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Derivation of a new banded waveguide model topology for sound synthesis

Alex M. Fink[a] **and Andreas S. Spanias**
School of Arts, Media and Engineering, SenSIP Center, School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, Arizona 85287
alex.fink@asu.edu, spanias@asu.edu

Perry R. Cook
Departments of Computer Science and Music, Princeton University, Princeton, New Jersey 08544
prc@princeton.edu

Abstract: Banded waveguide (BWG) synthesis is an efficient method for real-time physical modeling of dispersive and multidimensional sounding objects, affording simulation of complex interactions, such as bowing. Current implementations, however, use nonphysical design parameters and produce a range of outputs that do not match equivalently designed modal and digital waveguide (DWG) models. This letter proposes a new topology for implementing BWG models without arbitrary parameters. The impulse response of the proposed model is identical to that of equivalent Karplus-Strong type and lumped modal models. Test of a nonlinear bi-directional bowed-string model demonstrates improved attack characteristics relative to prior BWG models.

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1. Introduction

In applications such as gaming and musical performance, there is a need to produce the sounds of interactions with a diversity of simulated objects in real-time. Sample playback-based methods maintain a high level of realism, but at the expense of requiring many recordings to capture a range of objects and interactions.[1] Alternatively, physical modeling methods, in simulating the vibrations of various media, afford variation of the perceived properties of simulated objects and interactions. While several techniques are able to accurately simulate vibrating objects,[1] real-time realization requires methods that are computationally efficient.

Digital waveguide (DWG) modeling and lumped modal modeling are two examples of commonly used efficient physical modeling methods. DWG modeling encompasses a variety of physical modeling techniques, generally simulating wave propagation via digital delays.[1–3] DWG models typically maintain efficiency by lumping the effects of loss, dispersion, and reflection, simulated using digital filters.[1] Complex, nonlinear interactions, such as bowing, may be readily simulated in DWG models.[1] Modeling of stiff (i.e., highly dispersive) or multidimensional objects using DWG methods, though, can be computationally prohibitive.[4,5] The use of allpass filters for frequency-dependent delay enables efficient simulation of weak stiffness in DWG models,[1–3,5] but extension to very dispersive systems (such as stiff bars) requires high-order filters which can be difficult to design.[6]

Modal modeling includes techniques that decompose vibrations into components called *modes*.[7,8] Each mode represents the vibrations of an object at a certain

[a]Author to whom correspondence should be addressed.

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frequency, decaying at some rate. Modal models may also consider mode shapes, describing the local magnitude of each mode along a vibrating object; however, commonly used *lumped* modal models disregard shape. Lumped models may be implemented via resonant second-order digital filters, representing interaction and observation of an object at specified points.[8] Such lumped models, however, may incur significant computational cost when physically simulating complex interactions, particularly if the points of interaction or observation are dynamically varied.[4,5,9]

To resolve the difficulty in simulating complex, nonlinear interactions with stiff and multidimensional objects, banded waveguide (BWG) models have been proposed and implemented for sound synthesis.[4,5,10] Viewed as a hybrid of modal and DWG models, BWG models separately simulate propagation at the resonant frequencies of an object via band-limited waveguides, modeling the closed wavetrains that give rise to resonance in a sounding object.[4,5] Band-limitation in most previous BWG models[10] uses constant peak-gain resonant biquad filters[11] for their computational simplicity. Such implementations, however, result in outputs (due to both linear and nonlinear excitations) that may significantly differ from DWG and modal models designed to the same specifications, as a result of arbitrary, nonphysical parameters.[12]

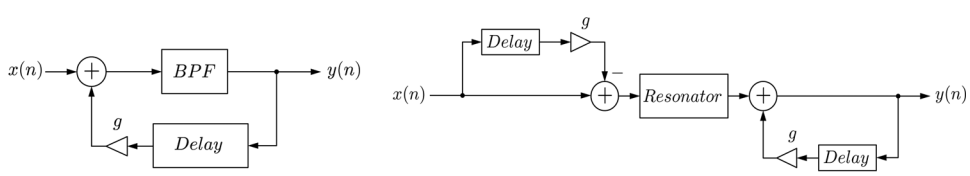
In this letter, we propose a new topology (with no arbitrary parameters) for BWG models, designed via *factorization* of DWG models, that maintains the order of computational complexity of previous topologies. The impulse response of the proposed BWG model is identical to that of equivalently designed filtered-delay-loops and lumped modal models. Preliminary results also demonstrate some improvement for nonlinear interactions when compared to DWG models.

