JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 39, No. 4, April 2016

Engineering Notes

Minimizing Proton Displacement Damage Dose During Electric Orbit Raising of Satellites

Atri Dutta,* N. Jeremy Kasdin,† and Edgar Choueiri‡
Princeton University, Princeton, New Jersey 08544
and
Philippe Francken§
SES S.A., 6815 Betzdorf, Luxembourg

DOI: 10.2514/1.G000503

Nomenclature

a, b	=	parameters describing the analytical radiation flux model, varied units
B_r, B_θ, B_ϕ	=	components of the geomagnetic flux density in spherical reference frame, $N/A \cdot m$
B_0	=	equatorial geomagnetic flux density, $N/A \cdot m$
D_d	=	displacement damage dose, MeV/g
E	=	energy level of protons, MeV
E_u	=	upper bound of the energy level of protons, MeV
E_ℓ	=	lower bound of the energy level of protons, MeV
$\mathcal{E}$	=	discrete set of all energy levels of protons, MeV
$\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_\phi$	=	unit vectors for the spherical coordinate system
$\boldsymbol{f}$	=	vector corresponding to the right-hand side of the equations of motion, varied units
g	=	function of the energy level and flux of protons, $/(cm^2 \cdot MeV^2 \cdot s)$
L	=	distance of magnetic field line at the equator, nondimensional
M	=	magnetic dipole moment of the Earth, $A \cdot m^2$
m	=	mass of the satellite, kg
R	=	radius of the Earth, km
r	=	radial distance of a satellite from the center of the Earth, km
S_P	=	nonionizing energy loss due to protons, $MeV \cdot cm^2/g$
T	=	thrust magnitude, N
t_f	=	final time for the orbit-raising maneuver, days
u	=	radial velocity of the satellite, m/s
v	=	transverse velocity of the satellite, m/s
w	=	out-of-plane velocity component of the satellite, m/s
$\boldsymbol{x}$	=	state vector for a satellite, varied units
α	=	direction (in-plane) of the thrust vector of the satellite, deg
β	=	direction (out-of-plane) of the thrust vector of the satellite, deg
δE	=	change in energy level across a segment, days
δt	=	time duration corresponding to a trajectory segment, days
ζ	=	defect constraint corresponding to an equation of motion, varied units
θ	=	azimuthal angle corresponding to the position of a satellite, deg
$\mathcal{K}$	=	user-defined constant, nondimensional
λ	=	latitude of a location in space, deg
μ	=	gravitational parameter for the Earth, km^3/s^2
μ_0	=	magnetic permeability of free space, N/A^2
τ	=	timelike variable for the direct optimization scheme
ϕ	=	colatitude corresponding to the position of a satellite, deg
Ψ_P	=	flux of protons at a location in space, $/(cm^2 \cdot MeV \cdot s)$
$\Psi_{P'}$	=	derivative of proton flux with respect to energy level of protons, $/(cm^2 \cdot MeV \cdot s)$

Subscripts

i	=	indices for nodes for the energy grid
k	=	indices for nodes for the time grid

I. Introduction

ALTHOUGH electric propulsion systems have been used in deep space missions for performing interplanetary transfers, their use has primarily been restricted to station-keeping purposes for Earth-orbiting satellites. Geosynchronous equatorial orbit (GEO) satellites have mostly used chemical thrusters to transfer to GEO from an initial orbit into which it is launched by an appropriate launch vehicle. The use of electric thruster for raising the orbit of a GEO satellite is only a recent phenomenon; in March 2015, the first ever all-electric satellites incorporating the Boeing 702-SP architecture were launched using the Falcon 9 launcher of SpaceX. The use of electric propulsion systems during orbit raising, typically starting from a low Earth orbit or a geosynchronous transfer orbit, results in significant fuel savings owing to the superior propellant management of electric thrusters compared to their chemical counterparts. This reduction of fuel expenditure (that translates into the realization of a smaller and lighter satellite) comes at the cost of a higher transfer time to GEO because of the low thrust levels generated by electric thrusters. Apart from delaying the deployment of the satellite by months compared to chemical orbit raising, the sluggish transfer causes the satellite to spend a long time within the Van Allen radiation belts where impacting protons and electrons cause degradation of the solar arrays of the satellite [1–3]. To deploy a satellite with the same beginning-of-life power at GEO, the satellite needs to start the transfer with a heavier solar array: either by launching the satellite with a larger solar array or by providing sufficient shielding material to the solar arrays. In either case, there is an associated mass penalty that reduces the mass benefits that can be achieved by electric thrusters during the transfer. Furthermore, the long exposure to the Van Allen radiation may be hazardous for other onboard electronics. This Note deals with the problem of designing transfer trajectories to the GEO such that the satellite encounters minimum radiation damage along its path.

A number of studies have looked at the general problem of electric orbit raising and have captured the tradeoffs among transfer time, mass savings, and radiation exposure for a variety of mission

Received 10 January 2014; revision received 19 October 2015; accepted for publication 23 October 2015; published online 11 January 2016.

*Postdoctoral Research Associate, Department of Mechanical and Aerospace Engineering; currently Assistant Professor, Wichita State University, Aerospace Engineering; atri.dutta@wichita.edu. Member AIAA.

†Professor, Department of Mechanical and Aerospace Engineering; jkasdin@princeton.edu. Associate Fellow AIAA.

‡Professor, Department of Mechanical and Aerospace Engineering; choueiri@princeton.edu. Fellow AIAA.

§Vice President, Risk Management and Innovation, Betzdorf Castle; Philippe.Francken@ses.com.

scenarios [4–8]. However, all these studies have considered minimum-time trajectories to the GEO and then evaluated the extent of radiation damage for these trajectories [5,7–13]. Note that the damage caused to solar arrays by Van Allen radiation is dependent on a number of factors: fluence of impacting particles, energies of impacting particles, the type of solar cells used, and the material of shielding used, where the fluence of particles captures the important parameters radiation flux and transfer time. For a given type of solar cell and associated shielding, a minimum-time trajectory would correspond to minimum solar cell damage only if the radiation flux is uniform for all particle energies throughout the Van Allen belt. Obviously, this is not the case (see Fig. 1, which shows the proton flux over circular equatorial orbits of varying radius) because the radiation flux varies with altitude and latitude, and a minimum-time trajectory cannot be guaranteed to traverse the regions of lower radiation flux. In other words, a minimum-time trajectory does not necessarily lead to minimum radiation damage of the solar arrays.

