sum_{v_{ud}} \boldsymbol{\theta}_{dvk}(J_{ud} - I_{ud})}{\sum_{v_{ud}} \boldsymbol{\theta}_{dvk}}. \quad (10)$$

A high value of Imp_k indicates that issue k is associated with an increase in log-likelihood, while a low value is associated with a decrease in log-likelihood.

We measured this for each issue in the 111th House. As this was an empirical question about the entire House, we fit the model to all votes (in contrast to the analysis above, which fit the model to five out of six folds, for each of the six folds).

We illustrate Imp_k for all issues in Figure 12. All issues increased log-likelihood; those associated with the greatest increase tended to be related to procedural votes. For example, *women*, *religion*, and *military personnel* issues are nearly unaffected by lawmakers’ offsets. For those issues, a global political spectrum (i.e., a single dimension) capably explains the lawmakers’ positions.

5.2. Exploring the Issue Adjustments

The purpose of our model is to adjust lawmaker’s ideal points according to the issues under discussion. In this section, we demonstrate a number of ways to explore this information.

We begin with a brief summary of the main information obtained by this model. During posterior inference, we jointly estimate the mean \tilde{x}_u, \tilde{z}_u of all lawmakers’ positions lawmakers’ issue-adjusted ideal points. These issue adjustments \tilde{z}_u adjust how we expect lawmakers to vote given their un-adjusted ideal points \tilde{x} . We illustrate this for *finance* (a substantive issue) and *congressional sessions* (a procedural issue) in Figure 13 and summarize all issues in Figure 14. Upon a cursory inspection, it is clear that in some issues, lawmakers’ adjustments are relatively sparse, i.e. only a few lawmakers’ adjustments are interesting. In other issues—such as the procedural issue *congressional sessions*—these adjustments are more systemic.

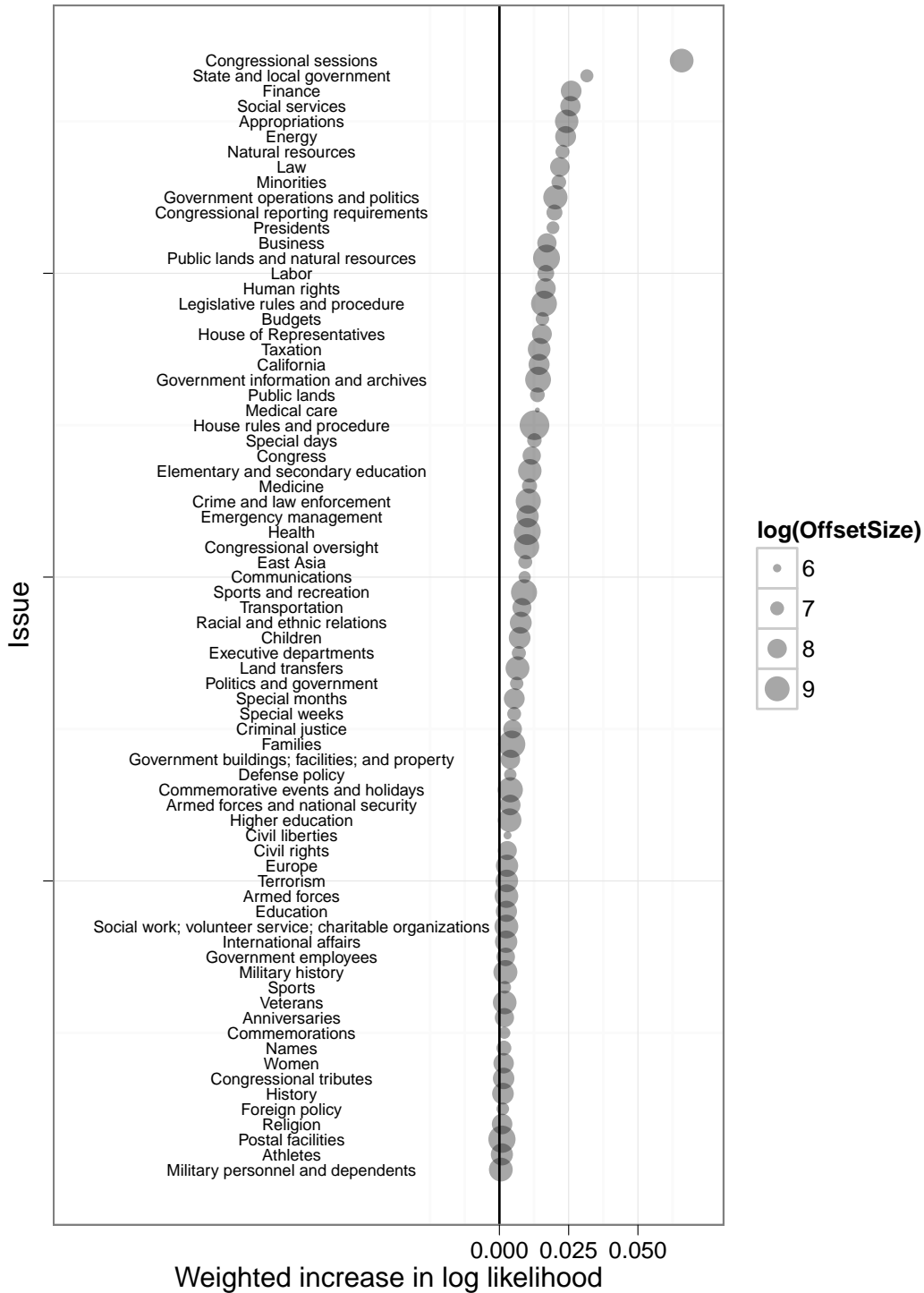


Figure 12: Log-likelihood increases when using adjusted ideal points most for procedural and strategic votes and less for issues frequently discussed during elections. Imp_k is shown on the x-axis, while issues are spread on the y-axis for display. The size of each issue k is proportional to the logarithm of the weighted sum $\sum_{v_{ud}} \theta_{dk}$ of votes about the issue.