The rest of this letter is organized as follows. Previous forms of BWG models are discussed in Sec. 2. Section 3 introduces the proposed BWG topology and its derivation, properties, and simulation results. Conclusions are drawn in Sec. 4.

2. Previous BWG models

Various topologies of BWG models have been put forward,[4,5,9,12,13] with many applications[4,9,10,14] using that shown in Fig. 1(a). Here, the bandpass filter (BPF) is a second-order constant peak-gain filter.[11] With knowledge of an object's modal data, the design of such a BWG is straightforward. The gain g_m is set to the round-trip loss of propagation (related to the mode decay and propagation length). The delay length is set to correspond to the round-trip propagation distance at the associated modal frequency. The BPF is centered about the modal frequency, with bandwidth arbitrarily chosen to sufficiently reject adjacent resonant frequencies due to the feedback path.[4,5] Note that this is a band-limited equivalent of a Karplus-Strong[1–3] or single delay-loop (SDL) model[15] without modeling of excitation position, bridge coupling, etc. Bi-directional and more complex models may be implemented as discussed in Sec. 3.3 and elsewhere.[4,5] For simplicity, except where noted, we primarily consider "filtered-delay-loop"[1] or "string loop"[15] structures with propagation and other elements lumped in a single feedback path; we refer to such structures as SDL models.

Designed by modal data, the ideal impulse response of such an individual BWG is an exponentially decaying sinusoid. However, using a second-order constant peak-gain filter as the BPF in a BWG, as in Fig. 1(a), the impulse response of this



(a) Common, previously used topology. (b) The proposed topology.

Fig. 1. Banded waveguide models.

BWG realization will not be an exponentially decaying sinusoid. Varying the bandwidth of the BPF produces differing results, with the output never matching the desired modal response. As bandwidth is decreased, the decay of the BWG's output is increased as the BPF's own decay time is lengthened, without regard to the BWG's desired decay due to round-trip attenuation. Increasing the bandwidth can result in insufficient suppression of the feedback path's other harmonics, producing a waveform with visible artifacts. This differing response has been noted[9] and used by some for timbral effect,[14] but such a difference is generally not desirable in physical modeling.

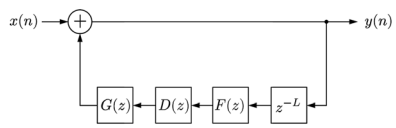
Subjected to nonlinear excitations (such as bowing), previous BWG models further exhibit differences from ideal responses. Previous work[12] has compared a DWG bowed-string model to an equivalently designed BWG model, with the BWG model's output showing a significantly slower attack. Perfect reconstruction filterbanks have been proposed for band-limiting in BWG models;[12] however, the filterbank design is prohibitively complex for real-time simulation.

3. Proposed BWG topology

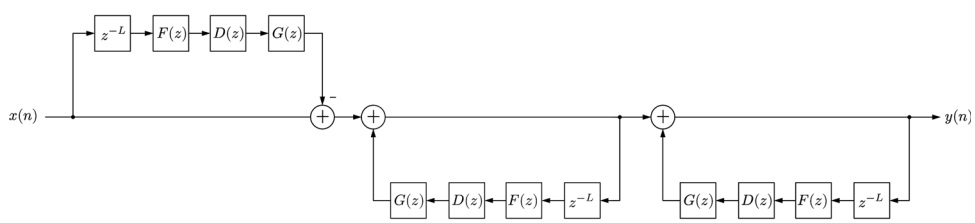
The proposed BWG topology is shown in Fig. 1(b) for a single BWG. Here, the BPF is implemented by a feedforward comb filter cascaded with a second-order resonant filter, outside of the feedback path. The delay lines (which may require a fractional delay) of the feedforward and feedback components should be of identical length. Further details are described in the model derivation which follows.