Dutta et al. [14] present a preliminary mathematical framework that determines orbit-raising trajectories minimizing the total radiation fluence of all particles encountered during the transfer. The current Note improves upon that work in two important ways. First, the optimization solver is extended to minimize the displacement damage dose [15] of particles encountered during the transfer because it is a truer measure of the radiation damage for a given solar cell type and shielding material. In fact, minimizing total radiation fluence of all particles does not necessarily lead to minimum radiation damage because the extent of damage caused by particles depends on their respective energy levels. For details on the computation of solar array degradation using displacement damage dose method, see [15–17]. Recognizing the fact that protons are the most hazardous of the impacting particles in the Van Allen belt and are the primary cause of solar array degradation of satellites during the transfer, the proton displacement damage dose is specifically minimized in the current Note. Second, the analytic model for the Van Allen belt radiation flux [18] is used to make the optimization process more efficient than using lookup tables to compute the objective function as in [14]. The analytical models used in the numerical simulations of the Note are based on the AP-8 models for minimum solar cycle (AP8MIN) data; however, our methodology is extendible to incorporate newer AP-9 models of the radiation flux.

The primary contribution of this Note is the development of a novel mathematical framework to minimize the displacement damage dose due to protons encountered by a satellite during electric orbit raising to the GEO. Our methodology is based on a discretization of the satellite trajectory and control variables at different times (standard procedure for a direct optimization technique) as well as the discretization of the energy levels encountered during the orbit-raising maneuver. Hence, the energy level of encountered protons is an additional independent variable in our trajectory optimization problem. Note that in our treatment of the minimum-radiation problem, we ignore the impact of slowing down of the protons when they pass through the shielding material. This is equivalent to consideration of zero shielding thickness for the solar arrays. Hence, the radiation damage predicted by our current methodology is not accurate when compared to radiation analysis software such as SCREAM [17]. However, note that the consideration of proton energy levels as variables will allow the extension of the currently developed mathematical framework in future to consider the impact of shielding material and thereby determine more accurately the radiation damage for the low-thrust trajectories. The Note is organized as follows. Section II presents the mathematical formulation for the minimum-radiation-damage orbit-raising problem. Section III presents a direct optimization based methodology using discretized states, control, and energy levels to set up a nonlinear programming problem (NLP) that minimizes the displacement damage dose along the trajectory. Section IV illustrates by numerical examples that minimum-radiation trajectories are considerably different from the minimum-time trajectories, specifically when starting from nonequatorial orbits.

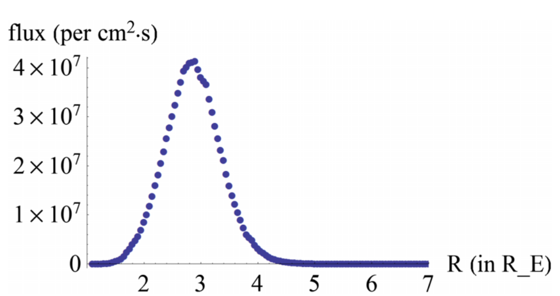

Fig. 1 Variation of radiation flux in Van Allen belt with radial distance (based on AP8MIN data).

II. Mathematical Formulation of Optimal Electric Orbit-Raising Problem

To develop the mathematical formulation of the electric orbit-raising problem, this section outlines the mathematical models of the satellite dynamics, geomagnetically trapped radiation, and solar array degradation during the orbit-raising maneuver. Next, this section describes the optimal control problem that needs to be solved to obtain the optimal trajectory of the satellite from the initial injection orbit to the GEO.

A. Electric Orbit-Raising Trajectory

Let us consider a satellite in an arbitrary initial orbit, which can be circular or elliptical and may have inclination with respect to the Earth's equatorial plane. Let $\boldsymbol{x}(t)$ denote the state of the satellite at any time t. The state vector is composed of the satellite's position vector, the inertial velocity vector, and the mass $m(t)$. At $t = 0$, the satellite is at its initial orbit into which it has been injected by a suitable launcher. The initial state of the satellite $\boldsymbol{x}(0)$ is known a priori and given by the position and velocity of the satellite in the injection orbit and the initial mass $m(0)$. It is assumed that the satellite uses its electric propulsion system to transfer from the starting injection orbit to the GEO. The GEO therefore provides constraints for the position and velocity at final time, where the final time t_f of the transfer is free. The dynamics of the satellite are described in a spherical reference frame. Figure 2 depicts this spherical reference frame and its orientation with respect to the Cartesian reference frame fixed to the Earth in terms of the azimuthal angle θ and the polar angle ϕ. Let us denote by u, v, and w the components of the inertial velocity along the unit vectors $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, and $\boldsymbol{e}_\phi$. Also, the state vector of the satellite is now given by $\boldsymbol{x}(t) \equiv (r(t), \theta(t), \phi(t), u(t), v(t), w(t), m(t))$. The equations of motion are summarized as

$$\dot{r} = u \tag{1a}$$

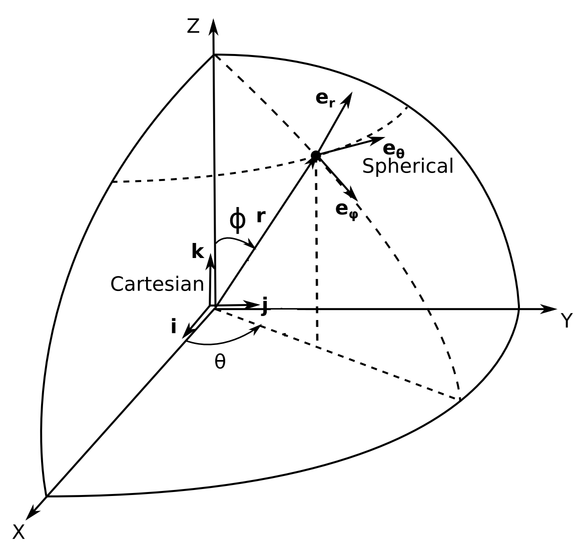

Fig. 2 Reference system to describe satellite motion and the geomagnetically trapped radiation flux.