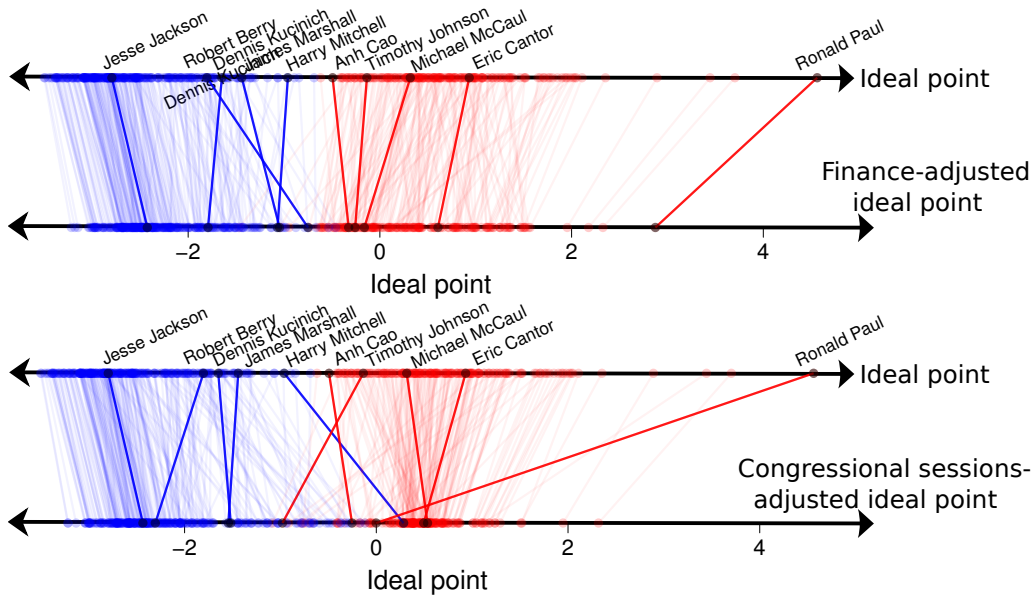


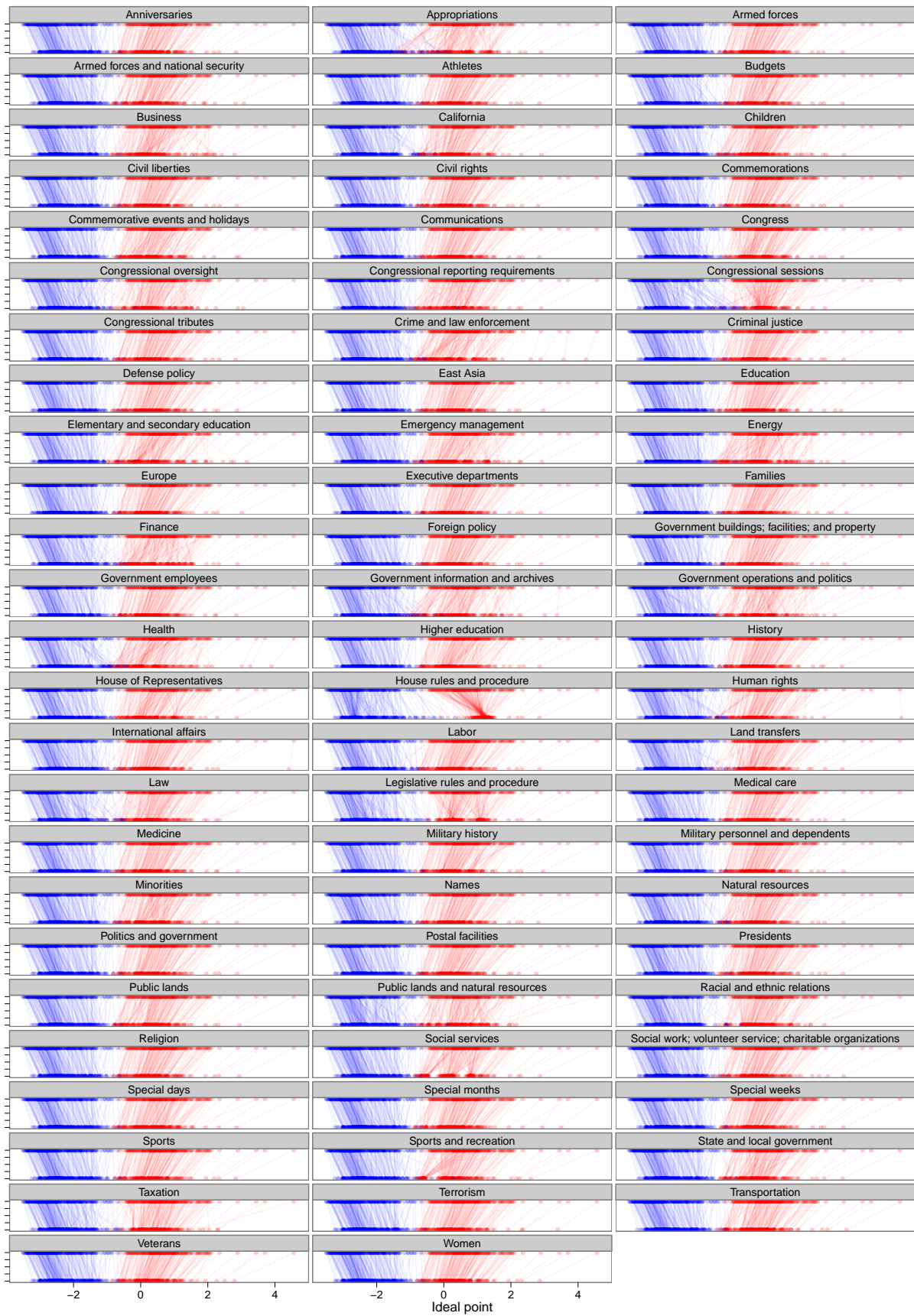
Figure 13: Ideal points x_u and issue-adjusted ideal points $x_u + z_{uk}$ from the 111th House for the substantive issue *finance* and the procedural issue *congressional sessions*. Democrats are blue and Republicans are red. Votes about *finance* and *congressional sessions* were better fit using issue-adjusted ideal points. For procedural votes such as *congressional sessions*, lawmakers become more polarized by political party, behavior predicted by procedural cartel theory (Cox and McCubbins 1993).