3.1 Derivation by factorization of DWG models

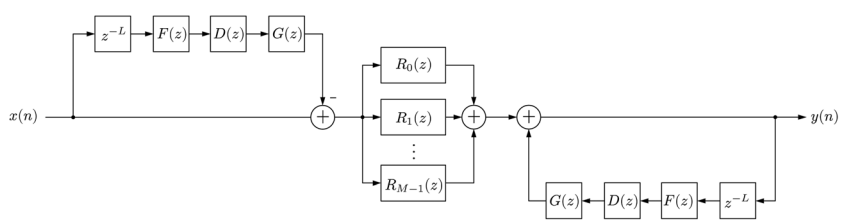
To derive the proposed BWG topology, with the ideal of producing an output that matches a DWG-type or modal model, first consider a generic SDL model of the form in Fig. 2(a), where losses, dispersion, fractional delay, and integer delay are represented by the elements labeled $G(z)$, $D(z)$, $F(z)$, and z^{-L}, respectively. Introduction of an identical



(a) A generic SDL model.



(b) A SDL-equivalent model.



(c) A SDL-equivalent model using a parallel implementation of a feedback loop.

Fig. 2. Derivation steps.

model and its inverse in series, as in Fig. 2(b) will result in a functionally identical model. This model may be further modified by replacing one of the feedback loops by a parallel implementation of the partial fraction expansion of its transfer function, as in Fig. 2(c). The feedforward and feedback loops on the left and right side, respectively, of Fig. 2(c) may be duplicated and distributed with identical copies being placed directly before and after each resonant filter, $R_m(z)$. For computational simplicity, since each filter $R_m(z)$ is bandpass in nature, with pole angles at $\pm\Theta_m$, we choose to approximate the feedforward and feedback paths by a delay and scalar gain. [The delay is equal to the phase delay of $H(z) = G(z)D(z)F(z)z^{-L}$ at $z = e^{j\Theta m}$; the gain is $|H(e^{j\Theta m})|$.] This results in a model consisting of parallel BWGs of the form of Fig. 1(b), with all outputs summed and all BWGs receiving the same input.

3.2 Synthesis and complexity

Following this derivation, a BWG model implemented in this manner may be made to have an impulse response that exactly matches an equivalent SDL or lumped modal model. Each individual BWG, even with the approximations used, produces an output with the desired response. With the same approximation carried out on the feedback and feedforward filters, they are still inverses of one another. Thus, the impulse response of each BWG is the same as that of the resonant filters, an exponentially decaying sinusoid with the desired modal parameters.

The computational complexity of this proposed topology is $O(M)$, where M is the number of modes. This is the same order of complexity as lumped modal models and previous BWG topologies,[4,5] though the actual number of computations is slightly increased. Thus, if one only desires to simulate linear interactions at a point, identical results may be achieved more efficiently via a lumped modal model; however, BWG models are additionally useful in simulating nonlinear interactions, such as bowing, discussed below.

3.3 Improved attack with nonlinear excitations

Subjected to nonlinear excitations that depend on the state of the system (such as bowing), the proposed model is not guaranteed to produce results identical to similar DWG models. In bowing simulations, this is due to a nonlinear bow model that connects all of the feedback paths (tapped immediately after delay and attenuation) to the system input, as well as the approximation of the feedback and feedforward filters by only a delay and gain. (Note that these approximations are essential to maintain computational simplicity.) Simulation of highly dispersive and multidimensional objects under such excitations, however, is the impetus for BWG models.[4,5]

To examine the behavior of the proposed topology subject to one such nonlinear excitation, we have simulated a bowed string tuned to 441 Hz using a bi-directional "digital waveguide bowed-string" model[1] and bi-directional BWG models[4,5] using the proposed BPFs and a "classical" implementation[4,5] (with varying BPF bandwidths). This model is of the transverse vibrations in one dimension of a string with no stiffness and lumped losses, chosen as a test case for its simplicity. The BWG models are implemented by parallel configurations of the DWG model, with appropriate BPFs, and the loss filters replaced by scalar gains. The BPFs of the proposed model are derived using the procedure of Sec. 31 with only the "string loop"[15] of the DWG model. (Note that this will cause the impulse response of the BWG model to vary from the ideal.) The bow model uses a table[1] and receives, as input, the sum of the waves in every BWG traveling into the bow. For brevity, a single "classical" model using only biquad filters for band-limitation is considered here. An implementation with BPFs set to a bandwidth of 100 Hz was used, chosen by the authors, among other simulations, as producing the best sounding steady-state output. These simulations of a DWG model, a classical BWG model, and the proposed model may be heard in Mm. 1, 2, and 3, respectively.