$$\dot{\theta} = \frac{v}{r \sin \phi} \tag{1b}$$

$$\dot{\phi} = \frac{w}{r} \tag{1c}$$

$$\dot{u} = -\frac{\mu}{r^2} + \frac{v^2 + w^2}{r} + \frac{T}{m} \sin \alpha \cos \beta \tag{1d}$$

$$\dot{v} = -\frac{uv + vw \cot \phi}{r} + \frac{T}{m} \cos \alpha \cos \beta \tag{1e}$$

$$\dot{w} = \frac{-uw + v^2 \cot \phi}{r} + \frac{T}{m} \sin \beta \tag{1f}$$

$$\dot{m} = -\frac{T}{c} \tag{1g}$$

where T is the magnitude of the total thrust provided by the electric propulsion system; α is the angle that the projection of the thrust vector onto the $r-\theta$ plane makes with the $\boldsymbol{e}_r$ axis; β is the angle that the thrust vector makes with the $r-\theta$ plane; and c is the characteristic constant of the electric propulsion system and equals $g_0 I_{sp}$, with g_0 being the acceleration due to gravity at the surface of the Earth and I_{sp} being the specific impulse of the electric propulsion system.

B. Geomagnetically Trapped Radiation

To make the optimization framework efficient, the focus is on the use of analytic descriptions of the Earth's magnetic field and the radiation flux in the Van Allen belts. To this end, the geomagnetic field is modeled by considering a magnetic dipole centered with the Earth and with an axis parallel to the Earth's magnetic axis passing through its center of mass. The tilted dipole approximation results in the following components of the Earth's magnetic field in the spherical reference frame [19]:

$$B_r = -\frac{2B_0}{(r/R)^3} \cos \phi \tag{2a}$$

$$B_\phi = -\frac{B_0}{(r/R)^3} \sin \phi \tag{2b}$$

$$B_\theta = 0 \tag{2c}$$

where R is the radius of the Earth, and B_0 is the equatorial value of geomagnetic flux density given by

$$B_0 = \frac{\mu_0 M}{4\pi R^3} \tag{3}$$

with M being the magnetic dipole moment of the Earth and μ_0 being the magnetic permeability of free space. The magnitude of the geomagnetic field for a spin axis aligned dipole can therefore be written as

$$B = \frac{B_0}{(r/R)^3}(1 + 3\cos^2 \phi)^{1/2} \tag{4}$$

The geometry of the magnetic field lines can be expressed by the following relation:

$$r = LR \sin^2 \phi \tag{5}$$

where the parameter L represents the distance to the field line at $\phi = \pi/2$, that is, at the equator. Substituting Eq. (5) in Eq. (4), the relationship between B and L is given by

$$B = \frac{B_0}{(r/R)^3}\left(4 - \frac{3r}{LR}\right)^{1/2} \tag{6}$$

McIlwain's (B, L) coordinate system [20] is used to conveniently compute properties of the trapped charged particles because it is a means of converting the three-dimensional space into a two-dimensional space based on the fact that the dipole field is axially symmetric. The omnidirectional radiation flux (defined as the flux of all particles averaged over all directions) at any location owing to charged particles, namely protons and electrons, can be computed using McIlwain's coordinates for this location, and a suitable model of radiation flux, for instance, the NASA AP-8 and AE-8 models [21], provides data for the flux values for various energies in terms of the (B, L) coordinates, described in terms of tabulated values. However, usage of tabulated values does not lend itself well for optimization purposes. To this end, approximate analytical expressions of the Van Allen belt radiation flux are derived [18] in terms of the McIlwain coordinate L and the latitude λ, where

$$\lambda = \frac{\pi}{2} - \phi \tag{7}$$

Furthermore, it is known that the inner Van Allen belt that is composed of protons is significantly more hazardous compared to the outer belt that is composed of electrons only. Hence, in this study, only the radiation damage caused by protons is considered. The omnidirectional flux of protons of energy E can be expressed by the following analytical form [18]:

$$\Psi_P(L, \lambda, E) = a(L, E)e^{-b(L,E)\lambda^2} \tag{8}$$

where $a(L, E)$ and $b(L, E)$ are functions of L and E. Given this relationship, the rate of change of the radiation flux with respect to the energy levels (other parameters remaining constant) is given by

$$\Psi'_P(L, \lambda, E) = \frac{\partial \Psi}{\partial E} = \frac{\partial a}{\partial E} e^{-b(L,E)\lambda^2} - \Psi_P(L, \lambda, E)\frac{\partial b}{\partial E}\lambda^2 \tag{9}$$

Note that there is a one-to-one relationship between the spherical coordinates and McIlwain's coordinates, and hence $\Psi(E, \boldsymbol{r})$ is written instead of $\Psi(L, \lambda, E)$, where λ is the latitude at any place and is related to the colatitude by $\lambda + \phi = \pi/2$. In other words, it is considered that

$$\Psi_P(E, \boldsymbol{r}) \equiv \Psi_P(L, \lambda, E) \quad \text{and} \quad \Psi'_P(E, \boldsymbol{r}) \equiv \Psi'_P(L, \lambda, E) \tag{10}$$

Radiation data models like AP-8 or AP-9 can be used to find the values of the parameters defining the function a and b that best fits the radiation data. As already indicated, in this Note, we will use the AP-8 data to prove the value of our approach, but this methodology can be extended to the case of AP-9 data too. We can write these functions as [18]

$$a(L, E) = a_0 e^{a_1 E + a_2 (a_3 + L)^2} \tag{11}$$

and

$$b(L, E) = b_0 + b_1 E + b_2 L + b_3 EL + b_4 L^2 + b_5 L^3 \tag{12}$$

where the constants in the aforementioned relationships are given by: $a_0 = 2.094 \times 10^8\ \text{cm}^{-2} \cdot \text{s}$, $a_1 = -1.673\ \text{MeV}^{-2}$, $a_2 = -2.07$, $a_3 = -2.825$, $b_0 = -0.00971$, $b_1 = 0.0000982\,\text{MeV}^{-1}$,