Adjustments by issue and party. Figure 15 illustrates the distribution across lawmakers of the posterior issue adjustments (denoted \tilde{z}_{uk}) for issues with the highest and lowest variance. This figure shows the distribution for the four issues with the greatest variation in \tilde{z}_{uk} (across lawmakers) and the four issues with the least variation. Note the systematic bias in Democrats’ and Republicans’ issue preferences: they become more partisan on certain issues, particularly procedural ones.

Controlling for ideal points. We found that posterior issue adjustments can correlate with the ideal point of the lawmaker—for example, a typical Republican tends to have a Republican offset on taxation. In some settings, we are more interested in understanding when a Republican deviates from behavior suggested by her ideal point. We can shed light on this systemic issue bias by explicitly controlling for it. To do this, we fit a regression for each issue k to explain away the effect of a lawmaker’s ideal point \mathbf{x}_u on her offset \mathbf{z}_{uk} :

$$\mathbf{z}_k = \beta_k \mathbf{X} + \boldsymbol{\varepsilon},$$

where $\beta_k \in \mathbb{R}$. Instead of evaluating a lawmaker’s observed offsets, we use her residual $\hat{z}_{uk} = \mathbf{z}_{uk} - \beta_k \mathbf{x}_u$, which we call the *corrected issue adjustment*. By doing this, we can evaluate lawmakers in the context of other lawmakers who share the same ideal points: a positive offset \hat{z}_{uk} for a Democrat means she tends to vote more conservatively about issue k than others with the same ideal point (most of whom are Democrats).



26
 Figure 14: Ideal points x_u and issue-adjusted ideal points $x_u + z_{uk}$ from the 111th House for all issues.

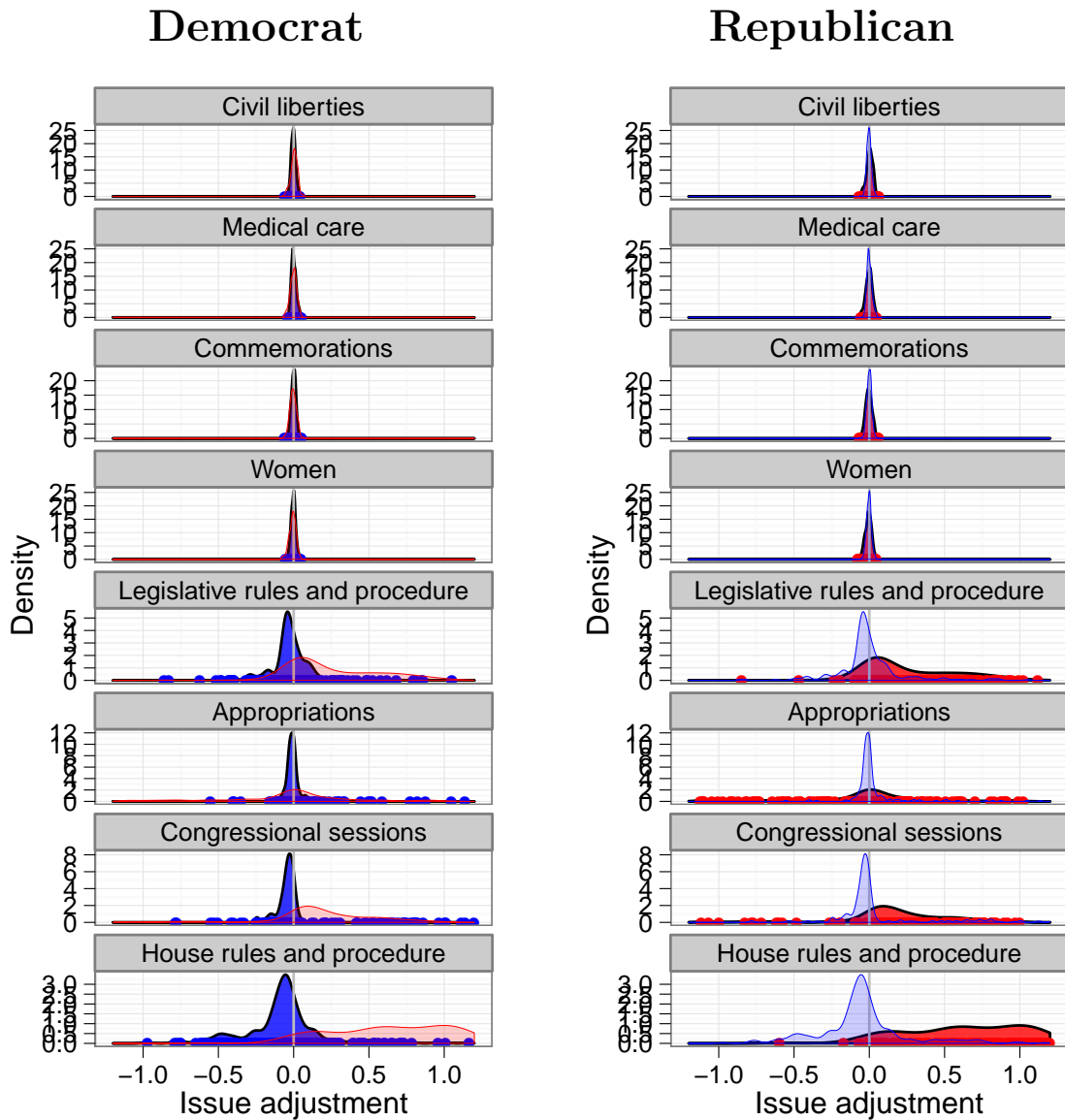
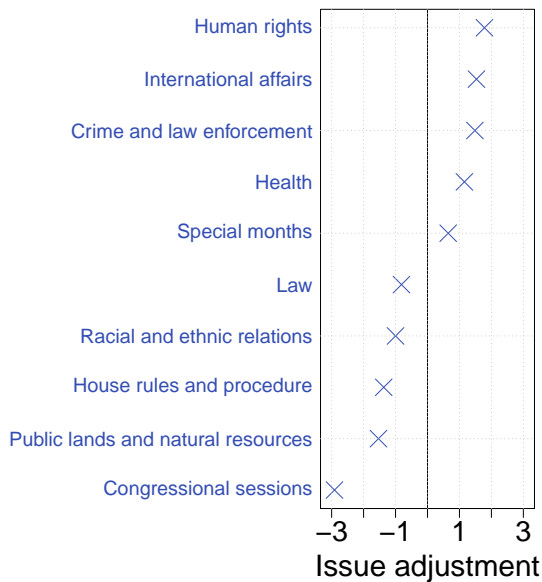
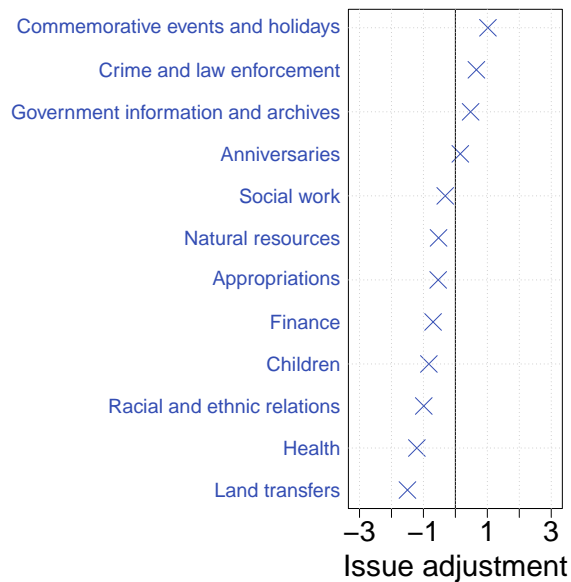