Mm. 1. Bowed string simulation using a DWG model. This is a file of type "wav" (532 KB).

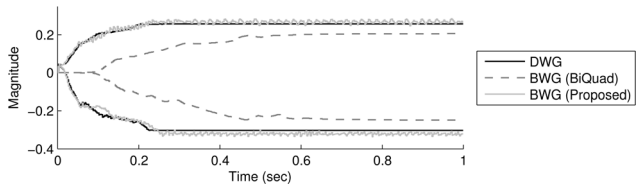


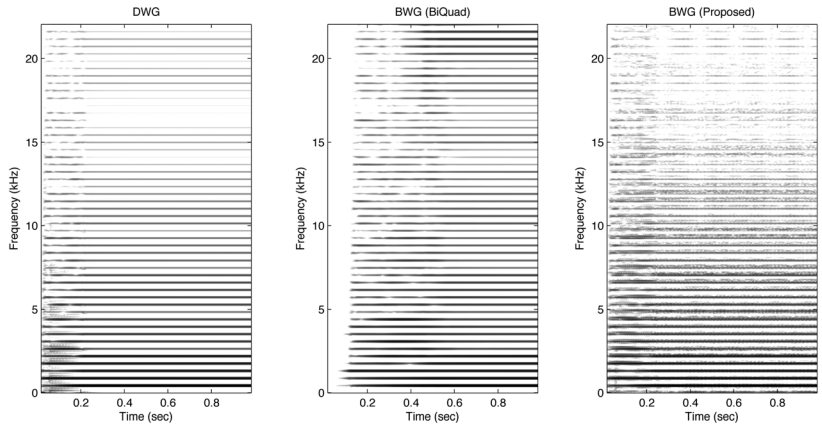
Fig. 3. Amplitude envelopes of the output waveforms. Note the improved attack characteristics using the proposed topology.

Mm. 2. Bowed string simulation using a classical BWG model. This is a file of type "wav" (532 KB).

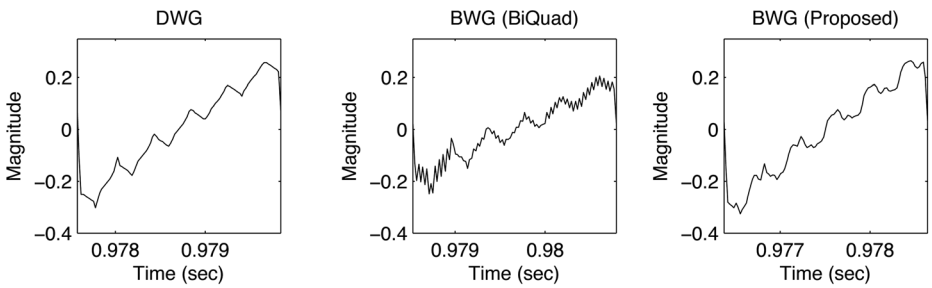
Mm. 3. Bowed string simulation using the proposed BWG model. This is a file of type "wav" (532 KB).

The amplitude envelopes of the output waveforms of all tested models are shown in Fig. 3. The proposed model clearly shows an attack similar to that of the DWG model, but it also has a tremolo-like effect on the steady-state sustained oscillations. The biquad model exhibits a much slower attack. Increasing the bandwidth of the BPFs can reduce the attack time, but this can greatly affect the output's timbre and does not retain the shape of the attack.

The spectrograms of the simulated outputs appear in Fig. 4(a). Note that the proposed model exhibits more energy between the harmonics, as compared to the other



(a) Simulation spectrograms. Black represents 0 dB; white represents -60 dB and below.