$b_2 = 0.01484$, $b_3 = 0.0001561\ \mathrm{MeV}^{-1}$, $b_4 = -0.004581$, and $b_5 = 0.0004356$. Using Eqs. (11) and (12), it follows from Eq. (9) that

$$\Psi'_P(L, \lambda, E) = [a_1 - \lambda^2(b_1 + b_3 L)]\Psi_P(L, \lambda, E) \tag{13}$$

C. Displacement Damage Dose

The solar array of the maneuvering satellite degrades owing to the impacting particles encountered in the Van Allen belts. The amount of degradation is a complex function of the type of solar cells, amount and material of the shielding provided, the energies of radiation encountered along the path, and the number of particles for each encountered energy level. As given in [15], the power loss is defined by the quantity displacement damage dose that is defined as

$$D_d = \int_{E_\ell}^{E_u} \frac{\partial \Phi(E, \boldsymbol{r})}{\partial E} S_P(E)\, \mathrm{d}E \tag{14}$$

where $\Phi(E, \boldsymbol{r})$ is the differential particle fluence for the optimal trajectory, and $S_P(E)$ is the nonionizing energy loss. Note that [15] uses the total differential $d\Phi/dE$ to compute the displacement damage dose for a given spacecraft trajectory, where the trajectory does not change. But this Note deals with a trajectory optimization problem in which the trajectory is a variable to be determined. Hence, Φ in reality is a function of $\boldsymbol{r}$ and E, and the partial differential $\partial\Phi/\partial E$ is used to compute the displacement damage dose.

The fluence of protons of all energy levels greater than or equal to E encountered along the orbit-raising trajectory is given by

$$\Phi_P(E, \boldsymbol{r}) = \int_{t=0}^{t_f} \Psi_P(E, \boldsymbol{r})\, \mathrm{d}t \tag{15}$$

Note that the differential of the fluence is needed to compute the displacement damage dose. By definition,

$$\frac{\partial}{\partial E}\Phi_P(E, \boldsymbol{r}) = \lim_{\epsilon \to 0} \frac{1}{\epsilon}[\Phi_P(E + \epsilon, \boldsymbol{r}) - \Phi_P(E, \boldsymbol{r})] \tag{16}$$

where $\epsilon > 0$ represents a small change in the energy level E. Using Eq. (15),

$$\frac{\partial}{\partial E}\Phi_P(E, \boldsymbol{r}) = \lim_{\epsilon \to 0} \frac{1}{\epsilon}\left(\int_0^{t_f} \Psi_P(E + \epsilon, \boldsymbol{r})\, \mathrm{d}t - \int_0^{t_f} \Psi_P(E, \boldsymbol{r})\, \mathrm{d}t\right) \tag{17}$$

Taking the limit within the integral and then rearranging terms,

$$\frac{\partial}{\partial E}\Phi_P(E, \boldsymbol{r}) = \int_0^{t_f} \lim_{\epsilon \to 0} \frac{1}{\epsilon}[\Psi_P(E + \epsilon, \boldsymbol{r}) - \Psi_P(E, \boldsymbol{r})]\, \mathrm{d}t \tag{18}$$

which equals (by considering the definition of derivatives of the flux)

$$\frac{\partial}{\partial E}\Phi_P(E, \boldsymbol{r}) = \int_0^{t_f} \Psi'_P(E, \boldsymbol{r})\, \mathrm{d}t \tag{19}$$

representing $(\partial\Psi_P/\partial E)$ by Ψ'_P for the sake of brevity. Note that analytical expressions for the omnidirectional radiation flux $\Psi_P(E)$ and its partial derivative $\Psi'_P(E)$ are given by Eqs. (8) and (9). The displacement damage dose for the orbit-raising trajectory is then given by

$$D_d = \int_{E_\ell}^{E_u} \int_0^{t_f} \Psi'_P(E, \boldsymbol{r}) S_P(E)\, \mathrm{d}t\, \mathrm{d}E \tag{20}$$

D. Optimal Orbit-Raising Trajectory

The next step is to summarize the optimal control problem, the solution of which yields the electric orbit-raising trajectory corresponding to the minimum radiation damage:

$$\min J = D_d = \int_0^{t_f}\left[\int_{E_\ell}^{E_u} \Psi'_P(E, \boldsymbol{r}) S_P(E)\, \mathrm{d}E\right] \mathrm{d}t \tag{21}$$

The constraints of the optimal control problem are given by the equations of the motion for the satellite as depicted in Eq. (1) and the boundary conditions

$$r(0) = r_0,\ u(0) = u_0,\ v(0) = v_0,\ \phi(0) = \phi_0,\ \theta(0) = 0,\ m(0) = m_0 \tag{22}$$

at the initial injection orbit and the

$$r(t_f) = r_f, \qquad u(t_f) = 0, \qquad v(t_f) = v_f, \qquad \phi(t_f) = \frac{\pi}{2} \tag{23}$$

at the final time.

III. Solution Methodology

This section discusses the methodology that is followed for solving the optimal control problem. A direct optimization approach is followed and is known to be more robust numerically in terms of initial guesses and solution convergence [22]. The developed solver in [12] is extended by discretizing the energy spectrum of protons encountered during the transfer to compute the displacement damage dose associated with the transfer.

A. Direct Transcription and Collocation

As already mentioned, a direct optimization scheme is used to solve the optimization problem for each scenario. The scheme converts the trajectory optimization problem to a parameter optimization problem using direct transcription and collocation and then uses a nonlinear programming (NLP) problem solver (IPOPT [23], LOQO [24]) to determine the solution. The time variable is discretized using a nondimensional timelike variable τ:

$$0 = \tau_1 < \tau_2 < \ldots < \tau_n = 1 \tag{24}$$

and final time t_f is also a parameter in the problem, that is,

$$t_k = t_f \tau_k, \qquad k \in \{1, 2, \ldots, n\} \tag{25}$$

The state $\boldsymbol{x}$ and control $\boldsymbol{u}$ variables of a continuous trajectory are also discretized based on the selected time grid:

$$\boldsymbol{x}_k = \boldsymbol{x}(t_k), \qquad \boldsymbol{u}_k = \boldsymbol{u}(t_k) \tag{26}$$