Figure 15: Histogram of issue adjustments for selected issues. Democrats are in the left column, and Republicans are in the right column. Both Democrats and Republicans tend to have small issue adjustments for traditional issues. Their issue adjustments differ substantially for procedural issues. A more-dispersed distribution of issue adjustments does not mean that these lawmakers tend to feel differently from one another about these issues. Instead, it means that lawmakers deviate from their ideal points more.



Ronald Paul (House Republican)



Donald Young (House Republican)

Figure 16: Significant issue adjustments for exceptional senators in Congress 111. Each illustrated issue is significant to $p < 0.05$ by a permutation test.

Most issues had only a moderate relationship to ideal points. *House rules and procedure* was the most-correlated with ideal points, moving the adjusted ideal point $\beta_k = 0.26$ right for every unit increase in ideal point. *public land and natural resources* and *taxation* followed at a distance, moving an ideal point 0.04 and 0.025 respectively with each unit increase in ideal point. *health*, on the other hand, moved lawmakers $\beta_k = 0.04$ left for every unit increase in ideal point. At the other end of the spectrum, the issues *women*, *religion*, and *military personnel* were nearly unaffected by lawmakers’ offsets.

Extreme lawmakers. We next use these corrected issue adjustments to identify lawmakers’ exceptional issue preferences. To identify adjustments which are significant, we turn again to the same nonparametric check described in the last section: permute issue vectors’ document labels, i.e. $(\theta_1, \dots, \theta_D) \mapsto (\theta_{\pi_i(1)} \dots \theta_{\pi_i(D)})$, and refit lawmakers’ adjustments using both the original issue vectors and permuted issue vectors, for permutations π_1, \dots, π_{20} . We then compare a corrected issue adjustment \hat{z}_{uk} ’s absolute value with corrected issue adjustments estimated with permuted issue vectors $\theta_{\pi_i(d)k}$. This provides a nonparametric method for finding issue adjustments which are more extreme than expected by chance: an extreme issue adjustment has a greater absolute value than all of its permuted counterparts. We use these to discuss several unique lawmakers.

Using corrected issue adjustments, we identified several of the most-unique lawmakers. We focused this analysis on votes from 2009-2010, the most recent full session of Congress, using $\lambda = 1$. We fit the variational approximation to all votes in the House and computed lawmakers’ corrected issue adjustments \hat{z}_{uk} , which are conditioned on their ideal points as described in Section 5.2. Figure 16 illustrates those issue preferences which were significant

under 20 permutation replications ($p < 0.05$) for several lawmakers from this Congress.

- **Ron Paul.** We return to Ron Paul, one of the most unique House Republicans, and a lawmaker who first motivated this analysis. Paul’s offsets were very extreme; he tended to vote more conservatively than expected on *health*, *human rights* and *international affairs*. He voted more liberally on social issues such as *racial and ethnic relations*, and broke with behavior expected under a procedural cartel (congressional sessions). The issue-adjusted training accuracy of Paul’s votes increased from 83.8% to 87.9% with issue offsets, placing him among the two most-improved lawmakers with this model.