(b) Example steady-state pseudo-periods.

Fig. 4. Simulation results. (a) Note the vibrato-like effect using the proposed topology, as well as the improved attack characteristics. (b) Note the high-frequency vibrations using a previous model, as compared to the smoother waveform from the proposed topology.

models. This produces sustained oscillations with a somewhat "harsher" timbre, as compared to the biquad model. Additionally, the tremolo-like effect appears most prominent with the higher harmonics. Example pseudo-periods of the steady-state oscillations of each model are shown in Fig. 4(b) for comparison.

4. Conclusion

A new topology for implementing BWG models has been presented. The proposed model may be designed to produce outputs identical to that of equivalent SDL and lumped modal models when subjected to linear excitations. A test of a bowed-string model has shown improvement in the simulation's attack characteristics using the proposed BWG implementation, as compared to previous models. Further evaluation is needed to quantify any improvements of the proposed topology and assess the applicability to other models (particularly stiff and multidimensional objects) and excitations. Additionally, alterations of the proposed topology, including approximating steps, could be considered.

Acknowledgments

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References and links

[1]J. O. Smith III, *Physical Audio Signal Processing* (W3K Publishing, 2010).
[2]D. A. Jaffe and J. O. Smith III, "Extensions of the Karplus-Strong plucked-string algorithm," Comput. Music J. **7**(2), 56–69 (1983).
[3]K. Karplus and A. Strong, "Digital synthesis of plucked-string and drum timbres," Comput. Music J. **7**(2), 43–55 (1983).
[4]G. Essl, "Physical wave propagation modeling for real-time synthesis of natural sounds," Ph.D. dissertation, Princeton University, Princeton, NJ, 2002.
[5]G. Essl, S. Serafin, P. R. Cook, and J. O. Smith III, "Theory of banded waveguides," Comput. Music J. **28**(1), 37–50 (2004).
[6]D. Rocchesso and F. Scalcon, "Bandwidth of perceived inharmonicity for physical modeling of dispersive strings," IEEE Trans. Speech Audio Processing **7**(5), 597–601 (1999).
[7]J.-M. Adrien, "The missing link: Modal synthesis," in *Representations of Musical Signals*, edited by G. D. Poli, A. Piccialli, and C. Roads (MIT Press, Cambridge, MA, 1991), pp. 269–298.
[8]K. van den Doel and D. K. Pai, "Modal synthesis for vibrating objects," in *Audio Anecdotes III: Tools, Tips, and Techniques for Digital Audio* (A. K. Peters, Boca Raton, FL, 2006), pp. 99–120.
[9]S. Serafin, "The sound of friction: Real-time models, playability and musical applications," Ph.D. dissertation, Stanford University, Stanford, CA, 2004.
[10]G. Essl, S. Serafin, P. R. Cook, and J. O. Smith III, "Musical applications of banded waveguides," Comput. Music J. **28**(1), 51–63 (2004).
[11]K. Steiglitz, *A DSP Primer* (Prentice-Hall, Englewood Cliffs, NJ, 1996).
[12]A. Fink, H. Thornburg, and A. Spanias, "An individually tunable perfect reconstruction filterbank for banded waveguide synthesis," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* (Dallas, Texas, 2010), pp. 37–40.
[13]U. Uzmen, "Çizim tabletleri İçin fiziksel modelleme temelli sentezleyici (Physical modeling-based synthesizer employing a drawing tablet)," Master's thesis, Yildiz Teknik Üniversitesi, Istanbul, Turkey, 2008.
[14]J. E. Bower, S. Lindroth, J. Burchett, S. D. Feller, D. J. Brady, and R. Brady, "Sound-Sense: Sonifying pyroelectric sensor data for an interactive media event," in *Proceedings of the International Conference on Auditory Display* (July 2005), pp. 113–120.
[15]M. Karjalainen, V. Välimäki, and T. Tolonen, "Plucked-string models: From the Karplus-Strong algorithm to digital waveguides and beyond," Comput. Music J. **22**(3), 17–32 (1998).