Using a trapezoidal discretization scheme, one can approximate the dynamic constraints as follows:

$$\boldsymbol{\zeta}_k = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k - \frac{\Delta t_k}{2}[\boldsymbol{f}(\boldsymbol{x}_{k+1}, \boldsymbol{u}_{k+1}) + \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k)] = 0 \tag{27}$$

Note that one has a set of defects corresponding to each of the equations of motion given in Eq. (1), that is, $\boldsymbol{\zeta}_k \equiv (\boldsymbol{\zeta}^r, \boldsymbol{\zeta}^\theta, \boldsymbol{\zeta}^\phi, \boldsymbol{\zeta}^u, \boldsymbol{\zeta}^v, \boldsymbol{\zeta}^w)$, where the superscript indicates the equation of motion that the defect corresponds to. If these defects are driven to zero, then the dynamic constraints (equations of motion) will hold approximately at each of the segments created by the discretization process. The set $(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n; \boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n; t_f)$ represents the decision variables of the resulting parameter optimization problem. During the optimization process (using an NLP solver), the defects are driven to zero. Using the discretization scheme, one can also write down the objective function (radiation fluence) using the values of L and λ evaluated at discretized nodes using Eqs. (5) and (7).

B. Discretized Energy Levels

To compute the objective function representing the radiation damage caused to the solar arrays of a satellite, consider a discrete set of energy levels $\mathcal{E} = \{E_1, E_2, \ldots, E_m\}$, such that $E_1 = E_\ell$ and

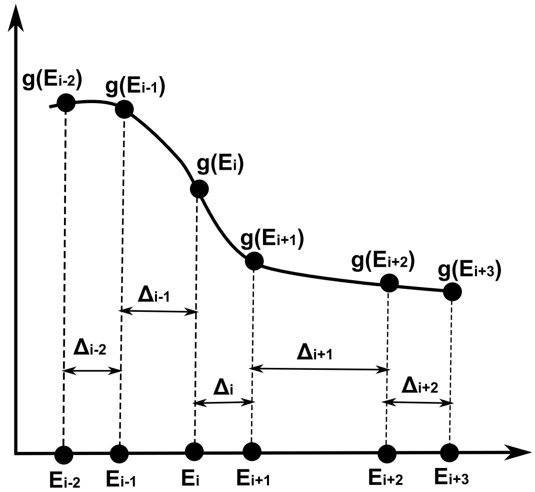

Fig. 3 Function of *E* evaluated for a set of discretized energy levels.

$E_m = E_u$, with E_ℓ and E_u being respectively the lower and the upper bounds of the set of *m* energy levels under consideration (see Fig. 3). Based on this discretization,

$$\int_{E_\ell}^{E_u} g(E, \boldsymbol{r}(t))\,\mathrm{d}E \approx \sum_{i=1}^{m-1} \frac{1}{2}[g(E_i, \boldsymbol{r}) + g(E_{i+1}, \boldsymbol{r})]\Delta E_i \tag{28}$$

Using this relation, the displacement damage dose can be written as

$$D_d \approx \frac{1}{2}\int_0^{t_f} \sum_{i=1}^{m-1} (E_{i+1} - E_i)[\Psi_P'(E_i, \boldsymbol{r}) S_P(E_i) + \Psi_P'(E_{i+1}, \boldsymbol{r}) S_P(E_{i+1})]\,\mathrm{d}t \tag{29}$$

Finally, considering the discretization of the trajectory states,

$$D_d \approx \frac{1}{4}\sum_{k=1}^{n}\sum_{i=1}^{m-1}(t_{k+1} - t_k)(E_{i+1} - E_i) \cdot [\Psi_P'(E_i, \boldsymbol{r}_k) S_P(E_i) + \Psi_P'(E_i, \boldsymbol{r}_{k+1}) S_P(E_i) + \Psi_P'(E_{i+1}, \boldsymbol{r}_k) S_P(E_{i+1}) + \Psi_P'(E_{i+1}, \boldsymbol{r}_{k+1}) S_P(E_{i+1})]$$

Based on the discretization of the trajectory and the energy levels, one can write the radiation flux in Eq. (8) as follows:

$$\Psi_P(L_k, \lambda_k, E_i) = a(L_k, E + i)e^{-b(L_k, E_i)\lambda_k^2} \tag{30}$$

Similarly, using Eq. (21), the rate of change of the radiation flux with energy level can be expressed as

$$\Psi_P'(L_k, \lambda_k, E_i) = [a_1 - \lambda_k^2(b_1 + b_3 L_k)]\Psi_P(L_k, \lambda_k, E_i) \tag{31}$$

C. Nonlinear Programming Problem Solution

Equations (25–31) represent the nonlinear programming problem that needs to be solved to yield the optimal trajectory to the GEO. If *n* nodes are selected to represent the trajectory, and *m* energy levels are similarly selected to represent the energy spectrum, then there are $12n$ NLP variables corresponding to the discretization of variables $r, \theta, \phi, u, v, w, m, T, \alpha, \beta, L, \lambda, m$ NLP variables corresponding to the discretization of variable *E* and 1 NLP variable corresponding to the final time. Hence, there are altogether $12n + m + 1$ NLP variables for the entire optimal control problem. Apart from the boundary conditions in the problem, there are $7(n-1)$ constraints corresponding to the equations of motion, $2n$ constraints that relate *L* and λ to the spherical coordinates, and $n^2 m$ constraints that define the radiation fluence corresponding to the analytical model relating the radiation fluxes to the variables λ and *E*.