The issue-adjusted improvement Imp_K (Equation 10) when restricted to Paul’s votes indicate significant improvement in *international affairs* and *East Asia* (he tends votes against U.S. involvement in foreign countries); *congressional sessions*; *human rights*; and *special months* (he tends to vote against recognition of months as special holidays).

- **Donald Young.** One of the most exceptional legislators in the 111th House was Donald Young, Alaska Republican. Young stood out most in a topic used frequently in House bills about naming local landmarks. In many cases, Young voted against the majority of his party (and the House in general) on a series of largely symbolic bills and resolutions. For example, in the *commemorative events and holidays* topic, Young voted (with only two other Republicans and against the majority of the House) *not to* commend “the members of the Agri-business Development Teams of the National Guard and the National Guard Bureau for their efforts... to modernize agriculture practices and increase food production in war-torn countries.”

Young’s divergent symbolic voting was also evident in a series of votes against naming various landmarks—such as post offices—in a topic about such symbolic votes. Yet Donald Young’s ideal point is -0.35, which is not particularly distinctive (see Figure 2): using the ideal point alone, we would not recognize his unique voting behavior.

Procedural Cartels. Above we briefly noted that Democrats and Republicans become *more partisan* on procedural issues. Lawmakers’ more partisan voting on procedural issues can be explained by theories about partisan strategy in the House. In this section we summarize a theory underlying this behavior and note several ways in which it is supported by issue adjustments.

The sharp contrast in voting patterns between procedural votes and substantive votes has been noted and studied over the past century (Jr. 1965; Jones 1964; Cox and McCubbins 1993; Cox and Poole 2002). Cox and McCubbins (1993) provide a summary of this behavior: “parties in the House—especially the majority party—are a species of ‘legislative cartel’ [which usurp the power] to make rules governing the structure and process of legislation.” A defining assumption made by Cox and McCubbins (2005) is that the majority party delegates an agenda-setting monopoly to senior partners in the party, who set the procedural agenda in the House. As a result, the cartel ensures that senior members hold agenda-setting seats (such as committee chairs) while rank-and-file members of the party support agenda-setting decisions.

This *procedural cartel theory* has withstood tests in which metrics of polarity were found to be *greater* on procedural votes than substantive votes (Cox and McCubbins 1993; Cox and

Poole 2002; Cox and McCubbins 2005). We note that issue adjustments support this theory in several ways. First, lawmakers’ systematic bias for procedural issues was illustrated and discussed in Section 5.2 (see Figure 15): Democrats systematically lean left on procedural issues, while Republicans systematically lean right. Importantly, this discrepancy is more pronounced among procedural issues than substantive ones. Second, lawmakers’ positions on procedural issues are more partisan than expected under the underlying un-adjusted ideal points (see Section 5.2 and Figure 13). Finally, more extreme polarity and improved prediction on procedural votes (see Section 4.3 and Figure 10) indicate that that issue adjustments for procedural votes are associated with *more extreme* party affiliation—also observed by Cox and Poole (2002).

6. SUMMARY

We developed and studied the issue-adjusted ideal point model, a model designed to tease apart lawmakers’ preferences from their general political position. This is a model of roll-call data that captures how lawmakers vary, issue by issue. It gives a new way to explore legislative data. On a large data set of legislative history, we demonstrated that it is able to represent votes better than a classic ideal point model and illustrated its use as an exploratory tool.

This work could be extended in several ways. One of the most natural way is to incorporate lawmakers’ stated positions on issues – which may differ from how they actually vote on these issues; in preliminary analyses, we have found little correlation to external sources. We might also study lawmakers’ activities outside of voting to understand their issue positions. For example, lawmakers’ fund-raising by industry area might (or might not) be useful in predicting their positions on different issues. Additional work includes modeling how lawmakers’ positions on issues change over time, by incorporating time-series assumptions as in Martin and Quinn (2002).

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APPENDIX A. POSTERIOR INFERENCE

In this appendix we provide additional details for *A Textual Issue Model for Legislative Roll Calls*. We begin by detailing the inference algorithm summarized in Section 3.

A.1. Optimizing the variational objective

Variational bounds are typically optimized by gradient ascent or block coordinate ascent, iterating through the variational parameters and updating them until the relative increase in the lower bound is below a specified threshold. Traditionally this would require symbolic expansion of the ELBO $\mathcal{L}_\eta = \mathbb{E}_q [p(x, v, \mathbf{z}, \boldsymbol{\theta}, a, b) - q_\eta(x, v, \mathbf{z}, a, b)]$, so that the bound can be optimized with respect to the variational parameters η . This expectation cannot be analytically expanded with our model. One solution would be to approximate this bound. Especially when there are many variables, however, this approximation and the resulting optimization algorithm are complicated and prone to bugs.

Instead of expanding this bound symbolically, we update each parameter with stochastic optimization. We repeat these updates for each parameter until the parameters have converged. Upon convergence, we use the variational means \tilde{x} and \tilde{z} to inspect lawmakers' issues and bill parameters \tilde{a} and \tilde{b} to inspect items of legislation.

Without loss of generality, we describe how to perform the m th update on the variational parameter \tilde{x} , assuming that we have the most-recent estimates of the variational parameters \tilde{x}_{n-1} , \tilde{z}_{n-1} , \tilde{a}_{n-1} , and \tilde{b}_{n-1} . To motivate the inference algorithm, we first approximate the ELBO \mathcal{L} with a Taylor approximation, which we optimize. At the optimum, the Taylor approximation is equal to the ELBO.

Writing the variational objective as $\mathcal{L}(\tilde{x}) = \text{KL}(q_{\tilde{x}}||p)$ for notational convenience (where all parameters in η except \tilde{x} are held fixed), we estimate the KL divergence as a function of \tilde{x} around our last estimate \tilde{x}_{m-1} with its Taylor approximation

$$\mathcal{L}(\tilde{x}) \approx \mathcal{L}(\tilde{x}_{n-1}) + \left(\frac{\partial \mathcal{L}}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} \right)^T \Delta \tilde{x} + \frac{1}{2} \Delta \tilde{x}^T \left(\frac{\partial^2 \mathcal{L}}{\partial \tilde{x} \partial \tilde{x}^T} \Big|_{\tilde{x}_{n-1}} \right) \Delta \tilde{x}, \quad (11)$$

where $\Delta \tilde{x} = \tilde{x} - \tilde{x}_{n-1}$. Once we have estimated the Taylor coefficients (as described in the next paragraph), we can perform the update

$$\tilde{x}_n \leftarrow \tilde{x}_{n-1} - \left(\frac{\partial^2 \mathcal{L}}{\partial \tilde{x} \partial \tilde{x}^T} \Big|_{\tilde{x}_{n-1}} \right)^{-1} \left(\frac{\partial \mathcal{L}}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} \right). \quad (12)$$

We approximated the Taylor coefficients with Monte Carlo sampling. Without loss of generality, we will illustrate this approximation with the variational parameter \tilde{x} . Let \tilde{x}_{n-1} be the current estimates of the variational mean, $q_{\tilde{x}_{n-1}}(x, \mathbf{z}, a, b)$ be the variational posterior at this mean, and $\mathcal{L}_{\tilde{x}_{n-1}}$ be the ELBO at this mean. We then approximate the gradient with

Monte Carlo samples as

$$\begin{aligned}
\frac{\partial \mathcal{L}_{\tilde{x}}}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} &= \frac{\partial}{\partial \tilde{x}} \int q_{\tilde{x}}(x, \mathbf{z}, a, b) (\log p(x, \mathbf{z}, a, b, v) - \log q_{\tilde{x}}(x, \mathbf{z}, a, b)) dx dz da db \Big|_{\tilde{x}_{n-1}} \quad (13) \\
&= \int \frac{\partial}{\partial \tilde{x}} (q_{\tilde{x}}(x) (\log p(x, \mathbf{z}, a, b, v) - \log q_{\tilde{x}}(x, \mathbf{z}, a, b))) d\tilde{x} \Big|_{\tilde{x}_{n-1}} \\
&= \int q_{\tilde{x}}(x) \frac{\partial \log q_{\tilde{x}}(x)}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} (\log p(x, \mathbf{z}, a, b, v) - \log q_{\tilde{x}}(x, \mathbf{z}, a, b)) d\tilde{x} \\
&\approx \frac{1}{M} \sum_{m=1}^M \left(\left(\frac{\partial \log q_{\tilde{x}}(x_{n-1,m}, z_{n-1,m}, a_{n-1,m}, b_{n-1,m})}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} \right) \right. \\
&\quad \times \left(\log p(x_{n-1,m}, z_{n-1,m}, a_{n-1,m}, b_{n-1,m}, v) \right. \\
&\quad \left. \left. - C - \log q_{\tilde{x}_{n-1}}(x_{n-1,m}, z_{n-1,m}, a_{n-1,m}, b_{n-1,m}) \right) \right),
\end{aligned}$$

where we have taken the gradient through the integral using Leibniz's rule and used M samples from the current estimate of the variational posterior. The second Taylor coefficient is straightforward to derive with similar algebra:

$$\begin{aligned}
\frac{\partial^2 \mathcal{L}_{\tilde{x}}}{\partial \tilde{x} \partial \tilde{x}^T} \Big|_{\tilde{x}_{n-1}} &\approx \frac{1}{M} \sum_{m=1}^M \left(\left(\frac{\partial \log q_{n-1}(x_{n-1,m})}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} \right) \left(\frac{\partial \log q_{m-1}(x_{n-1,m})}{\partial \tilde{x}} \Big|_{\tilde{x}_{n-1}} \right)^T \quad (14) \\
&\quad \times \left(\log p(x_{n-1,m}, \mathbf{z}_{n-1,m}, a_{n-1,m}, a_{n-1,m}, v) \right. \\
&\quad \left. - C - \log q_{\tilde{x}_{n-1}}(x_{n-1,m}, z_{n-1,m}, a_{n-1,m}, b_{n-1,m}) - 1 \right), \\
&\quad + \left(\left(\frac{\partial^2 \log q_{n-1}(x_{n-1,m})}{\partial \tilde{x} \partial \tilde{x}^T} \Big|_{\tilde{x}_{n-1}} \right) \right. \\
&\quad \times \left(\log p(x_{n-1,m}, \mathbf{z}_{n-1,m}, a_{n-1,m}, b_{n-1,m}, v) \right. \\
&\quad \left. \left. - C - \log q_{\tilde{x}_{n-1}}(x_{n-1,m}, z_{n-1,m}, a_{n-1,m}, b_{n-1,m}) \right) \right) \Bigg),
\end{aligned}$$

where we increase M as the model converges. Note that C is a free parameter that we can set without changing the final solution. We set C to the average of $\log p(x_{n-1,m} | \dots) - \log q_{n-1}(x_{n-1,m})$ across the set of M samples.

Quasi-Monte Carlo samples. Instead of taking *iid* samples from the variational distribution q_{M-1} , we used quasi-Monte Carlo sampling Niederreiter (1992). By taking non-*iid* samples from q_{m-1} , we are able to decrease the variance around estimates of the Taylor coefficients. To select these samples, we took M equally-spaced points from the unit interval, passed these through the inverse CDF of the variational Gaussian $q_{n-1}(x)$, and used the resulting values as samples. Note that these samples produce a biased estimate of Equation 11. This bias decreases as $N \rightarrow \infty$.