The model of the optimization problem defined by the aforementioned variables and constraints is set up in A Mathematical Programming Language (AMPL) [25] and then the NLP solver LOQO [24] is used to solve the problem. LOQO is an interior-point-based method and is suitable for performing large-scale nonlinear optimization. The minimum-time solution is provided as the initial guess to the NLP solver. The minimum-time solutions are derived by solving a much simpler NLP as depicted in [12]. The minimum-time optimization solver needs an initial guess, and the following initial guesses are used to yield converged solution: radial position *r* is considered to be a cubic polynomial of the timelike parameter τ, transverse velocity $v = \sqrt{\mu/r}$, thrust is considered to have a component only along the $\boldsymbol{e}_\theta$ direction, thrust magnitude is set to the maximum value, and final time $t_f = \mathcal{K}(1 - \sqrt{r(0)})m(0)/T$ when the transfer starts from the equatorial orbit, where $\mathcal{K}$ is a user-defined constant. Furthermore, nondimensionalized quantities are used for the numerical problems, by considering one distance unit (DU) to be the radius of the GEO and the gravitational parameter μ to be equal to 1 $\mathrm{DU}^3/\mathrm{TU}^2$. This choice of the nondimensionalized quantities implies that the time period of a satellite in the GEO is 2π time unit (TU).

IV. Numerical Examples

This section presents solutions obtained using our methodology for low-thrust orbit raising from an initial circular injection orbit to the GEO. The obtained solutions are compared with the minimum-time solutions for transfer starting from both equatorial and nonequatorial orbits. The minimum-time solutions are derived using the tool developed in [12]. The proton non-ionizing energy loss (NIEL) values S_P for different energy levels used in the simulations correspond to Gallium Arsenide (GaAs) solar cells and are determined from the Table 1 of NIEL values provided in [16].

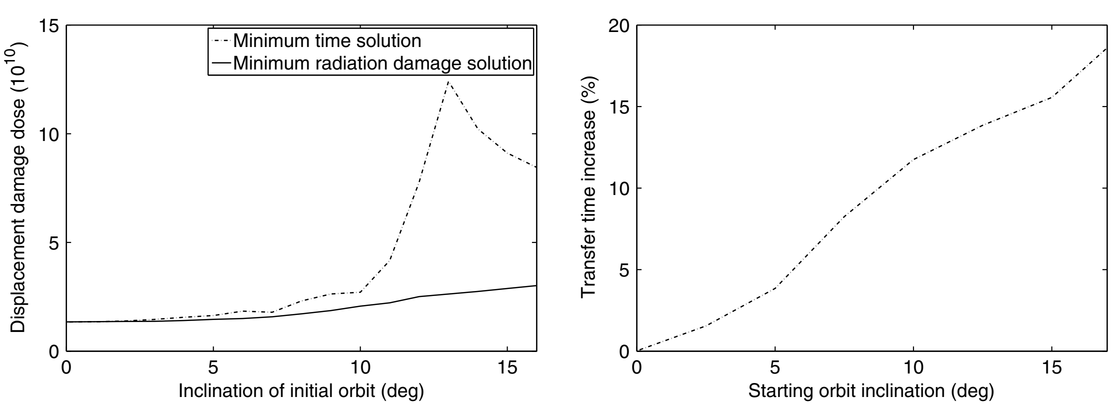

Fig. 4 Minimum-time and minimum-radiation solutions for varying inclinations: a) comparison of displacement damage dose, and b) percentage increase in transfer time for minimum-radiation solutions.

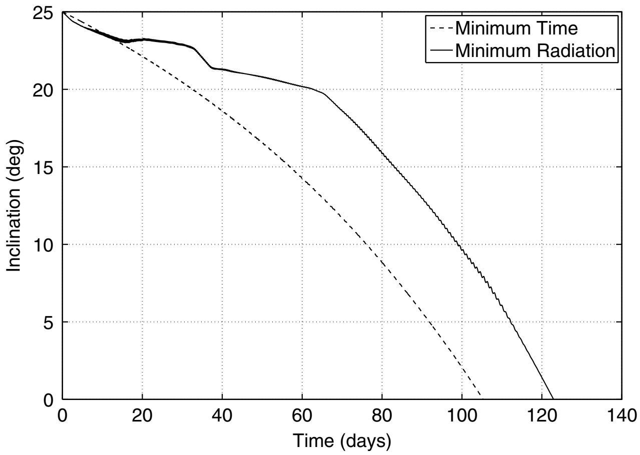

Fig. 5 Variation of osculating inclination angle with time.

A. Minimum-Radiation Solution from Equatorial Circular Orbits

The orbit-raising problem for transferring a satellite of mass 4500 kg from an equatorial circular orbit of altitude 10,000 km to the GEO is considered. It is assumed that the satellite is equipped with four BPT-4000 thrusters, each of which has a thrust of 290 mN and a specific impulse of 1788 s. The power level is held constant during the transfer to operate the four thrusters at maximum thrust level. The minimum-time solution for this problem yields a displacement damage dose of 1.2×10^{11} MeV/g after 79.18 days. This solution is used as an initial guess for the minimum-radiation-damage problem, in which the displacement damage dose due to protons of energies greater than 2 MeV are considered. This problem converges to a solution that is essentially the minimum-time solution. The transfer time approximately equals that of the minimum-time problem, and the dose is almost the same. The minute difference in the transfer time of 0.02% is only a numerical artifact of the problem, and the solution does not show any plane changes. Varying the type of thruster and the starting altitude of the orbit yields a similar behavior for the minimum-radiation-damage problem. Note here that the displacement damage dose is a function of both transfer time and the radiation flux. The satellite can potentially reduce the radiation flux by making plane changes (moving away from the equatorial plane) during the transfer, but such a change would also increase the transfer time because the satellite has to come back to an equatorial destination orbit in the end. In other words, the displacement damage dose incorporates two conflicting objectives with the transfer time dominating when the maneuver starts from an equatorial orbit. Hence, it is concluded that the solutions of the minimum-radiation problem and the minimum-time problem are essentially the same if the transfer starts from an equatorial circular injection orbit.

B. Minimum-Radiation Orbit Raising from Starting Inclined Orbits

We now consider that the starting orbit for the orbit-raising maneuver is inclined. We increase the value of inclination of the starting orbit in increments of 1 deg and solve both the minimum-time and minimum-radiation problem for each case. For all these cases, the altitude remains at 10,000 km at the beginning of transfer, and the power available during the transfer is considered to be enough to operate four BPT-4000 thrusters. The energy spectrum used to compute the displacement damage has a lower bound of $E_\ell = 3$ MeV. The starting mass of the satellite is 4500 kg. It is found that the difference between the minimum-time solution and the minimum-radiation damage solution becomes more prominent with increasing inclination of the initial injection orbit. This comparison is depicted in Fig. 4a. Another important observation is that the maximum displacement damage dose for the minimum-time solutions occurs at around 13 deg rather than the equator, owing to the fact that the Earth's magnetic axis is tilted with respect to its rotational axis. However, the angle at which the displacement damage dose becomes maximum is a bit off from the actual tilt angle (around 10 deg), likely due to our use of an approximate analytical model for the radiation flux of protons in the Van Allen belt. Figure 4b shows the increment of transfer time for minimum-radiation transfers, when compared to the minimum-time transfers.