When we update the variational parameter \tilde{x}_u , we do not need to sample all random variables, but we do need a sample of all random variables in the Markov blanket of x_u . The cumulative distribution is of course ill-defined for multivariate distributions, so the method in

the last paragraph is not quite enough. For a quasi-Monte Carlo sample from the multivariate distribution of x_u 's Markov blanket, we selected M samples using the method in the previous paragraph for each marginal in the Markov blanket of x_u . We then permuted each variable's samples and combined them for M multivariate samples $\{x_{n-1,m}, \dots, b_{n-1,m}\}_m$ from the current estimate q_{n-1} of the variational distribution.

Estimating $\frac{\partial \log q_m}{\partial x}$. We estimate the gradients of $\log q$ above based on the distribution of the variational marginals. We have defined the variational distribution to be factorized Gaussians, so these take the form

$$\begin{aligned} \left. \frac{\partial \log q_{n-1}(x_{n-1,m})}{\partial \tilde{x}} \right|_{\tilde{x}_{n-1}} &= \frac{x_{n-1,m} - \tilde{x}_{n-1}}{\sigma_x^2} \\ \left. \frac{\partial^2 \log q_{n-1}(x_{n-1,m})}{\partial \tilde{x}^2} \right|_{\tilde{x}_{n-1}} &= -\frac{1}{\sigma_x^2}. \end{aligned} \tag{15}$$

We finally address practical details of implementing issue-adjusted ideal points.

A.2. Algorithmic parameters.

We fixed the variance σ_x^2 to $\exp(-5)$. Allowing σ_x to vary freely provides a better variational bound at the expense of accuracy. This happens because the issue-adjusting model would sometimes fit poor means to some parameters when the posterior variance was large: there is little penalty for this when the variance is large. Low posterior variance σ_x^2 is similar to a non-sparse MaP estimate.

These updates were repeated until the exponential moving average $\Delta_{\text{est},i} \leftarrow 0.8\Delta_{\text{est},i-1} + 0.2\Delta_{\text{obs},i}$ of the change in KL divergence dropped below one and the number N of samples passed 500. If the moving average dropped below one and $N < 500$, we doubled the number of samples.

When performing the second-order updates described in Section 3, we skipped variable updates when the estimated Hessian was not positive definite (this disappeared when sample sizes grew large enough). We also limited step sizes to 0.1 (another possible reason for smaller coefficients).

A.3. Hyperparameter settings

The most obvious parameter in the issue voting model is the regularization term λ .

The main parameter in the issue-adjusted model is the regularization λ , which is shared for all issue adjustments. The Bayesian treatment described in the Inference section of this paper demonstrated considerable robustness to overfitting at the expense of precision. With $\lambda = 0.001$, for example, issue adjustments z_{uk} remained on the order of single digits, while experiments with MaP estimates yielded adjustment estimates over 100.

We report the effect of different λ by fitting the issue-adjusted model to the 109th Congress (1999-2000) of the House and Senate for a range $\lambda = 0.0001, \dots, 1000$ of regularizations. We performed 6-fold cross-validation, holding out one sixth of votes in each fold, and

109th U.S. Senate sensitivity to λ								
Model	Lambda							
	1e-4	1e-3	1e-2	1e-1	1	10	100	1000
Ideal	-0.188	-0.189	-0.189	-0.189	-0.189	-0.190	-0.189	-0.189
Issue (LDA)	-0.191	-0.191	-0.188	-0.186	-0.188	-0.189	-0.189	0.198
Permuted Issue	-0.242	-0.245	-0.231	-0.221	-0.204	-0.208	-0.208	-0.208

109th U.S. House sensitivity to λ								
Model	Lambda							
	1e-4	1e-3	1e-2	1e-1	1	10	100	1000
Ideal	-0.119	-0.119	-0.119	-0.119	-0.120	-0.119	-0.119	-0.119
Issue (LDA)	-0.159	-0.159	-0.158	-0.139	-0.118	-0.119	-0.119	0.119
Permuted Issue	-0.191	-0.192	-0.189	-0.161	-0.122	-0.120	-0.120	-0.120

Figure A.1: Average log-likelihood of heldout votes by regularization λ . Log-likelihood was averaged across folds using six-fold cross validation for Congress 109 (2005-2006). The variational distribution represented votes with higher heldout log-likelihood than traditional ideal points for $1 \leq \lambda \leq 10$. In a model fit with permuted issue labels (Perm. Issue), heldout likelihood of votes was worse than traditional ideal points for all regularizations λ .

calculated average log-likelihood $\sum_{v_{ud} \in V_{\text{heldout}}} \log p(v_{ud} | \tilde{x}_u, \tilde{z}_u, \tilde{a}_d, \tilde{b}_d)$ for votes V_{heldout} in the heldout set. Following the algorithm described in Section 3, we began with $M = 21$ samples to estimate the approximate gradient (Equation 11) and scaled it by 1.2 each time the Elbo dropped below a threshold, until it was 500. We also fixed variance $\sigma_x^2, \sigma_z^2, \sigma_a^2, \sigma_b^2 = \exp(-5)$. We summarize these results in Table A.1.

The variational implementation generalized well for the entire range, representing votes best in the range $1 \leq \lambda \leq 10$. Log-likelihood dropped modestly for $\lambda < 1$. In the worst case, log-likelihood was -0.159 in the House (this corresponds with 96% heldout accuracy) and -0.242 in the Senate (93% heldout accuracy).