To understand why the minimum-time trajectories differ from the minimum-radiation trajectories, we consider the specific example of orbit raising starting from a circular orbit of 10,000 km altitude with 25 deg inclination. We assume that the spacecraft employs five BPT-4000 thrusters, each providing a maximum thrust of 290 mN at a specific impulse of 1788 s. When the minimum-time problem is solved, the resulting transfer time is 105.021 days, and the corresponding displacement damage dose is 7.36×10^{10} MeV/g. Solving the minimum-radiation problem yields a reduced displacement damage dose of 3.86×10^{10} MeV/g, at the cost of an increased transfer time of 122.60 days. To understand the difference between the two solutions, we plot the osculating inclination angle during the transfers with respect to time in Fig. 5. As seen from the figure, the spacecraft avoids changing plane as much as possible during the initial phase of the transfer to avoid the regions where charged protons are more abundant. The majority of the inclination change happens at comparatively higher altitudes so as to reduce the impact of radiation.

If we start the transfer of a 4500 kg satellite from a higher altitude, say 24,000 km, the amount of total dose decreases mostly because the protons become less abundant in these regions. Even for transfers starting at these altitudes, the minimum-radiation solution is significantly different from the minimum-time solution. We consider a starting inclination of 15 deg and solve both the minimum-time problem and the minimum-radiation-damage problem for this case. Also, we use the following propulsion system parameters for the transfer: four BPT-4000 thrusters providing a maximum thrust of 290 mN each at a specific impulse of $I_{sp} = 1788$ s. For the minimum-time case, the satellite starts performing plane changes from the very beginning of the transfer; the change in inclination is approximately linear in this case. Figure 6a depicts the change in the polar angle for this transfer. For the case of minimum-radiation transfer, the satellite stays away from the more radiation-dense areas by delaying the plane change until it has reached a sufficient altitude because the radiation flux also decreases with increasing altitude from the Earth. The difference in the optimal trajectories are illustrated in Fig. 6, which depicts the variation in the polar angle ϕ for the minimum-time solution and the minimum-radiation-damage solution. Note here that

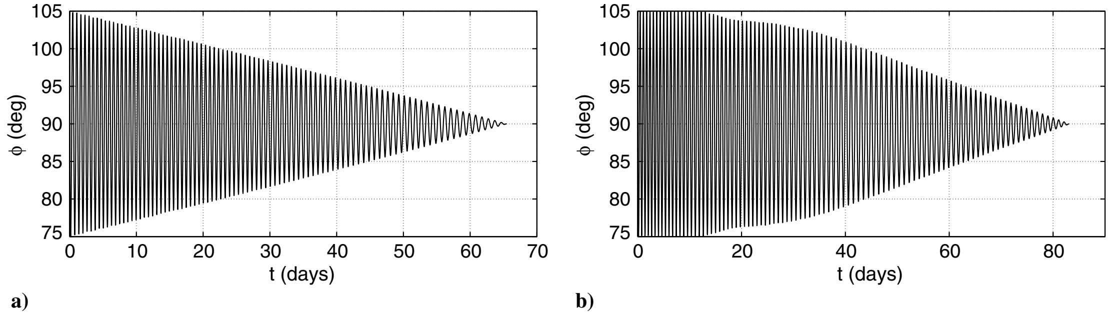

Fig. 6 Variation of the polar angle with time during the transfer for a) minimum-time solution, and b) minimum-radiation-damage solution.

there is no benefit for the satellite to do unnecessary plane changes to move to regions of even higher latitudes with less severe radiation flux, owing to the effect of the increased transfer time on the displacement damage dose. Hence, the satellite only makes plane changes that are necessary for the transfer and only delays it sufficiently to reduce the radiation damage. In this case, the displacement damage dose decreases by almost an order of magnitude (88%) at the cost of increasing the transfer time by 22%. The total dose for the minimum-time solution (65.27 days) is 1.242×10^9 MeV/g, and that for the minimum-radiation solution (83.08 days) is 1.407×10^8 MeV/g.

V. Conclusions

This Note develops a formulation to determine orbit-raising trajectories that minimize the radiation damage dose incurred by a satellite during electric orbit raising to the geosynchronous equatorial orbit. The proposed formulation considers the discretization of the spacecraft trajectory as well as the energy levels for the protons in the Van Allen belts, and the resulting nonlinear programming problem is solved using the solver LOQO. Furthermore, analytical models of the radiation flux of protons in the Van Allen belts, based on AP8MIN models, is used to facilitate the optimization process. The slowing down of the proton spectrum, that is the effect of shielding, is ignored in the computation of the displacement damage dose. The minimum-time solutions are provided as initial guesses to our developed solver. Orbit-raising examples to GEO starting from circular injection orbits are considered. When the satellite starts from an equatorial orbit, it is found that minimizing displacement damage dose is the same as minimizing time. However, when the satellite starts from an inclined circular orbit, the minimum-radiation-damage solution differs from the minimum-time solution. At the beginning of the transfer, the satellite stays in the lower regimes of radiation flux and thereby restricts doing as much plane change as a minimum-time transfer. The proposed methodology can be extended to include newer and accurate AP-9 models of the Van Allen belt as well as the effect of shielding thickness to compute the solar array power degradation accurately and to enable the computation of trajectories that minimize the power degradation during the transfer.

Acknowledgments

This work was performed under grant number AGMT dated 1 April 2011 from SES Engineering. We also thank Stan Russo of SES for his help and suggestions.

References

[1] Smits, F., "The Degradation of Solar Cells Under Van Allen Radiation," *IEEE Transactions on Nuclear Science*, Vol. 10, No. 1, 1963, pp. 88–96. doi:10.1109/TNS.1963.4323248

[2] Flanagan, P. F., Horsewood, J. L., and Pines, S., "Precision Orbit Raising Trajectories," *11th Electric Propulsion Conference*, AIAA Paper 1975-0354, March 1975.

[3] Dailey, C., and Lovberg, R., "Shuttle to GEO Propulsion Trade Offs," *18th Joint Propulsion Conference*, AIAA Paper 1982-1245, June 1982.

[4] Redding, D., "Avoiding the Van Allen Belt in Low-Thrust Transfer To Geosynchronous Orbit," *21st Aerospace Sciences Meeting*, AIAA Paper 1983-0195, Jan. 1983.

[5] Ennix, K. A., Dickey, M. R., Klucz, R. S., and Matuszak, L. W., "Electric Vehicle Analyzer," *21st International Electric Propulsion Conference*, AIAA Paper 1990-2573, July 1990.

[6] Free, B., "High Altitude Orbit Raising with On-Board Electric Power," *Proceedings of the International Electric Propulsion Conference*, IEPC Paper 1993-205, Electric Rocket Propulsion Soc., Fairview Park, OH, Sept. 1993, pp. 1861–1877.

[7] Fitzgerald, A., "The Effect of Solar Array Degradation in Orbit-Raising with Electric Propulsion," *Proceedings of the International Electric Propulsion Conference*, IEPC Paper 1993-207, Electric Rocket Propulsion Soc., Fairview Park, OH, Sept. 1993, pp. 1889–1895.

[8] Biagioni, L., "Evolutionary Optimization of Launch Vehicle Electric Propulsion Integration for GEO Missions," *36th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*, AIAA Paper 2000-3418, July 2000.

[9] Oleson, S. R., and Myers, R. M., "Launch Vehicle and Power Level Impacts on Electric GEO Insertion," *32nd Joint Propulsion Conference and Exhibit*, AIAA Paper 1996-2978, July 1996.

[10] Byers, D. C., and Dankanich, J. W., "Geosynchronous-Earth-Orbit Communication Satellite Deliveries with Integrated Electric Propulsion," *Journal of Propulsion and Power*, Vol. 24, No. 6, Nov.–Dec. 2008, pp. 1369–1375. doi:10.2514/1.35322

[11] Duchemin, O. B., Caratge, A., Cornu, N., Sannino, J.-M., Lassoudière, F., and Lorand, A., "Ariane 5-ME and Electric Propulsion: GEO Insertion Options," *47th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*, AIAA Paper 2011-6084, July–Aug. 2011.

[12] Dutta, A., Libraro, P., Kasdin, N. J., and Choueiri, E., "A Direct Optimization Based Tool to Determine Orbit-Raising Trajectories to GEO for All-Electric Telecommunication Satellites," *AIAA/AAS Astrodynamics Specialist Conference*, AIAA Paper 2012-4589, Aug. 2012.

[13] Kluever, C. A., "Optimal Geostationary Orbit Transfers Using Onboard Chemical-Electric Propulsion," *Journal of Spacecraft and Rockets*, Vol. 49, No. 6, 2012, pp. 1174–1182. doi:10.2514/1.A32213

[14] Dutta, A., Libraro, P., Kasdin, N. J., and Choueiri, E., "Minimizing Total Radiation Fluence During Time-Constrained Electric Orbit-Raising," *Proceedings of the International Symposium on Space Flight Dynamics*, Pasadena, CA, Nov. 2012.

[15] Messenger, S. R., Xapsos, M. A., Burke, E. A., Walters, R. J., and Summers, G. P., "Proton Displacement Damage and Ionizing Dose for Shielded Devices in Space," *IEEE Transactions on Nuclear Science*, Vol. 44, No. 6, Dec. 1997, pp. 2169–2173. doi:10.1109/23.659032

[16] Jun, I., Xapsos, M., Messenger, S. R., Burke, E. A., Walters, R. J., Summers, G. P., and Jordan, T., "Proton Nonionizing Energy Loss (NIEL) for Device Applications," *IEEE Transactions on Nuclear Science*, Vol. 50, No. 6, Dec. 2003, pp. 1891–1895. doi:10.1109/TNS.2003.821380

[17] Messenger, S. R., Jackson, E. M., Warner, J. H., and Walters, R. J., "SCREAM: A New Code for Solar Cell Degradation Prediction Using the Displacement Damage Dose Approach," *Proceedings of the 35th IEEE Photovoltaics Conference*, IEEE Publ., Piscataway, NJ, June 2012, pp. 1106–1111. doi:10.1109/PVSC.2010.5614713.

[18] Guo, C., "Modeling Earth's Radiation Belts to Find Minimal-Radiation Satellite Trajectories," Junior Paper, Physics Dept., Princeton Univ., Princeton, NJ.

[19] Pisacane, V. L., *The Space Environment and Its Effects on Space Systems*, Education Series, AIAA, Reston, VA, 2008, pp. 77–89.

[20] McIlwain, C. E., "Coordinates for Mapping the Distribution of Magnetically Charged Particles," *Journal of Geophysical Research*, Vol. 66, No. 11, Nov. 1961, pp. 3681–3691. doi:10.1029/JZ066i011p03681

[21] Jordan, C. E., "NASA Radiation Belt Models AP-8 and AE-8," Radex Rept. 1 GL-TR-89-0267, Bedford, MA, Sept. 1989.

[22] Conway, B. A. (ed.), *Spacecraft Trajectory Optimization: Conway, Techniques and Technology*, Cambridge Univ. Press, Cambridge, MA, 2010.

[23] Wachter, A., and Biegler, L. T., "On the Implementation of an Interior-Point Filter Line-Search Algorithm for Large-Scale Nonlinear Programming," *Mathematical Programming*, Vol. 106, No. 1, 2006, pp. 25–57. doi:10.1007/s10107-004-0559-y

[24] Vanderbei, R. J., "LOQO: An Interior Point Code for Quadratic Programming," *Optimization Methods and Software*, Vol. 11, Nos. 1–4, 1999, pp. 451–484.

[25] Fourer, R., Gay, D., and Kernighan, B., *AMPL: A Modeling Language for Mathematical Programming*, Brooks/Cole Publ., 2003.