We recommend a modest value of $\lambda = 1$, and no greater than $\lambda = 10$. At this value, the model outperforms ideal points in validation experiments on both the House and Senate, for a range of Congresses.

APPENDIX B. CORPUS PREPARATION

B.1. Issue labels

In the empirical analysis, we used issue labels obtained from the Congressional Research Service. There were 5,861 labels, ranging from *World Wide Web* to *Age*. We only used issue labels which were applied to at least twenty five bills in the 12 years under consideration. This filter resulted in seventy-four labels which correspond fairly well to political issues. These issues, and the number of documents each label was applied to, is given in Table B.2.

B.2. Vocabulary selection

In this section we provide further details of vocabulary selection.

Figure B.2: Issue labels and the number of documents with each label (as assigned by the Congressional Research Service) for Congresses 106 to 111 (1999 to 2010).

Issue label	No. bills	Issue label	No. bills
Women	25	Europe	44
Military history	25	Military personnel and dependents	44
Civil rights	25	Taxation	47
Government buildings; facilities; and property	26	Government operations and politics	47
Terrorism	26	Postal facilities	47
Energy	26	Medicine	48
Crime and law enforcement	27	Transportation	48
Congressional sessions	27	Emergency management	48
East Asia	28	Sports	52
Appropriations	28	Families	53
Business	29	Medical care	54
Congressional reporting requirements	30	Athletes	56
Congressional oversight	30	Land transfers	56
Special weeks	31	Armed forces and national security	56
Social services	31	Natural resources	58
Health	33	Law	60
Special days	33	History	61
California	33	Names	62
Social work; volunteer service; charitable organizations	33	Criminal justice	62
State and local government	34	Communications	65
Civil liberties	35	Public lands	68
Government information and archives	35	Legislative rules and procedure	69
Presidents	35	Elementary and secondary education	74
Government employees	35	Anniversaries	82
Executive departments	35	Armed forces	83
Racial and ethnic relations	36	Defense policy	92
Sports and recreation	36	Higher education	103
Labor	36	Foreign policy	104
Special months	39	International affairs	105
Children	40	Budgets	112
Veterans	40	Education	122
Human rights	41	House of Representatives	142
Finance	41	Commemorative events and holidays	195
Religion	42	House rules and procedure	329
Politics and government	43	Commemorations	400
Minorities	44	Congressional tributes	541
Public lands and natural resources	44	Congress	693

We first converted the words in each bill to a canonical form using the *Tree-tagger* part-of-speech tagger (Schmid 1994). Next we counted all phrases with one to five words. From these, we immediately eliminated phrases which occurred in more than 10% of bills or in fewer than 4 bills, or which occurred as fewer than 0.001% of all phrases. This resulted in a list of 40603 phrases (called n -grams in natural language processing).

We then used a set of features characterizing each word to classify whether it was good or bad to use in the vocabulary. Some of these features were based on corpus statistics, such as the number of bills in which a word appeared. Other features used external data sources, including whether, and how frequently, a word appeared as link text in a Wikipedia article. We estimated weights for these features using a logistic regression classifier. To train this classifier, we used a manually curated list of 458 “bad” phrases which were semantically awkward or meaningless (such as *the follow bill*, *and sec amend*, *to a study*, and *pr*). These were selected as negative examples in a L_2 -penalized logistic regression, while the remaining words we considered “good” words. We illustrate weights for these features in Figure B.3. The best 5,000 phrases under this model were used in the vocabulary.

Coefficient	Summary	Weight
$\log(\text{count} + 1)$	Frequency of phrase in corpus	-0.018
$\log(\text{number.docs} + 1)$	Number of bills containing phrase	0.793
anchortext.presentTRUE	Occurs as anchortext in Wikipedia	1.730
anchortext	Frequency of appearing as anchortext in Wikipedia	1.752
frequency.sum.div.number.docs	Frequency divided by number of bills	-0.007
doc.sq	Number of bills containing phrase, squared	-0.294
has.secTRUE	Contains the phrase <i>sec</i>	-0.469
has.parTRUE	Contains the phrase <i>paragra</i>	-0.375
has.strikTRUE	Contains the phrase <i>strik</i>	-0.937
has.amendTRUE	Contains the phrase <i>amend</i>	-0.484
has.insTRUE	Contains the phrase <i>insert</i>	-0.727
has.clauseTRUE	Contains the phrase <i>clause</i>	-0.268
has.provisionTRUE	Contains the phrase <i>provision</i>	-0.432
has.titleTRUE	Contains the phrase <i>title</i>	-0.841
test.pos	$\ln(\max(-\text{test}, 0) + 1)$	0.091
test.zeroTRUE	1 if test = 0	-1.623
test.neg	$\ln(\max(\text{test}, 0) + 1)$	0.060
number.terms1	Number of terms in phrase is 1	-1.623
number.terms2	Number of terms in phrase is 2	2.241
number.terms3	Number of terms in phrase is 3	0.315
number.terms4	Number of terms in phrase is 4	-0.478
number.terms5	Number of terms in phrase is 5	-0.454
$\log(\text{number.docs} + 1) * \text{anchortext}$	$\ln(\text{Number of bills containing phrase})$ $\times 1_{\{\text{Appears in Wikipedia anchortext}\}}$	-0.118
$\log(\text{count} + 1) * \log(\text{number.docs} + 1)$	$\ln(\text{Number of bills containing phrase} + 1)$ $\times \ln(\text{Frequency of phrase in corpus} + 1)$	0.246

Figure B.3: Features and coefficients used for predicting “good” phrases. Below, test is a test statistic which measures deviation from a model assuming that words appear independently; large values indicate that they occur more often than expected by chance. We define it as

$$\text{test} = \frac{\text{Observed count} - \text{Expected count}}{\sqrt{\text{Expected count under a language model assuming independence}}}.$$